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Structural damage identification with output-only measurements using modified Jaya algorithm and Tikhonov regularization method

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Abstract. The absence of excitation measurements may pose a big challenge in the application of structural damage identification owing to the fact that substantial effort is needed to reconstruct or identify unknown input force. To address this issue, in this paper, an iterative strategy, a synergy of Tikhonov regularization method for force identification and modified Jaya algorithm (M-Jaya) for stiffness parameter identification, is developed for damage identification with partial output-only responses. On the one hand, the probabilistic clustering learning technique and nonlinear updating equation are introduced to improve the performance of standard Jaya algorithm. On the other hand, to deal with the difficulty of selection the appropriate regularization parameters in traditional Tikhonov regularization, an improved L-curve method based on B-spline interpolation function is presented. The applicability and effectiveness of the iterative strategy for simultaneous identification of structural damages and unknown input excitation is validated by numerical simulation on a 21-bar truss structure subjected to ambient excitation under noise free and contaminated measurements cases, as well as a series of experimental tests on a five-floor steel frame structure excited by sinusoidal force. The results from these numerical and experimental studies demonstrate that the proposed identification strategy can accurately and effectively identify damage locations and extents without the requirement of force measurements. The proposed M-Jaya algorithm provides more satisfactory performance than genetic algorithm, Gaussian bare-bones artificial bee colony and Jaya algorithm.

Keywords: B-spline interpolation function; damage identification; force identification; Jaya algorithm; k-means clustering; Tikhonov regularization

1. Introduction

During the long-term service period, continuous health monitoring and early damage detection for civil structures are necessary to assess their performance, arrange maintenance and predict future service life etc. Sudden failure of structures, especially for large-scale and major infrastructure, would result in great casualties and property loss. Therefore, in the last decades, considerable structural damage identification methods have been proposed and investigated in the frequency domain or the time domain to evaluate the health status of a structure (Doebling *et al*. 1998).

Frequency domain methods can identify structural damages based on the change of modal information, such as natural frequencies (Wang *et al*. 2001), mode shapes (Dinh-Cong *et al*. 2021), frequency response functions (Esfandiari *et al*. 2020), modal strain energy (Daneshvar *et al*. 2022), modal flexibility (Yan and Ren 2014) without excitation measurements. However, modal characteristics may be insufficient to determine the existence, locations, and severities of structural damages. Frequencies are more likely affected by temperature variation than damages (Farrar and Doebling 1997). Besides, from a practical point of view, lower modes are insufficiently sensitive to minor or local damages. Higher modes are beneficial to improve identification accuracy but difficult to be accurately acquired because of their susceptibility to noise (Perry and Koh 2008). For time domain methods, like the least squares estimation (Caravani *et al*. 1977), the Kalman filter (Gao and Lu 2006), the particle filter (Xue *et al*. 2009), the dynamic response sensitivity-based model updating method (Li *et al*. 2015), etc., have been proposed and validated in many examples, while they generally require a good initial estimation of structural parameters and gradient information.

Structural damage identification in the time domain can be formulated as an optimization process in which the objective function is defined as the discrepancy between the measured responses from the real structure and the simulated values from the finite element model. The inverse identification could be addressed by minimizing the objective function using heuristic algorithms. A variety of state-of-the-art heuristic algorithms, such as particle swarm optimization (PSO) (Das and Dhang 2020), differential evolution (DE) (Seyedpoor *et al*. 2018), artificial bee

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colony algorithm (ABC) (Sun *et al*. 2013), butterfly optimization algorithm (BOA) (Zhou *et al*. 2021), pigeon colony algorithm (Yi *et al*. 2016), monkey algorithm (Yi *et al*. 2015), tree seeds algorithm (Ding *et al*. 2019), charged system search algorithm (Kaveh and Zolghadr 2015, Kaveh and Maniat 2015), simplified dolphin echolocation algorithm (Kaveh *et al*. 2016), cyclical parthenogenesis algorithm (Kaveh and Zolghadr 2017), enhanced heat transfer optimization algorithm (Kaveh and Dadras 2018), shuffled shepherd optimization algorithm (Kaveh *et al*. 2021a), have been widely developed, and fruitful research results on damage identification have been achieved. For example, Silva *et al*. (2016) constructed a novel unsupervised and nonparametric genetic algorithm to efficiently identify damages of Z-24 bridge and the Tamar bridge. Feng *et al*. (2021) used a kNN algorithm to detect stiffness loss of bridges. Nevertheless, most of these approaches require input excitation to predict structural responses. In many practical situations, input forces applied to structure, for example, wind load, seismic load and traffic load, are difficult or impossible to be directly measured, which limits the real-world applications of time domain methods to some extent.

To deal with the absence of force measurements, some studies have been attempted by approximating the input excitation as a stationary Gaussian white noise. For instance, Wang *et al*. (2020) and Zhang *et al*. (2022a) proposed an output-only identification method based on acceleration correlation functions and heuristic algorithms assuming the excitation in terms of white noise. Lei *et al*. (2018) utilized cross-correlation functions of acceleration responses and the extended Kalman filter for the structural damage identification on the ASCE benchmark building subject to ambient excitation. However, it may be not valid for the assumption of white noise random process in some cases. Different from these approaches, various force identification techniques have been developed, treating the unmeasured input force as unknowns to be identified. The inverse problem of force identification is typically ill-conditioned, and it can be solved by Tikhonov regularization method (Tang *et al*. 2022). The location and magnitude of impact force are identified by deconvolution and Tikhonov regularization (Kalhori *et al*. 2018). A novel fractional Tikhonov regularization method based on the improved super-memory gradient was developed to properly address force identification problems (Wang *et al*. 2018). Genetic algorithm (GA) and Latin hypercube sampling were combined with an improved L-curve method to reconstruct distributed loads applied to uncertain structures (Zhao *et al*. 2021). However, most of these investigations assume that the parameters of structural systems are known a priori, which might be unreasonable considering the material degradation, fatigue and deterioration effects.

In recent years, many researchers have attempted to simultaneously identify the unknown structural parameters and unmeasured external input force (Feng *et al*. 2015, Liu *et al*. 2016, Ni *et al*. 2022). In their methods, the measurement of external force is not required. For example, Xu *et al*. (2012) developed a weighted adaptive iterative least-squares estimation technique and its performance was validated by experimental tests on a four-story frame structure. Lu *et al*. (2011) presented acceleration response sensitivity-based finite element model updating method to identify both the input excitation and local damages. Sun *et al*. (2015) adopted an output-only method to iteratively identify structural parameter and input force using the damped Gauss-Newton method and Bayesian inference-based regularization. Jayalakshmi and Rao (2017) modified Tikhonov regularization method by combining Tikhonov regularization with truncated singular value decomposition for force identification. Subsequently, Jayalakshmi *et al*. (2018) compared two different time-domain algorithms to reconstruct the input excitation force acting on the structure and found that the inverse force identification algorithms based on the modified regularization technique can perform better performance than direct method. In this method, L-curve method was employed to select optimal regularization parameter. However, there are some limitations for classical L-curve method. The selection of regularization parameter has randomness in the first step, and a proper search range of optimal regularization parameter is needed. To deal with these issues, in this study, B-spline function is utilized to interpolate L-curve. The optimal regularization parameter is determined by the curvature values of B-spline curve.

Computational intelligence techniques have been extensively developed over the past two decades. Apart from abovementioned heuristic algorithms, PSO, DE, ABC, BOA, etc., a novel swarm intelligence algorithm named Jaya algorithm, inspired by the concept of moving toward the optimal solution and away from the worst solution, was proposed by Rao in 2016 for solving the constrained or unconstrained optimization problems (Rao 2016). Jaya algorithm has the advantage of simple structure, high stability and easy operation owing to it does need any algorithm-specific parameter. Therefore, Jaya algorithm has been successfully utilized in diverse optimization problems (Zitar *et al*. 2021), such as cost optimization of building (Aslay and Dede 2022), sizing optimization of skeletal structures (Kaveh *et al*. 2021b), parameters identification of airfoil systems (Ding *et al*. 2022), damage identification for the Guangzhou new TV tower (Ding *et al*. 2020), nonlinear system identification (Zhang *et al*. 2022a). The effectiveness of the Jaya algorithm over other existing heuristic algorithm was reported in Ref (Yu *et al*. 2017), while as an emerging population-based stochastic optimization algorithm, Jaya algorithm also suffers from premature convergence and is easy to be trapped into the local optimum. To improve the performance of exploration and exploitation, modified Jaya algorithm (M-Jaya) is proposed by integrating probabilistic clustering learning technique and nonlinear updating equation into the standard Jaya algorithm. Probabilistic clustering learning technique is introduced to effectively utilize current population information and accelerate the algorithm's convergence speed. Nonlinear updating equation is implemented to further improve the exploration capacity by randomly searching around the best solution.

In the present paper, the main contribution is that an output-only strategy is proposed to simultaneously identify unknown stiffness parameters and excitation force based on the modified Jaya algorithm and Tikhonov regularization method. Different from the traditional Tikhonov regularization, an improved L-curve method based on B-spline interpolation function is developed to select the appropriate regularization parameters. In each iteration of the strategy, Tikhonov regularization method is employed to identify the unknown external input forces in state space while structural parameters are updated with the proposed M-Jaya. The stiffness parameters and excitation force are iteratively identified by using different sets of dynamic acceleration response until convergence condition is satisfied. It is assumed that the acting point of unknown force and structural mass distribution are known. The effectiveness and applicability of the proposed strategy in identifying structural damages with partial output-only measurements is validated by not only numerical simulations on a truss structure but also experimental tests on a laboratory five-floor steel frame structure in the laboratory.

2. Force identification method

2.1 State-space representation

The equation of motion for a damped linear MDOF structural system can be written as follows

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = Lf(t) \tag{1}$$

where M, C, K stand for the mass, damping and stiffness matrices; $u(t)$, $\dot{u}(t)$, $\ddot{u}(t)$ mean the displacement, velocity and acceleration vectors under the external excitation $f(t)$, respectively; L denotes the input location matrix. Dynamic time history responses in Eq. (1) can be obtained by Newmark constant average acceleration method. Rayleigh damping model is employed, expressed as follows

$$C = aM + bK, \zeta_r = \frac{a}{2\omega_r} + \frac{b\omega_r}{2} \tag{2}$$

where a and b are two constant coefficients; ζ_r and ω_r represent the damping ratio and natural frequency corresponding to the r-th modes ($r = 1, 2$), respectively.

It is reasonable to assume that the system mass is known a priori since it can be acquired from material properties and geometries of structural members in the design drawings. The damaged stiffness matrix K_{dam} is given as

$$K_{dam} = \sum_{i=1}^{NE} (1 - \alpha_i)\, K_i^{\mathrm{ele}} \tag{3}$$

where K_i^{ele} indicates the i-th intact elemental stiffness matrix; NE denotes number of unknown stiffness parameters; α_i is stiffness reduction index for the i-th element. Stiffness parameter is $\theta_i = \{(1-\alpha_1), (1-\alpha_2), \ldots, (1-\alpha_{NE})\}$.

The representation of Eq. (1) in the state-space form can be described as

$$\dot{z}(t) = A_c z(t) + B_c f(t) \tag{4}$$

where state vector $z(t) = [u(t)\dot{u}(t)]^T$; Continuous system matrix A_c and input matrix B_c are given as follows

$$A_c = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ M^{-1}L \end{bmatrix} \tag{5}$$

where I means the identity matrix.

When partial acceleration responses are recorded, the output vector can be stated as $y(t) = R\ddot{u}(t)$. R is output influence matrix related to the location of sensors. Herein, the structural output $y(t)$ can be shown as

$$y(t) = C_c z(t) + D_c f(t) \tag{6}$$

where C_c is continuous output matrix $C_c = [-RM^{-1}K - RM^{-1}C]$; D_c is feedthrough matrix $D_c = RM^{-1}L$.

The discrete time form of Eq. (4) and Eq. (6) can be expressed as follows

$$z(l+1) = A_d z(l) + B_d f(l) \tag{7}$$

$$y(l) = C_d z(l) + D_d f(l) \tag{8}$$

where $z(l)$, $f(l)$, $y(l)$ stand for the discrete vectors at $t = l \times \Delta t (l = 0,1,2,\ldots,Z)$. Z and Δt denote the number of time step and the time interval, respectively.

Discrete system state space matrices A_d, B_d, C_d, D_d are stated as

$$\begin{aligned} A_d &= exp(A_c \Delta t), \quad & B_d &= {A_c}^{-1}(A_d - I)B_c, \\ C_d &= C_c, & D_d &= D_c \end{aligned} \tag{9}$$

Assuming the structural system is initially at rest state. Subsequently, substituting Eq. (7) into Eq. (8), the relation between output responses Y and input excitation F can be written as

$$Y = HF \tag{10}$$

where $Y = [y(0)y(1)y(2)\ldots y(Z-1)y(Z)]^T$; $F = [f(0)f(1)f(2)\ldots f(Z-1)f(Z)]^T$; the matrix H is calculated as (Sun *et al.* 2015)

$$H = \begin{bmatrix} D_d & 0 & 0 & \cdots & 0 \\ C_d B_d & D_d & 0 & \cdots & 0 \\ C_d A_d B_d & C_d B_d & D_d & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_d A_d^{N-1} B_d & C_d A_d^{N-2} B_d & \cdots & C_d B_d & D_d \end{bmatrix} \tag{11}$$

where H is lower block triangular Hankel matrix.

Force identification problem, as shown in Eq. (10), generally cannot be directly addressed owing to its ill-conditioned nature. Tikhonov regularization method is an effective method to solve the ill-posed inverse problem by minimizing the following equation

$$min J(x) = min(\|HF - Y\|_2^2 + \lambda \|I(F - F_0)\|_2^2) \tag{12}$$

where λ is regularization parameter.

If I is an identity matrix and F_0 is equal to zero, the Eq.

(12) can be simplified as

$$min J\,(x) = min(\|HF - Y\|_2^2 + \lambda\|F\|_2^2) \tag{13}$$

Then, the input excitation F can be identified by solving following equation (Xu *et al.* 2016)

$$F = (H^T H + \lambda I)^{-1} H^T Y \tag{14}$$

2.2 Improved L-curve method

To accurately identify unmeasured external force, it is essential to choose the proper regularization parameters λ, which is exactly the difficulty in using the Tikhonov regularization technique. In fact, the identified solution may be over-smoothened if setting a large regularization parameter, while it may lose stability if selecting a small regularization parameter. The L-curve method is a popular and effective method to determine the regularization parameter. At the corner of L-curve, the norm of regularized solution $\|F\|_2$ and the norm of the corresponding residual $\|HF - Y\|_2$ are well balanced. The specific curvature equation of L-curve can be given as

$$L\_C(\lambda) = \frac{|\rho'\eta'' - \rho''\eta'|}{((\rho')^2 + (\eta')^2)^{3/2}} \tag{15}$$

where $\eta = log(\|F\|_2)$, $\rho = log(\|HF - Y\|_2)$.

The optimal regularization parameter λ_{op} is determined through the following condition of maximum curvature in the L-curve method (Hansen and O'Leary 1993)

$$L\_C(\lambda_{op}) = \max_{\lambda>0} L\_C(\lambda) \tag{16}$$

However, there are some limitations of the classical L-curve method. First, the selection of regularization parameter has randomness in the first step, which may cause the most potential value to be omitted. Besides, an approximate search range of optimal regularization parameter λ_{op} should be provided in advance. Otherwise, the computational cost may be too high or the L-curve plot has difficulty in the selection of regularization parameter. To alleviate these issues, the improved L-curve method based on B-spline interpolation function is utilized, and its implementation procedures can be roughly divided into three steps. The L-curve is initially plotted according to the traditional L-curve method. Then, the region near the L-corner is selected for B-spline interpolation. Finally, the optimal regularization parameter is determined by the curvature of node points in this area.

The equation of curve $S(h)$ with B-spline interpolation can be expressed as follows

$$S(h) = \sum_{i=0}^{N_B-1} N_{i,p}(h) P_i\,, 0 \le h \le 1 \tag{17}$$

where $N_{i,p}(h)$ represents the p-degree B-spline basis function; h stands for the node; P_i means the i-th control point; N_B denotes the number of coordinates at the inflection point of L-curve.

The Cox-de Boor equation for the calculation of $N_{i,p}(h)$ can be described as (Yang and Xu 2019)

$$N_{i,0}(h) = \begin{cases} 1 if h_i \le h \le h_{i+1} \\ 0 otherwise \end{cases} \tag{18}$$

$$\begin{aligned} N_{i,p}(h) = &\frac{h - h_i}{h_{i+p} - h_i} N_{i,p-1}(h) \\ &+ \frac{h_{i+p+1} - h}{h_{i+p+1} - h_{i+1}} N_{i+1,p-1}(h) \end{aligned} \tag{19}$$

After interpolation operation, the curvature calculation of original L-curve is replaced by computing that of the B-spline curve. Obviously, less computing resources would be consumed. The first derivative of p-degree B-spline curve $S'(h)$ can be given as

$$\begin{aligned} S'(h) &= p \sum_{i=0}^{N_B-2} N_{i,p-1}(h) \frac{P_{i+1} - P_i}{h_{i+p+1} - h_{i+1}} \\ &= \sum_{i=0}^{N_B-2} N_{i,p-1}(h) Q_i \end{aligned} \tag{20}$$

where $Q_i = p \frac{P_{i+1} - P_i}{h_{i+p+1} - h_{i+1}}$.

It can be observed by comparing Eqs. (17) and (20) that the first derivative of p-degree B-spline curve is the (p-1)-degree B-spline curve. Accordingly, the second derivative of p-degree B-spline curve $S''(h)$ is equal to the first derivative of $S'(h)$

$$S''(h) = (p-1) \sum_{i=0}^{N_B-3} N_{i,p-2}(h) \frac{Q_{i+1} - Q_i}{h_{i+p} - h_{i+1}} \tag{21}$$

$S'(h)$ and $S''(h)$ are corresponding to coordinates (m', n') and (m'', n''), respectively. The curvature of L-curve can be calculated by $L\_C(\lambda) = \frac{|m'n'' - m''n'|}{((m')^2 + (n')^2)^{3/2}}$ (Zhao *et al.* 2021). The maximum curvature point of the curve is the optimal regularization parameter λ_{op}

$$L\_C(\lambda_{op}) = max\, L\_C(\lambda) \tag{22}$$

Substituting regularization parameter λ_{op} into Eq. (14), the unmeasured force F can be inversely identified.

3. Structural identification methods

3.1 Jaya algorithm

A novel population-based swarm intelligence algorithm was recently proposed by Rao in 2016, named Jaya algorithm, to solve the constrained or the unconstrained optimization problems. The main idea of Jaya algorithm is to get closer to success by moving toward the optimal solution and avoid failure by escaping from the worst solution. Different from other popular heuristic algorithms,

Algorithm 1 The pseudo-code of Jaya algorithm

Step 1. Initialization
  Define the population size *NP*, number of parameters *NE* and maximum number of iterations *MaxG*
  initialize population by Eq. (23)

Step 2. Individual updating
  While maximum number of iteration *MaxG* is not reached **do**
    Calculate fitness function and sort the population
    For individual *i* = 1 to *NP* **do**
      For variable *j*=1 to *NE* **do**
        Obtain two random numbers within [0, 1]
        Produce offspring by Eq. (24)
      End for
    End for
Step 3. Greedy selection
    Keep better solution with Eq. (25)
  End while

Step 4. Result output
  Output the optimal solution and value

Fig. 1 The pseudo-code of Jaya algorithm

algorithm-specific control parameters, such as crossover rate and mutation rate for DE, inertia weight and learning factors for PSO, sensor modality and power exponent for BOA, are not required for Jaya algorithm. There are four main steps, namely, initialization, individual updating, greedy selection and result output, as presented in Fig. 1.

The initial population of Jaya algorithm is randomly generated in the predefined lower and upper search space limits $[LB_{i,j}, UB_{i,j}]$ as follows

$$X_{i,j} = LB_{i,j} + rand(0,1) \times (UB_{i,j} - LB_{i,j}) \tag{23}$$

where $X_{i,j}$ denotes the *j*-th variable of the *i*-th candidate solution; $i \in (1,2,\ldots,NP), j \in (1,2,\ldots,NE)$, *NP* and *NE* represent the number of solutions and unknown parameters, respectively; $rand(0,1)$ stands for a random number generated within the range of [0, 1].

Then, evaluate the fitness function of all candidate solutions and determine the best solution X_{best} and the worst solution X_{worst} based on their fitness function values. The updating equation for a candidate X_i can be expressed as

$$\begin{aligned} X'_{i,j,G} = X_{i,j,G} + rand_1 \times (X_{best,j,G} - |X_{i,j,G}|) \\ - rand_2 \times (X_{worst,j,G} - |X_{i,j,G}|) \end{aligned} \tag{24}$$

where $X_{i,j,G}$ means the *j*-th variable of *i*-th solution at the *G*-th iteration; $X'_{i,j,G}$ and $|X_{i,j,G}|$ are the updated value and absolute value of $X_{i,j,G}$, respectively; $rand_1$ and $rand_2$ are two random numbers from the uniform distribution interval of [0, 1]. $rand_1 \times (X_{best,j,G} - |X_{i,j,G}|)$ and $rand_2 \times (X_{worst,j,G} - |X_{i,j,G}|)$ represent the tendency of $X_{i,j,G}$ towards the best solution and away from the worst solution.

In the step of greedy selection, the fitness function values of current solution $fit(X_{i,G})$ and new solution $fit(X'_{i,G})$ are compared. The candidate solution with better fitness value will survive to next iteration.

$$X_{i,G+1} = \begin{cases} X'_{i,G} if fit(X'_{i,G}) \geq fit(X_{i,G}) \\ X_{i,G} otherwise \end{cases} \tag{25}$$

Jaya algorithm would repeat step 2 and step 3 until the maximum number of iterations *MaxG* is reached. Finally, the optimal solution and value are output.

3.2 Modified Jaya algorithm

The performance of swarm intelligence algorithm depends on the balance between exploring the new regions of search space and exploiting those regions close to previously visited. It is reported that Jaya algorithm has the disadvantages of slow convergence speed and easy to be trapped into local optimal solution due to its relatively weak global search capacity. To deal with this issue, probabilistic clustering learning technique and nonlinear updating equation are introduced into Jaya algorithm.

3.2.1 Probabilistic clustering learning technique

K-means clustering technique is an effective method to utilize the population information and improve the convergence rate, which has been employed in Refs (Zhou *et al.* 2021, Ding *et al.* 2019). However, the classification results of K-means clustering may be different due to the random selection of initial clustering centers. A probabilistic clustering mechanism is proposed to automatically determine the appropriate clustering centers by making the distance between the different initial clustering centers as far as possible. The brief steps of the proposed probabilistic clustering mechanism are presented as follows

Step 1: randomly select a sample from the current colony $[x_1, x_2, \ldots, x_{np}]$ as initial clustering center c_1.

Step 2: calculate the shortest distance between each sample x_i and the existed clustering center c_j, i.e., the distance to the nearest clustering center, with following equation

$$Dis(x_i) = \|x_i - c_j\| = \sqrt{\sum_{\tau=1}^{dim} (x_{i,\tau} - c_{j,\tau})^2} \tag{26}$$

where *Dis* means the Euclidean distance. *dim* is the dimension of solution. Higher probability of being selected as the clustering center for sample x_i with large value of Euclidean distance. Then, calculate the probability value $PV(x_i)$ that each sample is selected as the next clustering center by following roulette wheel selection operation

$$PV(x_i) = \frac{Dis(x_i)^2}{\sum_{j=1}^{np} Dis(x_j)^2} \tag{27}$$

Calculate the cumulative probability of sample x_i

$$q_i = \sum_{j=1}^{np} PV(x_j) \tag{28}$$

where q_i represents the cumulative probability.

Step 3: repeat step 2 until k ($k = 0.1np$) clustering centers are selected.

Step 4: calculate the Euclidean distance between each individual and clustering center $c_j (j = 1,2,\ldots,g)$ and assign the remaining sample x_i to the cluster C_j if the condition $\|x_i - c_j\| \le \|x_i - c_g\|$ (c_g means any other clustering centers) is satisfied.

Step 5: update the new clustering centers $c_j^{'} (j = 1,2,\ldots,g)$

$$c_j^{'} = \frac{1}{n_i} \sum_{x_i \in C_j} x_i , j = 1,2,\cdots,g \tag{29}$$

where n_i stands for the number of samples belonging to the cluster C_j.

A new learning equation is proposed for an individual X_l^G

$$X_l^{G+1} = X_l^G + rand(0,1) \times \left(c_i^{'} - X_l^G\right) \tag{30}$$

where $c_i^{'}$ is the new clustering center and it denotes the mean population information.

Probabilistic clustering learning technique is proposed based on the above-mentioned probabilistic clustering mechanism and learning equation. It would be a promising technique to enhance the performance of Jaya algorithm.

3.2.2 Nonlinear updating equation

It can be observed from Eq. (24) that the new candidate solutions are generated related to the best solution X_{best}. In other words, the best solution would play a significant role during the searching process because it is capable of guiding and drawing other individuals to its own location. However, the identified best-so-far solution may be trapped into local optimal region under the adverse circumstance of solving complex multimodal optimization problems. Other individuals in the current population would be easily attracted to the region where the local best solution lies and result in premature convergence owing to falling into local optimum. To deal with this issue, a new nonlinear updating equation is introduced to refine the quality of the best solution as follows

$$X_{best,j,G}^{'} = X_{best,j,G} + \gamma_G \varphi_{i,j} \times \left(X_{best,j,G} - X_{q,j,G}\right) \tag{31}$$

where $X_{best,j,G}$ means the j-th variable of the best solution at the G-th iteration; $X_{best,j,G}^{\prime}$ is the offspring of $X_{best,j,G}$; $\varphi_{i,j}$ stands for a random number within the range of [-1, 1]; $X_{q,j,G}$ represents the j-th variable of a randomly selected q-th individual $q = 1,2,\ldots,np$; γ_G denotes a nonlinear factor, given as follows

$$\gamma_G = 1 - \left|\frac{G-\delta}{MaxG}\right|^v \tag{32}$$

where G and $MaxG$ indicate the current iteration and the maximum number of iterations; δ is an integer and v means the power exponent.

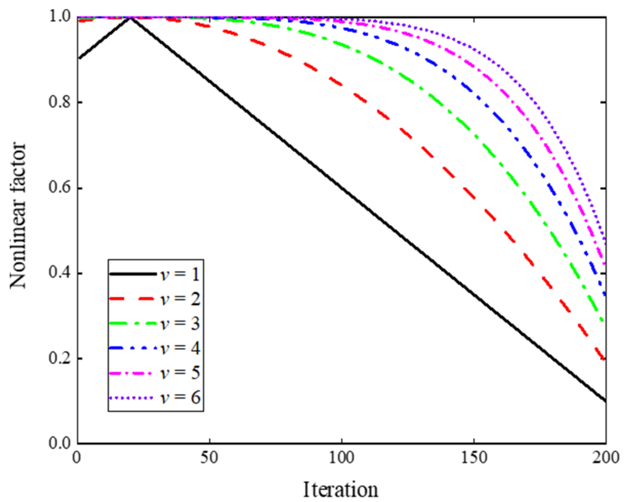

Fig. 2 The variation curves of nonlinear factor for different power exponents

The values of δ and v determine the variation curve of the nonlinear factor γ_G. If $MaxG$ = 200, δ = 20, the behavior of nonlinear factor is presented in Fig. 2 for different power exponents v. It can be noticed from Fig. 2 that the nonlinear factor γ_G has a large value in the initial stage, which is helpful to escape from the local optimal solution. Gradually, the nonlinear factor decreases its value as the iteration numbers increase with the result of converging to the final optimal solution.

3.2.3 Framework of modified Jaya algorithm

A modified Jaya algorithm (M-Jaya) is proposed by introducing two modifications, i.e., probabilistic clustering learning technique and nonlinear updating equation. Probabilistic clustering learning technique is implemented to effectively utilize the colony information and accelerate the convergence speed. Nonlinear updating equation is adopted to refine the quality of the best solution by randomly searching around it. The flowchart of modified Jaya algorithm is presented in Fig. 3. The proposed M-Jaya algorithm has simple structure and clear framework, so it is easy to operate.

3.3 Identification procedures

Most previous studies on force identification, structural parameters are assumed to be known a priori. Nevertheless, it may be difficult to directly determine structural stiffness parameters owing to damages possibly induced by earthquakes, aging or environmental corrosion, which would limit practical implementations of these approaches. In this section, an iterative approach is proposed to simultaneously identify the unmeasured input excitation using the force identification formulation described in Section 2 and the unknown structural stiffness using the proposed M-Jaya algorithm. The locations of external force and structural mass distribution are assumed to be known. The flowchart of the proposed identification strategy is illustrated in Fig. 4. The measured structural responses are grouped into measurement set 1 and set 2. As presented in Fig. 4, measurement set 1 is utilized to identify unmeasured

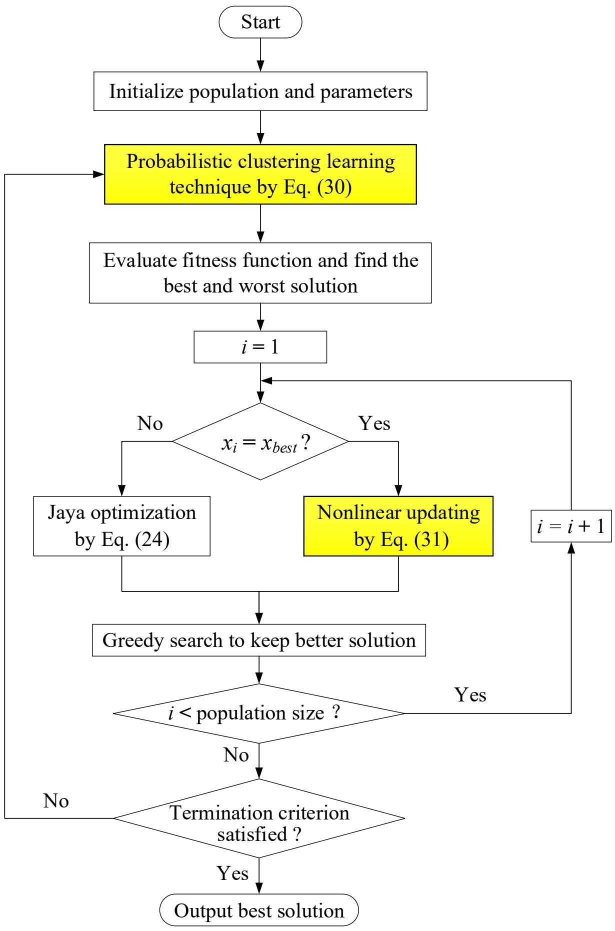

Fig. 3 The flowchart of proposed M-Jaya algorithm

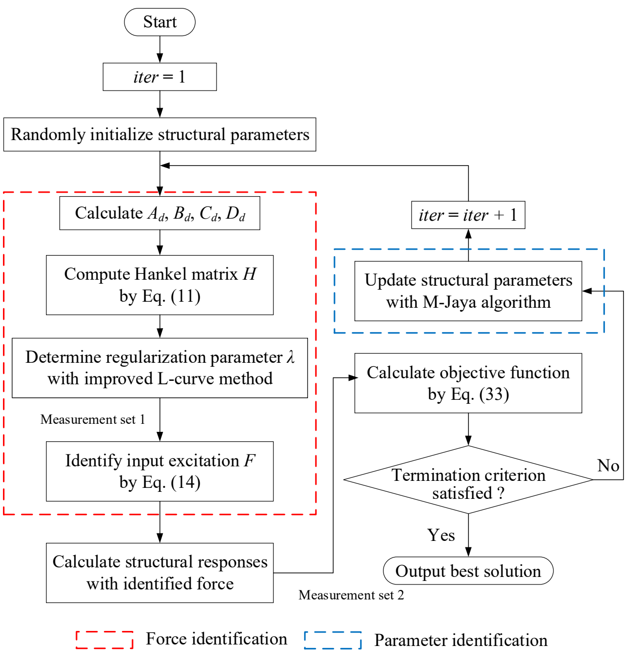

Fig. 4 The identification flowchart of structure parameters and input excitation

dynamic input time histories with Tikhonov regularization method while measurement set 2 is used to identify unknown structural parameters with optimization algorithm. It is noted that measurement set 1 and set 2 may have some common data but they cannot be the same with the purpose of iteratively updating excitation forces and structural parameters.

In addition, the proposed iterative identification procedures are further explained as follows:

Step 1: predefine parameters and randomly generate initial structural parameters in the given upper and lower search space limits.

Step 2: calculate A_d, B_d, C_d, D_d with Eq. (9) after constructing A_c, B_c, C_c, D_c, and compute the lower block triangular Hankel matrix H by Eq. (11).

Step 3: determine the proper regularization parameter λ with the proposed improved L-curve method.

Step 4: identify the unmeasured input excitation F applied to structure with Tikhonov regularization method using the responses of measurement set 1.

Step 5: calculate the structural responses R_{est} with the identified dynamic input time histories and update structural stiffness parameters with M-Jaya algorithm by minimizing the difference between R_{est} and the measured responses R_{mea} of set 2 as follows

$$fit = \frac{1}{r + \sum_{i=1}^{e} \sum_{j=1}^{s} \frac{|R_{est}(i,j) - R_{mea}(i,j)|^2}{E\left(R_{mea}^2(i)\right)}} \tag{33}$$

where fit means the fitness function value; r is a constant to avoid a potential zero denominator, whose value is set as 0.01, so the maximum fitness value is 100; e and s stand for the number of sensors used in measurement set 2 and time steps, respectively.

Step 6: repeat step 2 to step 5 until the maximum number of iterations $MaxG$ is reached or the following convergence criteria is satisfied

$$error^G = \frac{\sum_{i=1}^{NE} \frac{\left|K_i^G - K_i^{G-1}\right|}{K_i^G}}{NE} \times 100\% \leq Tol \tag{34}$$

where $error^G$ represents the mean absolute error at the G-th iteration; K_i^G and K_i^{G-1} are the identified i-th variable at the G-th and (G-1)-th iteration; NE means number of unknown parameters to be identified; Tol denotes the tolerance condition.

4. Numerical studies

Tikhonov regularization technique based on the improved L-curve method is utilized in synergy with modified Jaya algorithm to identify unmeasured external excitations so as to achieve output-only damage identification in the time domain. Numerical studies on a planar truss structure are implemented to validate the applicability and effectiveness of the proposed method in MATLAB 2018a on the Intel(R) Core i5-11320 CPU @ 3.20 GHz PC with 16.00 GB RAM. To solve parameter

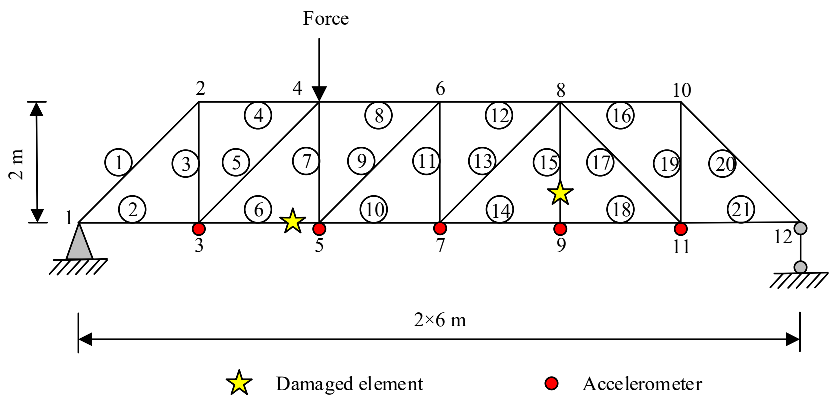

Fig. 5 The numerical model of a plane truss structure

Table 1 The parameter settings of GA, GBABC, Jaya, M-Jaya

Parameters	GA	GBABC	Jaya	M-Jaya
Population size *NP*	100	100	100	100
Maximum iterations *MaxG*	200	200	200	200
Tolerance condition *Tol*	0.001	0.001	0.001	0.001
Mutation rate	0.05			
Crossover rate	0.95			
Search tendency		0.3		
Integer δ				20
Power exponent v				4

identification problem, four different heuristic algorithms including GA (Zhang *et al.* 2010), Gaussian bare-bones artificial bee colony (GBABC) (Zhou *et al.* 2016), Jaya, M-Jaya are employed for comparisons, and their parameter settings are listed in Table 1.

As presented in Fig. 5, a 21-bar planar truss structure is adopted as a numerical example. The length of horizontal and vertical members is 2 m, and the cross-sectional area of each bar is 0.0009 m^2. The mass density and young's modulus of steel material are 7.8 × 10^3 kg /m^3 and 2.1 × 10^{11} N/m^2, respectively. Intermediate node has two degrees of freedom, while the boundary connections of simply-supported truss are modeled as a pin joint at node 1 and a roller joint at node 12. There is an unknown random excitation vertically applied at node 4 with the magnitude of 200 N, zero mean and unit standard deviation. Five accelerometers, as highlighted in Fig. 5, are installed on the structure to record dynamic acceleration responses for duration of 5 s with sampling rate of 200 samples/s. The acceleration measurements at nodes 3, 5, 9 are named as set 1 while those at nodes 5, 7, 11 are denoted as set 2.

Assuming there are 20% stiffness reduction in the 6th element and 30% stiffness reduction in the 15th element, namely, $\alpha_6 = 0.2$, $\alpha_{15} = 0.3$. To account for the adverse effect of noise on the identification of structural parameters and input excitation, Gaussian zero-mean white noise sequences are added into clean measurements $\ddot{u}_{\text{clean}}$ as follows

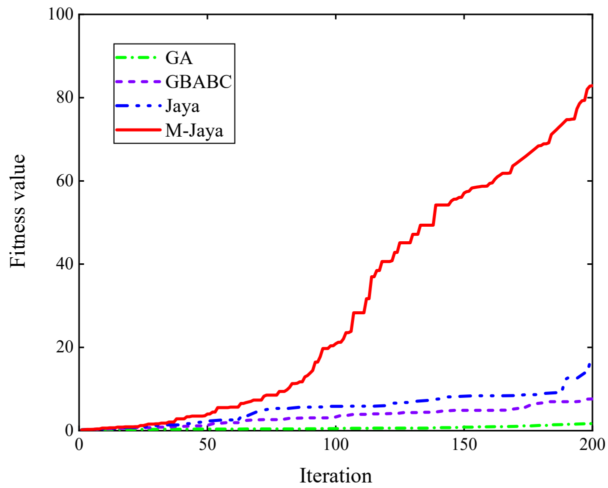

Fig. 6 Comparison of convergence for four different algorithms

$$\ddot{u}_{\text{mea}}=\ddot{u}_{\text{clean}}+N_l N_{\text{noise}}\text{RMS}(\ddot{u}_{\text{clean}}) \qquad (35)$$

where $\ddot{u}_{\text{mea}}$ means the noisy measurements; N_l denotes the level of noise; N_{noise} represents the randomly generated noise vector with zero mean and unit standard deviation in Gaussian distribution; $\text{RMS}(\ddot{u}_{\text{clean}})$ stands for the root-mean-square (RMS) of the noise free acceleration response. Three different noise levels, i.e., 0%, 5%, 10%, are considered in this numerical study. The average values of five independent Monte Carlo simulations are adopted as the final identification results.

The evolutionary process of the fitness function values with GA, GBABC, Jaya, M-Jaya algorithms are presented in Fig. 6. It is easily observed that the proposed M-Jaya algorithm achieves much fast convergence speed than other three algorithms. The final fitness value of GA, GBABC, Jaya and M-Jaya are 1.77, 7.62, 16.26 and 82.85, respectively. The identified result by M-Jaya is closest to the predefined maximum fitness function value, which means the proposed M-Jaya can provide more satisfactory identification results than GA, GBABC and Jaya.

Fig. 7 and Table 2 present the identified stiffness damages and errors using four heuristic algorithms under three levels of noise. In the noise free case, damage

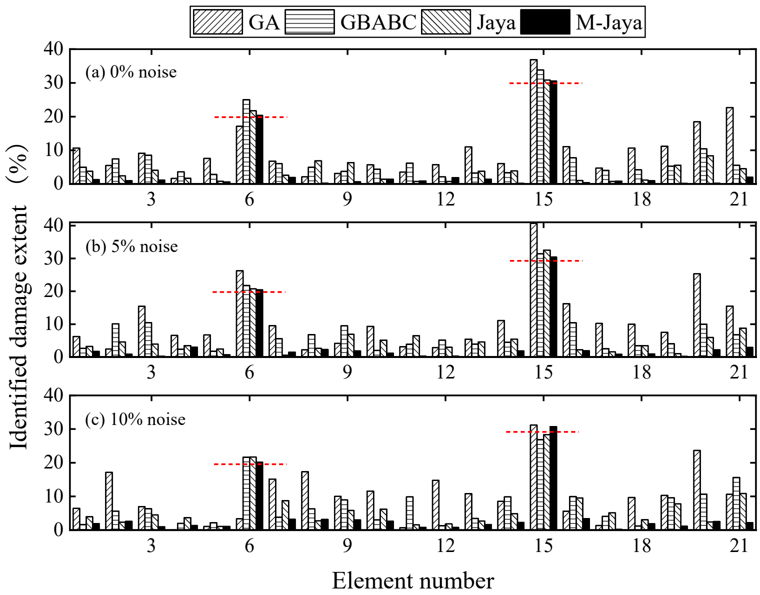

Fig. 7 Identified results of simply supported truss with four heuristic algorithms: (a) 0% noise; (b) 5% noise; (c) 10% noise

Table 2 The final identified errors of stiffness for 21-bar truss under three levels of noise (%)

Methods	0% noise		5% noise		10% noise	
	Mean error	Max error	Mean error	Max error	Mean error	Max error
GA	7.66	22.70	8.89	25.35	9.51	23.64
GBABC	5.10	10.44	5.18	10.47	5.43	15.59
Jaya	2.98	8.33	3.74	8.78	4.23	10.87
M-Jaya	0.84	2.01	1.22	3.01	1.80	3.43

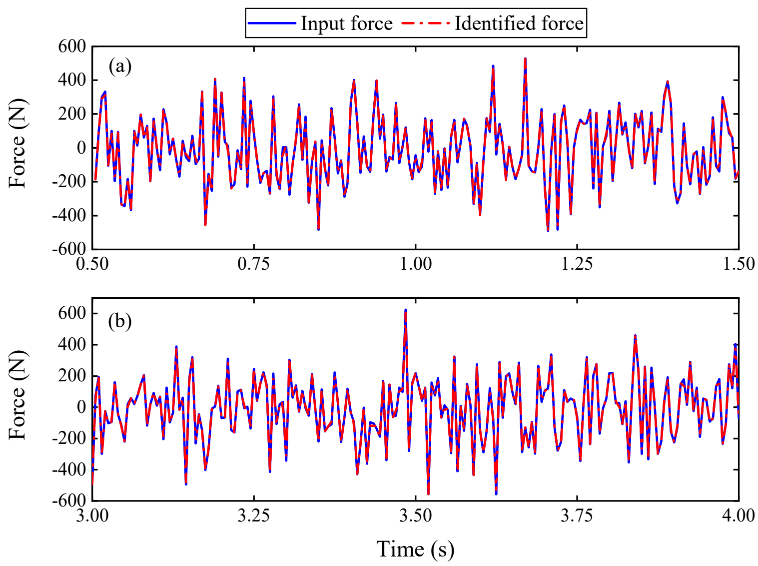

Fig. 8 The identified force time histories of truss structure for noise free case: (a) 0.5-1.5 s; (b) 3.0-4.0 s

locations and extents are successfully identified by GA,while it fails to provide correct results of element 6 under10% noise. Some large false identifications are clearly observed at the 9th, 20th and 21st elements for GBABC and Jaya, and moderate results are obtained with maximum error of 10.47% and 8.78% under 5% noise, 15.59% and10.87% under 10% noise, respectively. On the contrary, the proposed M-Jaya algorithm can not only accurately

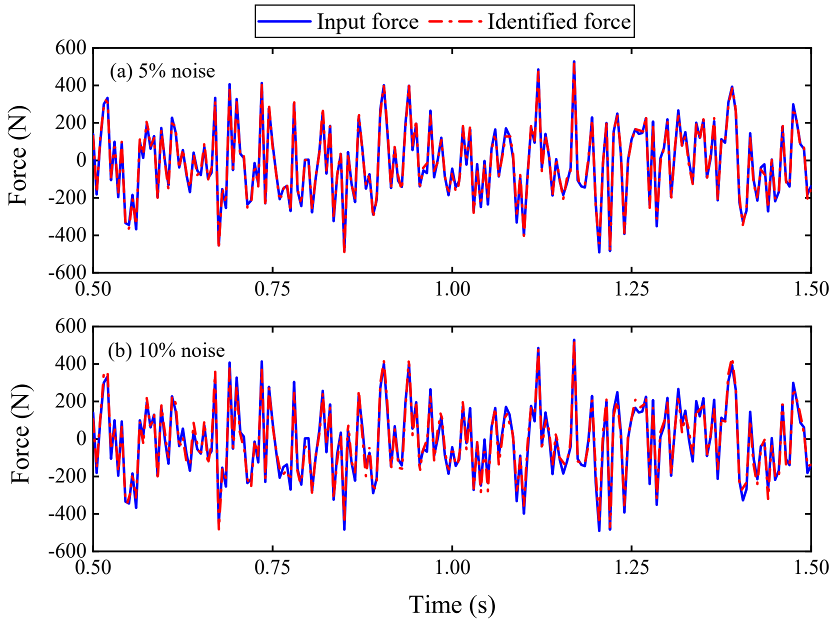

Fig. 9 The comparison between the identified and true force: (a) 5% noise; (b) 10% noise

locate and quantify the damages, i.e., $\alpha_6 = 0.2$, $\alpha_{15} = 0.3$, but also show superior robustness to noise, with less than 3.5% maximum error even for 10% noise case. These identification results indicate that the proposed strategy is able to accurately identify structural damages even with output-only noise-polluted measurements.

As presented in Fig. 8, the identified force time histories of 21-bar truss structure are compared with the actual input excitation. The zoomed view of input force from 0.5 s to 1.5 s and from 3.0 s to 4.0 s clearly shows that the identified force has excellent agreement with the exact white noise excitation. Fig. 9 shows the comparison between identified and actual input forces with 5% and 10% noisy responses.

To further evaluate the accuracy of force identification, two indicators of relative error (RE) and root mean square error (RMSE) are employed as follows

$$RE = \frac{\|F_{mea} - F_{est}\|_2}{\|F_{mea}\|_2} \times 100\% \tag{36}$$

$$RMSE(F_{mea}, F_{est}) = \sqrt{\frac{1}{s}\sum_{i=0}^{s-1}\left(F_{mea}(t_i) - F_{est}(t_i)\right)^2} \tag{37}$$

where F_{mea} and F_{est} are the real and predicted dynamic force time histories; s means the number of sampling points.

Herein, the underlying reason why the proposed method can achieve pleasant performance is illustrated. The identified errors of elemental damages and input excitation by the proposed M-Jaya algorithm and force identification method with noise free, 5% and 10% noise-polluted measurements after 1st, 50th, 100th and 200th iterations are

Table 3 Identified errors of stiffness and force by M-Jaya with respect to noise level and iteration number

Noise level	Iteration number	Stiffness		Force	
		Mean error (%)	Max error (%)	*RE* (%)	*RMSE* (N)
0% noise	1	20.36	42.15	25.27	56.85
	50	6.48	16.75	4.06	14.26
	100	2.46	6.52	2.44	4.59
	200	0.84	2.01	1.02	1.99
5% noise	1	18.76	43.61	30.46	62.51
	50	7.85	18.56	16.74	30.46
	100	3.12	8.44	10.59	23.85
	200	1.22	3.01	6.98	14.28
10% noise	1	16.46	36.86	40.48	83.39
	50	8.26	18.28	26.76	51.48
	100	4.97	8.75	19.27	36.72
	200	1.80	3.43	14.11	28.98

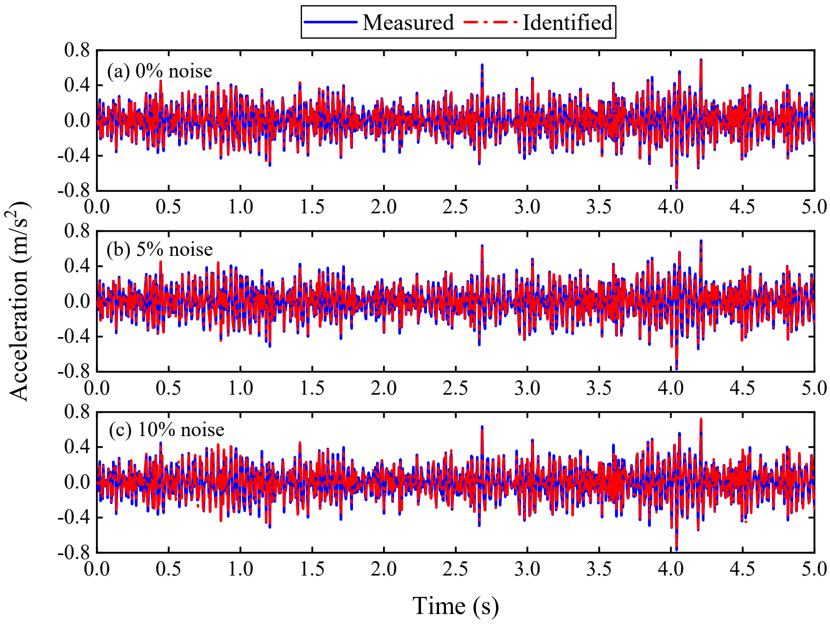

Fig. 10 The measured and predicted acceleration at node 7 under noise level: (a) 0%; (b) 5%; (c) 10%

listed in Table 3. For noise free case, after the 1st iteration, a large difference between the identified and actual parameters is observed with mean error of 20.36% and maximum error of 42.15% for stiffness, relative error of 25.27% and RMS error of 56.85 N for input force, due to the initially estimated parameters are randomly generated.

Subsequently, identified errors of structural damages and input force are iteratively decreased. After 200 iterations, the unmeasured input excitation and the unknown structural parameters approach their exact values. Similarly, the identification results are still good when 5% and 10% noise cases are considered. The predicted structural acceleration response is calculated using the identified stiffness and input excitation. Fig. 10 shows the comparison of identified and measured acceleration responses at node 7 under 0%, 5% and 10% noise levels. By Fig. 10, good agreements can be easily observed.

In summary, by numerical studies on the truss structure, iterative strategy for identification of structural damage and the external excitation based on the proposed M-Jaya algorithm and Tikhonov regularization method is successfully validated.

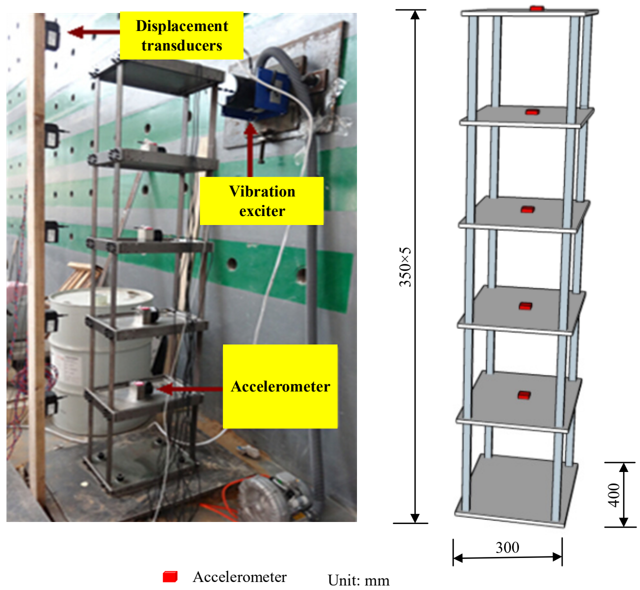

Fig. 11 Experimental tests on a five-story steel frame model

5. Experimental verification

Considering the superior performance of the proposed M-Jaya over other three algorithms, only M-Jaya algorithm is utilized in the following investigations. A series of experimental tests on a five-story steel frame structure are implemented to further verify the applicability and effectiveness of the proposed output-only strategy for the simultaneous identification of structural damages and external force.

5.1 Experimental setup

The experimental setup and detailed dimensions of the frame model are presented in Fig. 11. The total height, length and width of the frame structure are 1750 mm, 300 mm, 400 mm, respectively. Four identical bars with the dimension of 350 mm × 40 mm × 4 mm are utilized as columns, and the thickness of each story plate is 15 mm. All joints in this experiment are connected with bolts. The initial elastic modulus and mass density of steel material are estimated as 2.06×10^{11} N/m^2 and 7850 kg/m^3, respectively. Accordingly, the mass of the structural element can be calculated. The lumped mass of each floor including accelerometer is $M_1 = 24.99$ kg, $M_2 = 24.94$ kg, $M_3 = 24.93$ kg, $M_4 = 24.75$ kg, $M_5 = 24.80$ kg.

A sinusoidal excitation is horizontally applied at the top floor of steel frame model, induced by the dynamic

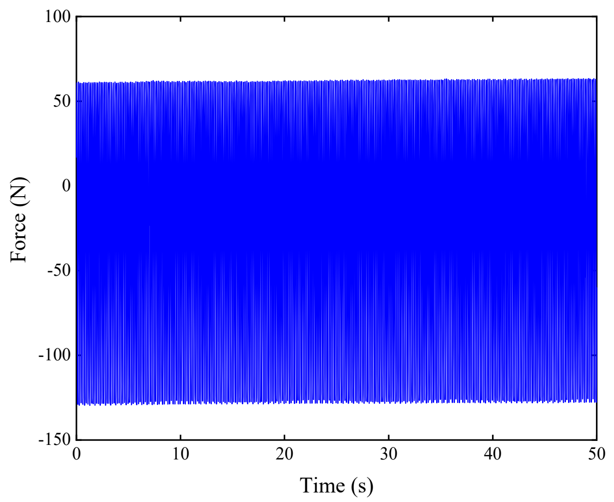

Fig. 12 A typical time history of input sinusoidal excitation

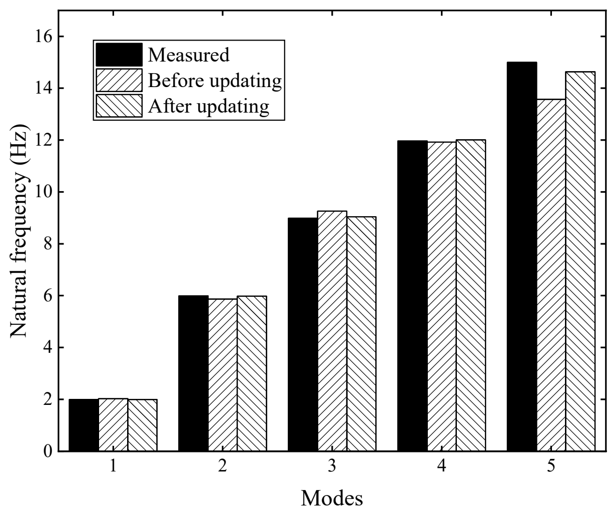

Fig. 13 The measured and updated natural frequencies of five-floor frame model

vibration exciter (Modal Shop 2100E11), and its typical time history is shown in Fig. 12. The shaker is tightly fixed on counterforce wall to provide expected load time histories. A power amplifier is used to produce sufficient power to actuate the vibration exciter and a force transducer (PCB208C02) is installed between the shaker and the frame to directly record external input excitation. The corresponding acceleration and displacement responses of all floors are measured by five model 991C accelerometers and five displacement transducers, respectively. These signals are recorded by the Quantum X data acquisition system with the sampling frequency of 100 Hz and sampling duration of 50 s. The acceleration measurements at floors 1, 3, 5 are regarded as set 1 while those on the 2nd, 3rd, 4th floors are denoted as set 2, and they are adopted in the identification of structural parameters and the input force, respectively. The measured displacements are only used for the comparison with the predicted responses based on the estimated structural parameters and input force, so as to evaluate the performance of the proposed strategy.

5.2 Initial model updating

The frame shown in Fig. 11 can be simplified as a 5-DOF shear-type system due to the comparatively strong floors and weak columns, while it would inevitably result in modeling errors considering the deviations of boundary condition, physical dimensions and material properties.

Hence, it is necessary to update initial model for the purpose of reducing the adverse effect of modeling errors on the identification of both structural damages and the unknown input excitation. Initial model updating is implemented by adjusting structural stiffness parameters so as to make the numerical model as close to the experimental model as possible. The objective function is established based on the discrepancy between the measured natural frequencies from the physical tests and the calculated ones from the finite element model as follows

$$obj(\theta) = \sum_{i=1}^{5} \frac{|w_i^c(\theta) - w_i^m|}{w_i^m} \tag{38}$$

where θ stands for stiffness parameters to be updated; w_i^c and w_i^m represent the calculated and measured natural frequencies under intact state, respectively.

Initial model updating can be transformed into an optimization problem to be solved by the proposed M-Jaya algorithm. The natural frequencies of steel frame structure before and after updating are shown in Fig. 13. There is a large difference between the measured and analytical natural frequencies before updating, and the maximum relative error is more than 9%. After model updating, as listed in Table 4, less than 2.5% relative error implies that the updated structural model is in good agreement with the real structure, so it can be considered as baseline for the following identification.

5.3 Identification results using output-only responses

There are two damage cases are considered in this experimental study. As presented in Fig. 14, damage case 1 is achieved by reducing the cross-section area of four columns at the 5th floor from the 40 mm × 4 mm to 36 mm × 4 mm. In the same way, damage case 2 is realized by reducing the cross-section area of four columns at the 4th floor from the 40 mm × 4 mm to 32 mm × 4 mm. As a result, equivalent stiffness of the 4th and 5th floors are reduced 20% and 10%, respectively. The alterations of the mass caused by these damages are directly neglected. The same parameter settings of M-Jaya algorithm in Table 1 are used, and the corresponding damage identification results with unknown external force are shown in Fig. 15.

In case 1, the identified damage extent at the 5th floor is 11.99%, which agrees well with the exact value of 10%. In case 2, the identified reduction of stiffness in the 4th floor is 22.41%, which matches well with the true value of 20%. In addition, less than 3% false identifications of elemental

Table 4 Measured and analytical natural frequencies of frame structure before and after updating

Mode	Measured (Hz)	Before updating Analytical (Hz)	Before updating Relative error (%)	After updating Analytical (Hz)	After updating Relative error (%)
1	1.997	2.026	1.452	1.990	0.351
2	5.989	5.863	2.104	5.977	0.200
3	8.986	9.259	3.038	9.043	0.634
4	11.967	11.920	0.393	12.010	0.359
5	14.993	13.574	9.464	14.637	2.374

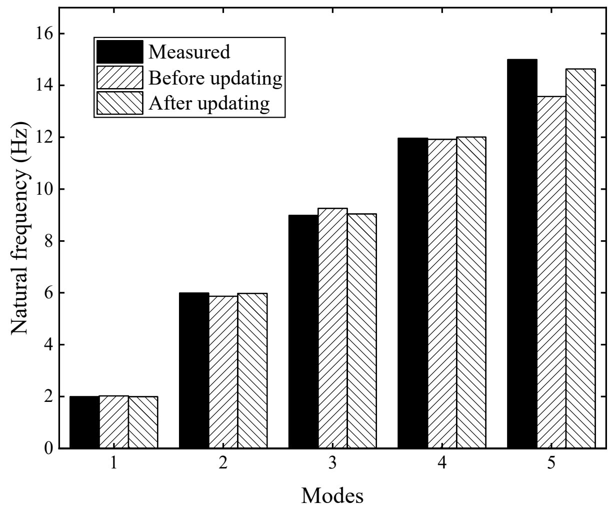

Fig. 13 The measured and updated natural frequencies of five-floor frame model

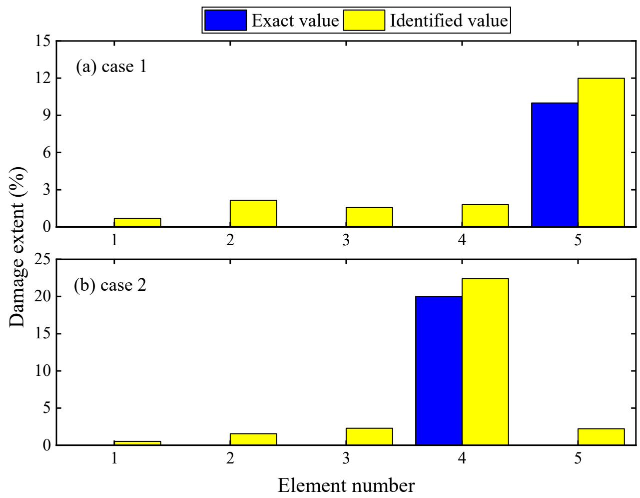

Fig. 15 Identified damage extent with output-only responses: (a) case 1; (b) case 2

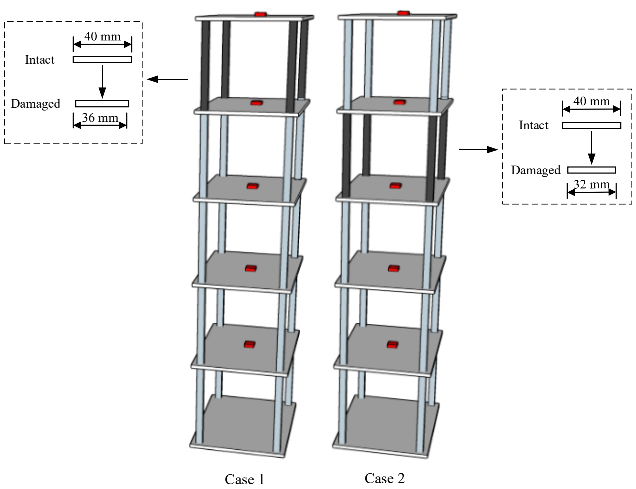

Fig. 14 Two damage cases of steel frame model

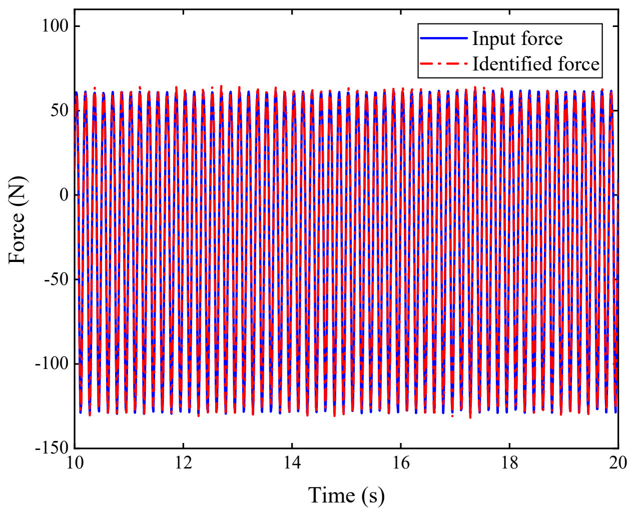

Fig. 16 Comparison of measured and identified input excitation from 10 to 20 s in damage case 2

stiffness are observed in Fig. 15, which indicates the proposed method can accurately identify both damage locations and severities with output-only acceleration responses. Furthermore, the identified external input force applied at the top floor in case 2, taking the force time histories from 10 to 20 s for example, is presented in Fig. 16, and it is compared with the corresponding measured excitation. It can be noticed that the identified forces have a good match with the measured values with small relative error of 2.89% and root mean square error of 2.04 N. Therefore, the proposed iterative strategy can simultaneously identify the structural damages and external excitation force.

The predicted time histories of acceleration responses in damage case 2 are calculated based on the estimated

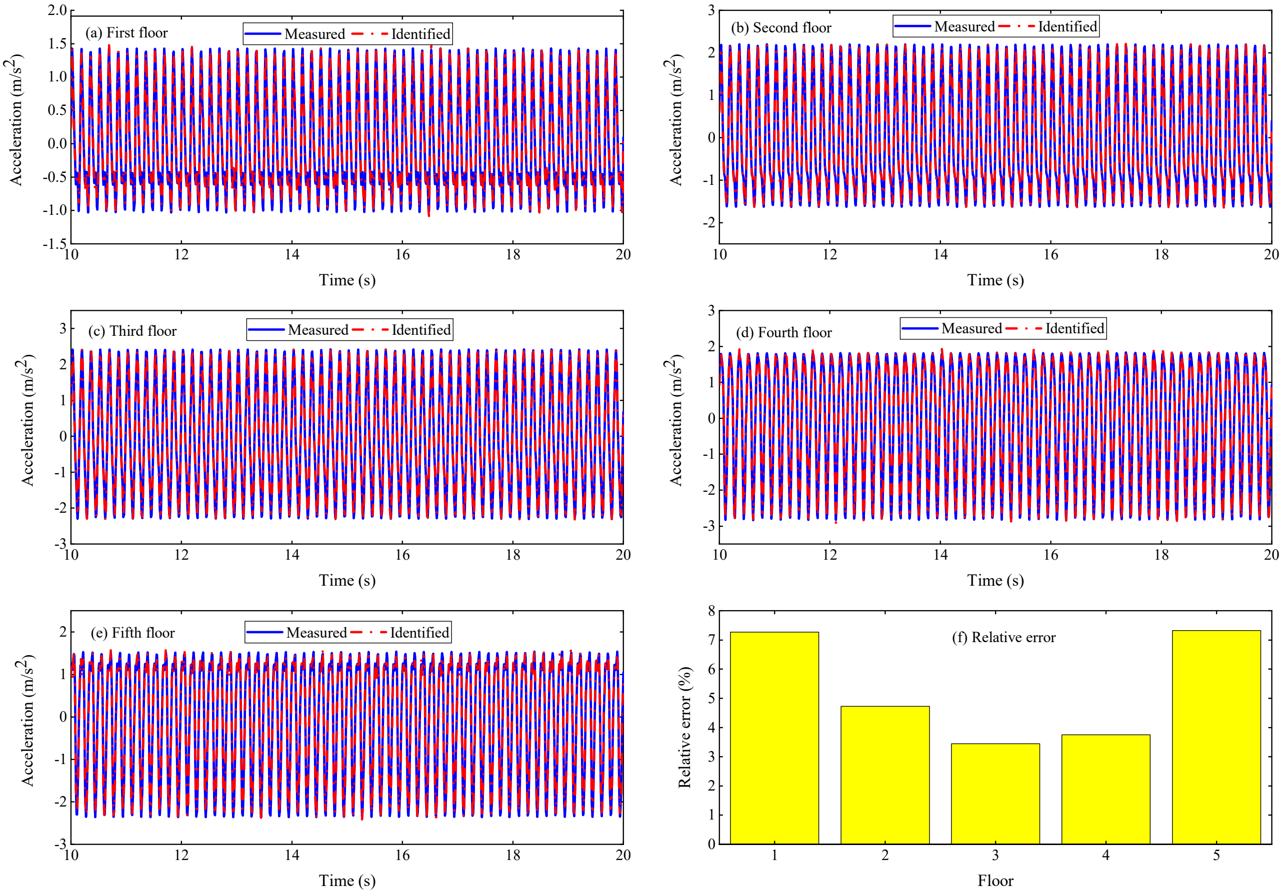

Fig. 17 The measured and predicted acceleration time histories of frame model in case 2: (a) first floor; (b) second floor; (c) third floor; (d) fourth floor; (e) fifth floor; (f) relative error

structural parameters and input excitation. Then, they are compared with measured responses. For clarity, only the results from 10 s to 20 s are presented in Fig. 17. Good agreement is observed between the identified and measured accelerations responses, with the relative errors of 7.27%, 4.73%, 3.45%, 3.75%, 7.32% for identified acceleration from the first floor to the fifth floor, which demonstrates the structural responses (displacement, velocity, acceleration, strain, etc.) could be accurately predicted. It is attractive and meaningful to acquire dynamic responses of large-scale or complex structures where sensors are difficult to be installed.

From the identified results of experimental studies on a five-floor frame structure, the proposed iterative strategy based on the modified Jaya algorithm and Tikhonov regularization method is able to accurately identify structural damages without force measurement.

6. Conclusions

In this paper, an iterative strategy, combining Tikhonov regularization method for force identification and modified Jaya algorithm for parameter identification, is proposed to identify structural damages without force measurements. To enhance the performance of Jaya algorithm, probabilistic clustering learning technique and nonlinear updating equation are introduced. An improved L-curve method based on B-spline interpolation function is presented to deal with the difficulty in selecting the proper regularization parameters for traditional Tikhonov regularization. Numerical studies on a simply-supported truss structure and experimental studies on a five-story steel frame model are conducted to validate the accuracy and effectiveness of the proposed approach in solving inverse output-only identification problem. Some interesting conclusions can be drawn as follows:

- Compared with GA, GBABC and Jaya algorithm, the proposed M-Jaya algorithm can achieve more favorable identification results owing to introducing probabilistic clustering learning technique to improve the convergence performance and nonlinear updating equation to refine the quality of the best solution.
- Different from the traditional Tikhonov regularization, an improved L-curve method based on B-spline interpolation function is developed and it successfully alleviates the ill-posedness problem of the force identification. The optimal regularization parameter could be easily determined by calculating the maximum curvature value of the B-spline curve.
- Numerical and experimental studies demonstrate that

the proposed iterative strategy can accurately identify structural damages and unknown input excitation simultaneously with limited output-only acceleration responses, and it has good robustness to measurement noise.

- Structural responses can be accurately predicted based on the estimated stiffness parameters and excitation force, which is meaningful to monitor the dynamic responses of large-scale or complex structures at locations where sensors are unavailable.
- It should be noted that some aspects are not considered in this paper, such as uncertainties in the temperature variation, boundary stiffness alternation, and modeling error. More investigations including using the proposed approach for the parameter identification of m substructure or nonlinear structure subjected to unknown ambient or moving load in real-life applications would be carried out in the future.

Declaration of competing interest

The authors declare no conflict of interest.

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