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A global-local hybrid strategy with adaptive space reduction search method for structural health monitoring

Guangcai Zhang [a], Chunfeng Wan [a,*], Songtao Xue [b,c,**], Liyu Xie [b]

[a] Key Laboratory of concrete and prestressed concrete structure of Ministry of Education, Southeast University, Nanjing, China
[b] Research Institute of Structural Engineering and Disaster Reduction, College of Civil Engineering, Tongji University, Shanghai, China
[c] Department of Architecture, Tohoku Institute of Technology, Sendai, Japan

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ABSTRACT

The difficulty in computational convergence poses challenges of application for traditional heuristic optimization algorithms to solve the optimization-based structural identification problem, especially for the large-scale and complex structural systems where considerable number of unknown parameters and degrees of freedom involved. Unlike the classic identification methods, in this paper, a novel hybrid strategy, coarsely exploring the relatively large search limits with the improved Jaya algorithm and adaptive search space reduction method in the global stage, and then fine-tuning the identified best solution with local optimization methods to the optimum in the local stage, is proposed and evaluated. The improved Jaya algorithm includes three improvements compared to its original version, fuzzy clustering competitive learning, experience learning and Cauchy mutation mechanisms. Gradient based Levenberg-Marquardt method, sequential quadratic programming method and non-gradient based Nelder-Mead simplex method are inserted as local mathematical optimizers to further enhance identification accuracy and efficiency. The superiority of proposed improved Jaya algorithm is validated in optimizing classical and CEC05 benchmark functions by comparing with several state-of-the-art algorithms. Furthermore, the effectiveness of proposed global-local hybrid method is verified by a numerical example of truss structure and an experimental test of the steel grid benchmark structure with incomplete set of noise-polluted measurements. The statistical results show that the improved Jaya algorithm and adaptive search space reduction method combined with sequential quadratic programming can achieve better performance in structural damage identification than other methods.

1. Introduction

Over the past two decades, considerable number of approaches have been proposed for structural health monitoring, especially for the structural identification [1,2]. Classical methods are generally based on sound mathematical knowledge, but gradient information and good starting point are generally required. Different from classical methods, some non-classical methods including neural networks [3], k-nearest neighbours algorithm [4], especially for the heuristic optimization algorithms with different inspiration sources, such as differential evolution (DE) [5], particle swarm optimization (PSO) [6], fruit

* Corresponding author at: Key Laboratory of concrete and pre-stressed concrete structure of Ministry of Education, Southeast University, China.
** Corresponding author at: Research Institute of Structural Engineering and Disaster Reduction, College of Civil Engineering, Tongji University, China.
E-mail addresses: guangcaizhang@seu.edu.cn (G. Zhang), wan@seu.edu.cn (C. Wan), xue@tongji.edu.cn (S. Xue), liyuxie@tongji.edu.cn (L. Xie).

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fly optimization algorithm [7], enhanced colliding body optimization algorithm [8], butterfly optimization algorithm [9], have been widely explored, becoming one of current research hotspots.

There is no restriction for the behaviors of objective function, such as monotonicity, derivability, modality, which facilitates the application of heuristic algorithms. In addition, these optimization algorithms have no requirements on a good initial estimate or proper gradient information and they cannot guarantee final success for the problem to be optimized. Tang et al. [10] utilized a DE strategy to estimate parameters of structural systems considering limited output data, noise polluted signals. Subsequently, Tang et al. [11] applied the big bang-big crunch algorithm into a series of parameter identification problems and observed that better performance was obtained compared with the mature algorithms of GA and PSO. Vosoughi and Gerist [12] proposed a hybrid FE-PSO–CGAs sensitivity base technique to detect damages of laminated composite beams, and the simulation results proved the effectiveness of proposed method. Ding et al. [13] presented a modified artificial bee colony algorithm and it can successfully identify structural damages even with high-level noise and temperature variation.

However, the difficulty in computational efficiency poses significant challenges of application for abovementioned heuristic algorithms to solve the optimization-based parameter identification problem, especially for the large-scale and complex structural systems where considerable number of unknown parameters and degrees of freedom involved. Motivated by this limitation, this paper proposed a more feasible approach, called global-local hybrid strategy. In the global search stage, an improved version of Jaya algorithm (I-Jaya) is developed. Compared with optimization algorithms such as PSO, GA, DE, the distinct feature of the Jaya algorithm is free from algorithm-specific parameters. The repeated trial-and-error procedures are not required to determine suitable algorithm parameters, which would significantly improve computing efficiency [14]. Despite the fact that Jaya algorithm can present favorable performance in some real-world optimization problems, such as structural optimization design [15] and structural damage identification [16], simple search mechanism makes Jaya algorithm still suffer from premature convergence and being trapped into local optimum when solving complex engineering problems. Some relevant researches have been attempted to enhance the performance of the standard Jaya algorithm. Shuffling process and k-means clustering technique are used to improve the convergence performance of Jaya algorithm respectively [17,18]. In the proposed improved Jaya algorithm, three modifications including fuzzy clustering competitive learning mechanism, experience learning mechanism and Cauchy mutation mechanism are introduced into basic Jaya algorithm. First, the fuzzy clustering competitive learning mechanism is implemented to effectively utilize the population information and accelerate the convergence rate. Second, the experience learning mechanism is adopted to keep the balance between exploiting the previously visited regions and exploring new search space during the search process. The last Cauchy mutation mechanism is implemented to alleviate the possibility of being trapped into local optimum by fine-tuning the quality of the best-so-far solution.

In the global search stage, an adaptive search space reduction method is also developed and employed. In fact, the search limits of unknown parameters have a significant influence on the convergence performance of heuristic algorithm. Although heuristic optimization algorithms can explore predefined search space, the general disadvantage of these methods is time-demanding owing to considerable number of potential solutions to be evaluated. Search space reduction method (SSRM) provides an appealing way to improve the accuracy and efficiency for structural parameter identification by reducing the resources spent on looking far outside the space where the optimal solution lies in. Search space reduction method was combined with GA and butterfly optimization algorithm, presenting good parameter's identification solutions by gradually reducing the search space limits of unknown parameters [19–21]. These researches indicate that it is feasible to improve convergence rate and identification accuracy by updating the limits of search space. It is noted that these methods simultaneously shrink upper and lower search space limits for all unknown parameters, which may lead to over-reduced search limits owing to sensitivity value of recorded responses with respect to each elemental stiffness is different. Instead, the proposed adaptive search space reduction method adaptively reduces the search space limits of parameters from the highest sensitivity to the smallest since more sensitive parameters are fast and easily identified.

It is known that local optimizers, such as gradient based method, i.e., Levenberg-Marquardt (LM) method, sequential quadratic programming (SQP) method, and non-gradient based method, i.e., Nelder-Mead simplex search (NM) method have more strong ability to capture the quick right approach to the nearest optimum than stochastic optimization algorithms, while these local search methods are prone to be trapped into local optimal if starting with a poor initial point. On the contrary, the proposed global method is insensitive to the initial point, while it might show unacceptable slow computational efficiency due to its stochastic nature of parameter searching, especially when the identified solution approaches to the neighborhood of the global optimum. It seems to be an attractive strategy by combining global search and local search methods to benefit from the advantage of each algorithm and alleviate their weakness.

Considering the advantages and disadvantages of heuristic algorithms and local optimizers, an attractive hybrid identification strategy for structural identification is proposed to overcome the local optima trap in the global stage and refine the identified solution in the local search stage. Specifically, fuzzy clustering competitive learning mechanism, experience learning mechanism and Cauchy mutation mechanism are integrated into Jaya algorithm to enhance its exploration and exploitation performance. In addition, adaptive search space reduction method is developed to adaptively reduce the search space limits of unknown parameters from the highest sensitivity to the smallest. In this way, computational time is significantly decreased, and the problem of over-reduced search limits in previous studies can be alleviated. Finally, three local optimization methods including LM method, SQP method, NM method are employed and compared. Computational efficiency and identification accuracy could be further improved. Numerical and experimental studies demonstrate the effectiveness of the hybrid identification strategy, coarsely exploring the relatively large search space with I-Jaya algorithm and adaptive search space reduction method (IJASR) in global search stage, then fine-tuning high-quality potential solution by implementing intensive local search starting from the best estimate of IJASR in local search stage.

2. Problem formulation

The equation of motion for a multi-degree-of-freedom linear structural system under external loads can be stated as follows

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = F(t) \tag{1}$$

where M, C, K stand for the $N \times N$ mass, damping and stiffness matrices; $\ddot{u}(t)$, $\dot{u}(t)$ and $u(t)$ denote the acceleration, velocity and displacement vectors with the same dimension of $N \times 1$; F(t) means a vector of input force acting on the structure system. N represents the DOFs of the structure. The Rayleigh damping model is utilized as an expression for the damping matrix C by a linear combination of the stiffness matrix K and the mass matrix M

$$C = \alpha M + \beta K,\ \varsigma_r = \frac{\alpha}{2\omega_r} + \frac{\beta\omega_r}{2} \tag{2}$$

where α and β are two constant coefficients. 5% damping ratio (ς_r) is considered for the first two modes ($r = 1$ and 2). Newmark's constant-average-acceleration method is employed to calculate structural dynamic responses. In Eq. (1), it is assumed that the mass matrix remains unchanged during the identification. The stiffness of each element and two damping coefficients are the unknown parameters to be identified.

The objective function is defined as minimizing the difference between the measured structural dynamic responses and the estimated structural responses from the finite element model as follows

$$obj(\theta) = \sum_{i=1}^{n_{sen}} \sum_{j=1}^{n_{time}} \frac{\left\| \widehat{\ddot{u}}_i\left(\theta, t_j\right) - \ddot{u}_i\left(t_j\right) \right\|^2}{E\left(\ddot{u}_i^2\left(t_j\right)\right)} \tag{3}$$

where ob$j(\theta)$ denotes the value of objective function, $\theta = \{\theta_1, \theta_2, \ldots, \theta_{Dim}\}$; $\ddot{u}_i(t_j)$ and $\widehat{\ddot{u}}_i(\theta, t_j)$are the measured and estimated structural responses; n_{sen} and n_{time} represent the number of measurements and recorded samples. $E(\ddot{u}_i^2(t_j)) = \frac{1}{n_{time}} \sum_{j=1}^{n_{time}} \ddot{u}_i^2(t_j)$ stands for mean squared value from i th measurement. The fitness function $fit(\theta)$ is expressed as the inverse formation of objective function

$$fit(\theta) = \frac{1}{\varepsilon + \mathrm{ob}j(\theta)} \tag{4}$$

where ε is a small value to avoid zero denominator, and it is set as 0.001 in this paper. The maximum fitness value is equal to ε^{-1} when the identified best structural parameter agrees with the actual value, i.e., obj=0. Thus, the problem of structural identification is summarized as

$$\begin{aligned} &\mathrm{maximize}\, fit(\theta), \theta = \{\theta_1, \theta_2, \ldots, \theta_{Dim}\} \\ &s.t. \theta \in \Gamma, \Gamma = \left\{\theta : \theta_n^{\min} \le \theta_n \le \theta_n^{\max}, \forall n = 1, 2, \ldots, \mathrm{D}im\right\} \end{aligned} \tag{5}$$

where θ_n is considered as the n-th parameter to be identified, and its corresponding lower and upper bounds are $\theta_n^{\min}$ and $\theta_n^{\max}$, respectively; Dim stands for the number of unknown parameters; Γ represents the Dim-dimensional search space.

By Eqs. (3-5), structural identification can be formulated as a linearly constrained nonlinear optimization problem where multiple local optimal solutions may exist, which poses some difficulties for traditional optimization techniques considering their poor optimization capacity. Instead, heuristic algorithms have the advantages of strong search ability, ease of implementation and loose initial conditions, etc. Therefore, it may be feasible to develop more powerful heuristic algorithms for solving the optimization-based structural identification problem.

3. Identification algorithms

3.1. Global search stage

3.1.1. Improved jaya algorithm

Jaya algorithm is a novel heuristic intelligent algorithm based on the concept of the feasible solution simultaneously approaching to the best solution and moving away from the worst solution during the search process, aiming to reach the best solution [22].

Individuals in the initial population are randomly generated in the predefined search space limits. Jaya algorithm generates offspring with the best-so-far solution and the worst-so-far solution as below

$$X_{i,j}^{G+1} = X_{i,j}^{G} + rand_1 \times \left(X_{best,j}^{G} - \left|X_{i,j}^{G}\right|\right) - rand_2 \times \left(X_{worst,j}^{G} - \left|X_{i,j}^{G}\right|\right) \tag{6}$$

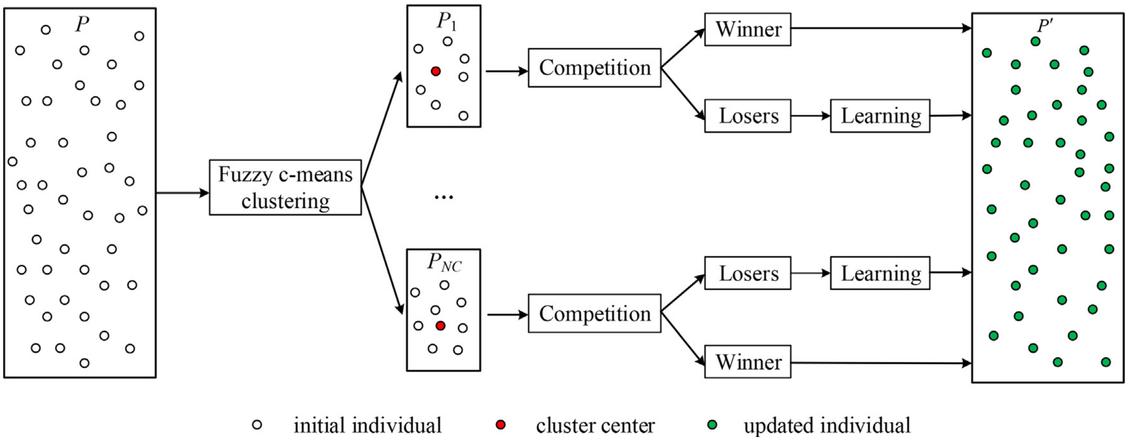

Fig. 1. The general process of fuzzy clustering competitive learning mechanism.

where $X_{i,j}^G$ and $X_{i,j}^{G+1}$ represent the value of the j-th variable for the i th individual at the G-th and $(G+1)$-th iterations, respectively; $|X_{i,j}^G|$ means the absolute value of $X_{i,j}^G$; $rand_1$ and $rand_2$ stand for random number within the range of [0, 1]; $X_{best,j}^G$ and $X_{worst,j}^G$ denote the value of the j-th variable for the best-so-far solution and the worst-so-far solution at the G-th iteration, respectively. The second term on the right side of the Eq. (6) implies the tendency to approach the current best candidate solution X_{best}^G while the third term shows the trend away from the current worst candidate X_{worst}^G.

3.1.1.1. Fuzzy clustering competitive learning mechanism. Fuzzy c-means clustering is a division-based clustering algorithm, and its idea is to maximize the similarity between samples classified into the same cluster and minimize the similarity between different clusters. Different from the hard division of k-means clustering algorithm, fuzzy c-mean clustering achieves soft fuzzy partitioning by introducing fuzzy theory to express the degree of a sample belonging to a certain cluster. A brief introduction about fuzzy c-means clustering is given below [23].

The clustering loss function to be minimized can be stated as follows [24]

$$\varphi_m = \sum_{j=1}^{n} \sum_{i=1}^{NC} \left(v_{ij}\right)^m \left\| X_j - c_i \right\|^2 \tag{7}$$

where m is fuzzy coefficient determining the fuzzy degree of clustering results, $m > 1$; v_{ij} denotes the degree of membership that sample X_j belongs to cluster i; X_j means the j-th sample, $j = 1, 2, ..., n$; c_i is the center of the i th cluster, $i = 1, 2, ..., NC$, $NC = 0.1 \times NP$; $\|X_j - c_i\|^2$is the Euclidean distance between the i th cluster center and the j-th data sample. The sum of membership degrees for every single sample to all clusters is 1 as following equation [24]

$$\sum_{i=1}^{NC} v_{ij} = 1, j = 1, 2, ..., n \tag{8}$$

Then, the Fuzzy c-means clustering iteratively optimizes the loss function φ_m by updating the degree of membership v_{ij} and cluster centers c_i

$$v_{ij} = \sum_{r=1}^{NC} \left(\frac{\left\| X_j - c_i \right\|}{\left\| X_j - c_r \right\|} \right)^{\frac{-2}{(m-1)}} \tag{9}$$

$$c_i = \left(\sum_{j=1}^{n} v_{ij}^m X_j \right) \left(\sum_{j=1}^{n} v_{ij}^m \right)^{-1} \tag{10}$$

If the variation of loss function is less than threshold, stop the iterative process.

As presented in Fig. 1, a novel mechanism of fuzzy clustering competitive learning is proposed, and its general process can be roughly divided into three steps

Step 1: All samples in population P are allocated into NC clusters, i.e., subpopulation P_1,P_2,..., P_{NC} by fuzzy c-means clustering.

Step 2: Assuming all samples compete within a same cluster. As a result of competition, the sample with best fitness value is denoted as winner X_w, while the rest samples lost this competition are named as losers X_l.

Step 3: The winner X_w could directly survive to the next generation whereas all losers X_l in a subpopulation P_i will be updated by following learning equation

$$X_l^{G+1} = X_l^G + rand_1 \times \left(X_w^G - X_l^G\right) + rand_2 \times \left(c_i - X_l^G\right) \tag{11}$$

where $rand_1$ and $rand_2$ mean random number in the interval of [0, 1]; c_i denotes the cluster center.

By Eq. (11), losers learn from the winner X_w and cluster center c_i. More specifically, the second term on the right side of the equation $rand_1 \times (X_w^G - X_l^G)$ is called cognitive component where samples lost competition update its position towards the local optimal through learning from winner X_w. In addition, the third term $rand_2 \times (c_i - X_l^G)$ is termed as social component where losers X_l learn from the mean position of the cluster c_i.

3.1.1.2. Experience learning mechanism. Candidate solutions in the population of Jaya algorithm are iteratively updated by considering both the optimal solution and the worst solution, which could facilitate improving the convergence rate and local search ability, while the population diversity and global search ability of Jaya algorithm might decrease during the search process. To this end, an experience learning mechanism based on the information of other candidates in the population is proposed to enhance the population diversity and global exploration ability [25]. Specifically, the new candidate $X'_{i,j}$ is generated based on the experience of another two different individuals X_{r1} and X_{r2} randomly selected from the population as follows

$$X'_{i,j} = \begin{cases} X_{i,j} + rand \times \left(X_{r1,j} - X_{r2,j}\right), if f(X_{r1}) < f(X_{r2}) \\ X_{i,j} + rand \times \left(X_{r2,j} - X_{r1,j}\right), otherwise \end{cases} \tag{12}$$

where $X_{i,j}$ means the value of the j-th variable for the current i th individual; $rand$ is a random number taken from the range of [0, 1].

3.1.1.3. Cauchy mutation mechanism. To address the problem that Jaya algorithm is prone to be trapped into local optimal solution, Cauchy mutation operation is introduced into basic Jaya algorithm. It is known that Cauchy distribution function has small peak at the origin point but long distribution at both ends. Thus, Cauchy mutation can generate a larger perturbation near the current best individual. Compared with Gaussian mutation, Cauchy mutation has stronger perturbation ability to make it easier to jump out of the local optimum [26]. The standard Cauchy distribution function $f_e(z)$ is formulated as follows

$$f_e(z) = \frac{1}{\pi} \frac{e}{e^2 + (z - z_0)^2}, e > 0, -\infty < z < +\infty \tag{13}$$

where e is the scale parameter defining the half width at half of the maximum value; z_0 means the positional parameter defining the distribution peak position. The best solution is updated using Eq. (16)

$$X'_{best} = X_{best} \times (1 + Cauchy(0, 1)) \tag{14}$$

where $Cauchy(0, 1)$ means a random number that obeys standard Cauchy distribution, $z_0 = 0$, $e = 1$.

The characteristics of Cauchy distribution enable it to generate random numbers far from peak position. Therefore, the stochastic perturbation with the Cauchy mutation contributes to keep the diversity of Jaya algorithm and alleviate the possibility of being trapped into the local optimum.

3.1.1.4. Framework of improved jaya algorithm. Three main improvements including fuzzy clustering competitive learning mechanism, experience learning mechanism and Cauchy mutation mechanism are introduced into original Jaya algorithm. The flowchart of improved Jaya algorithm is presented in Fig. 2. It can be observed that fuzzy clustering competitive learning mechanism is implemented before search stage, which is helpful to effectively utilize the population information and accelerate convergence rate. In addition, with the purpose of balancing the exploration and exploitation capacities in the search process, Jaya optimization by Eq. (6) and experience learning mechanism by Eq. (12) are implemented in a random manner. Cauchy mutation mechanism is conducted to update the best-so-far solution to reduce the risk of falling into a local optimum to some extent.

3.1.2. Adaptive search space reduction method

Previous search space reduction methods can enhance the computational efficiency and identification accuracy with the idea of reducing the time spent on looking far outside the area where the optimal solution lies in. However, the problem of over-reduced search limits may be caused by simultaneously reducing the search space limits for all parameters. It was reported that more sensitive parameters are prone to be exactly identified [27]. Accordingly, an adaptive search space reduction method is proposed and presented in following content.

Before the predefined number of runs to reduce search space reached, I-Jaya algorithm performs R_r independent runs with a large initial search space for sufficient exploration. At the beginning, the estimated value evidently deviates from their exactness. It is necessary to select reasonable number of runs R_r since search limits of structural parameters are reduced

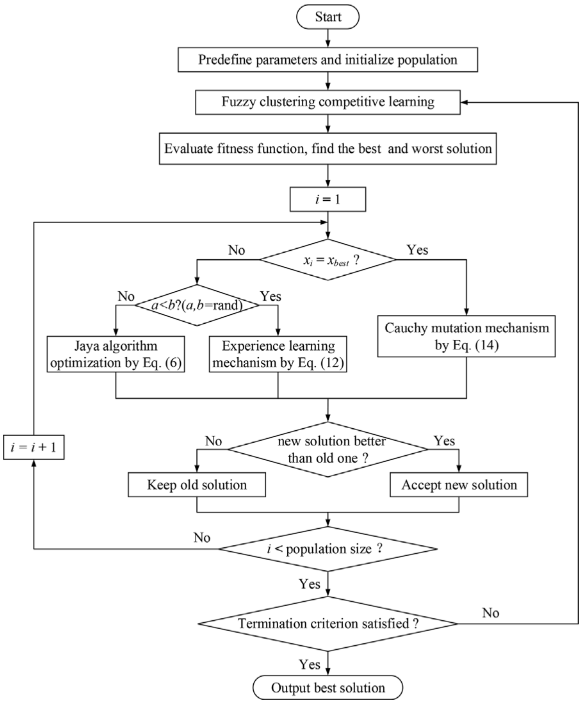

Fig. 2. The flowchart of I-Jaya algorithm.

based on a coarse estimation from previous runs. Subsequently, a set of identified solutions are sort out and the worst R_{w} ones are discarded. The remaining ($R_r - R_{\mathrm{w}}$) solutions can be expressed as

$$\hat{X} = \{\mathrm{X}_s | s = 1, 2, ..., R_r - R_{\mathrm{w}}\} \tag{15}$$

The weight coefficient w_s of solution X_s is calculated by

$$w_s = \frac{fit(X_s)}{\max\{fit(X_s)\}_{s=1,2,...,R_r-R_{\mathrm{w}}}} \tag{16}$$

where $fit(X_s)$ denotes the fitness value of X_s. The weight coefficient w_s is equal to 1 for X_s with the best fitness value.

Then, the weighted mean value μ_i and standard deviation σ_i of the i th parameter (i = 1, 2, ..., Dim) can be computed as follows

$$\mu_i = \left(\sum_{s=1}^{R_r-R_{\mathrm{w}}} w_s X_{si}\right)\left(\sum_{s=1}^{R_r-R_{\mathrm{w}}} w_s\right)^{-1} \tag{17}$$

$$\sigma_i = \sqrt{\left(\sum_{s=1}^{R_r-R_{\mathrm{w}}} w_s (X_{si} - \mu_i)^2\right)\left(\sum_{s=1}^{R_r-R_{\mathrm{w}}} w_s\right)^{-1}} \tag{18}$$

The new trial search range [Lb_i,Ub_i] of the i th parameter is defined as

$$\begin{cases} L\mathrm{b}_i = \mu_i - W \times \sigma_i \\ Ub_i = \mu_i + W \times \sigma_i \end{cases} \tag{19}$$

where X_{si} is the i th variable for the s-th solution; Dim denotes the number of parameters to be identified; W means the window width, which regulates the magnitude of reduced search space. It is noted that constant W should be reasonably set to satisfy requirements including being sufficiently small to achieve convergence but adequately wide to make the actual solution remain within the new trial search space.

To evaluate response sensitivity of unknown parameters, a sensitivity study is conducted by differentiating Eq. (1) with respect to elemental stiffness K_i

$$M\frac{\partial \ddot{u}(t)}{\partial K_i} + C\frac{\partial \dot{u}(t)}{\partial K_i} + K\frac{\partial u(t)}{\partial K_i} = -\frac{\partial K}{\partial K_i}(u(t) + \beta\dot{u}(t)) \tag{20}$$

where $\frac{\partial \ddot{u}(t)}{\partial K_i}$, $\frac{\partial \dot{u}(t)}{\partial K_i}$, $\frac{\partial u(t)}{\partial K_i}$ are the response sensitivity with respect to elemental stiffness K_i.

Response sensitivity with respect to damping coefficients α and β can be computed by

$$M\frac{\partial \ddot{u}(t)}{\partial \alpha} + C\frac{\partial \dot{u}(t)}{\partial \alpha} + K\frac{\partial u(t)}{\partial \alpha} = -M\dot{u}(t) \tag{21}$$

$$M\frac{\partial \ddot{u}(t)}{\partial \beta} + C\frac{\partial \dot{u}(t)}{\partial \beta} + K\frac{\partial u(t)}{\partial \beta} = -K\dot{u}(t) \tag{22}$$

Through Eqs. (20-22), the response sensitivity with respect to structural parameters are computed. After sensitivity analysis, all parameters are divided into several Parts. Structural parameters have similar amplitude of sensitivity value in each Part. The search space limits of parameters in the Part with the highest sensitivity are first reduced. Subsequently, the remaining parameters will be more easily identified if more sensitive parameters have converged to the neighborhood of exact value.

It is noted that the upper and lower limits of feasible solution in the next run should be ensured between the initial search space $[Lb_i,Ub_i]^{\mathrm{init}}$ and the minimum search limit $[Lb_i,Ub_i]^{\mathrm{min}}$

$$[Lb_i, Ub_i]^{new} = [Lb_i, Ub_i]^{init} \cap [Lb_i, Ub_i]^{tri} \cup [Lb_i, Ub_i]^{\min} \tag{23}$$

where $[Lb_i,Ub_i]^{\mathrm{tri}}$ means the trial search range; $[Lb_i,Ub_i]^{\mathrm{min}} = mean \times (1 \pm miniband)$. The minimum band *miniband* is defined as 0.1 to prevent premature caused by over-constrained search range.

3.2. Local search stage

Herein, three local mathematical optimizers including gradient based Levenberg-Marquardt method, sequential quadratic programming method, and non-gradient based Nelder-Mead simplex method, are briefly outlined, respectively.

3.2.1. Levenberg-Marquardt method

The Levenberg-Marquardt (LM) method is one of the most widely used algorithms to solve nonlinear least squares problem. Although Newton's method has fast convergence speed, it needs to calculate the Hessian matrix. In fact, it would be complicated to calculate the second-order derivatives, especially for high-dimensional problems. The Gauss-Newton method can obviously improve computational efficiency by replacing the calculation of Hessian matrix with Jacobian matrix. LM method was proposed by introducing an identity matrix I into Hessian matrix. More detailed description about LM method can be easily found in Ref. [28].

3.2.2. Sequential quadratic programming method

Sequential quadratic programming (SQP) method is considered one of the best methods for solving nonlinear programming (NLP) problems owing to quickly finding the optimal solution in the neighborhood. The basic idea of SQP method is to transform the original nonlinear programming problem into a sequence of quadratic programming subproblem, which can be obtained after linearizing the nonlinear constraints. Although SQP method has powerful gradient search ability and fast convergence speed, it is susceptible to be trapped into local suboptimum if a good initial value is not provided. More detailed description about SQP method can be found in Ref. [29].

3.2.3. Nelder-Mead simplex method

Nelder-Mead simplex (NM) method is a popular non-gradient based local search algorithm for solving unconstrained optimization problem. For a n-dimensional minimization optimization problem, the simplex is initialized by randomly generating $(n + 1)$ vertices and new simplex is obtained by replacing the worst vertices with newly generated vertices. There are four main procedures of NM method, i.e., reflection, expansion, contraction, shrinkage. More detailed description about NM method is illustrated in Ref. [30].

3.3. Hybrid identification strategy

Although heuristic optimization algorithms can explore predefined search space, the general disadvantage of these methods is time-demanding owing to considerable number of potential solutions to be evaluated. To improve the convergence rate of I-Jaya algorithm, an adaptive search space reduction method is developed to decrease computational time on evaluating the candidates far away from the optimal solution. The proposed I-Jaya algorithm and adaptive search space reduction method (IJASR) aims to coarsely explore the relatively large search space in the global stage. It is known that local optimizers, such as LM method, SQP method, NM method, have more strong ability to capture the quick right approach to the nearest optimum than stochastic optimization algorithms, while these local search methods are prone to be trapped into local optimal if starting with a poor initial point. On the contrary, the proposed IJASR method has powerful ability of global exploration, fast convergence speed in the early stage and insensitive to the initial point, while it might manifest unacceptable slow computational efficiency due to its stochastic nature of parameter searching, especially when the identified

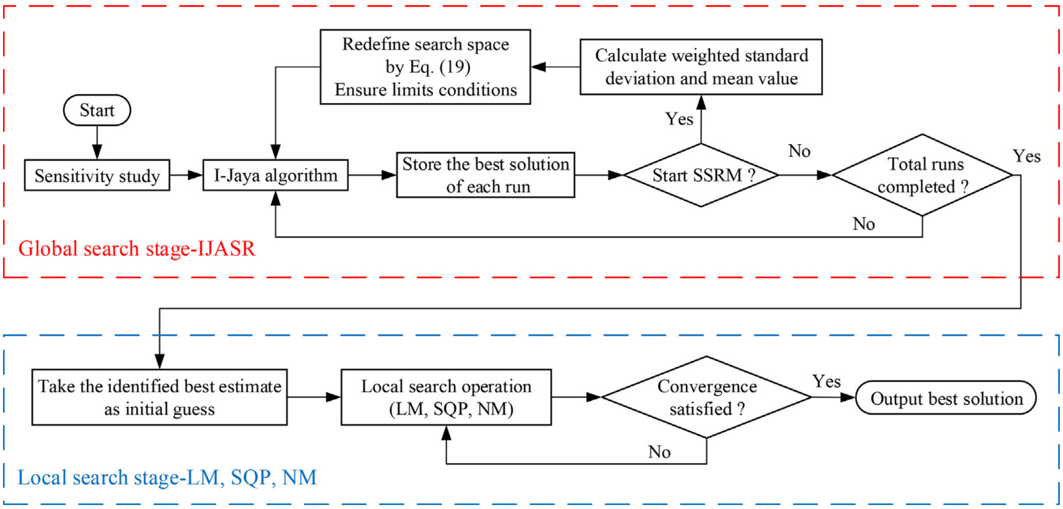

Fig. 3. Hybrid identification strategy based on IJASR and local search method.

solution approaches to the neighborhood of the global optimum. It seems to be an attractive strategy by combining IJASR and local search methods to benefit from the advantage of each algorithm and alleviate their weakness.

Fig. 3 shows the flowchart of hybrid method with IJASR and local mathematical optimizers. There are two different search stages, namely, the rough global search stage and the intensive local search stage. In the global search stage, a sensitivity study is first implemented to determine the sensitivity of unknown parameters. I-Jaya algorithm is independently performed for R_r runs in the same search space, followed by the calculation of weighted mean and standard deviation of the identified best solutions. Then, search space limits of unknown parameters are redefined by Eq. (19) in sequence according to their sensitivity value. If the maximum number of runs complete, the identified best solution is taken as initial value in the subsequent local search stage. Local optimizers LM, SQP and NM are utilized to identify high-quality potential solution by implementing intensive local search starting from the best estimate of IJASR method until predefined maximum iterations reached or the following convergence criteria Tol satisfied

$$Error_i^G = \frac{\sum_{i=1}^{Dim} \frac{\left|K_i^G - K_i^{G-1}\right|}{K_i^G}}{Dim} < Tol \tag{24}$$

where Dim is the number of unknowns to be identified. K_i^G and K_i^{G-1} stand for the identified parameter of the i th candidate after G and (G-1) iterations, respectively.

There are three different hybrid identification methods, denoted as IJASR-LM, IJASR-SQP, IJASR-NM, and their performance would be tested in the following studies.

4. Numerical studies

The performance of proposed I-Jaya algorithm is evaluated and compared with several state-of-the-art algorithms using a set of classical and CEC05 benchmark functions. In addition, a more complex 37-bar truss structure is utilized as the second example to verify the effectiveness, efficiency and robustness of the hybrid identification strategy based on IJASR and local optimization methods. All studies are conducted in MATLAB 2018a on the Intel(R) Core i5–13600KF CPU @ 3.50 GHz PC with 16.00 GB RAM.

4.1. Comparison of I-Jaya with other heuristic algorithms

4.1.1. Classical benchmark functions

To validate the performance of proposed I-Jaya algorithm, a representative set of classical mathematical benchmarks, as listed in Table 1, are employed in this section. For the comparison purpose, GA, DE, Jaya algorithms are adopted, and their parameters recommended from the literatures [31] and [32] are shown in Table 2. The population size and maximum number of iterations are 100 and 500. Termination threshold is set as 10^{-10}. If maximum iteration is reached or the objective function value is less than the termination threshold, the iteration process will be stopped. Due to the stochastic characteristic of heuristic algorithms, statistical solutions based on 30 independent runs are used.

The convergence curves of the mentioned four benchmark functions are presented in Fig. 4 and numbers of iteration for GA, DE, Jaya, I-Jaya are listed in Table 3. It can be observed that GA and DE have some difficulties in solving the Ackley and

Table 1
Four classical benchmark functions for tests.

Number	Name	Range	Dimension	Type
F1	Sphere	[−100, 100]	30	Uni-modal, Separable
F2	Ackley	[−32, 32]	30	Multi-modal, Non-separable
F3	Rastrigin	[−5.12, 5.12]	30	Multi-modal, Separable
F4	Griewank	[−600, 600]	30	Multi-modal, Non-separable

Table 2
Parameters of four algorithms adopted for structural identification.

Parameters	GA	DE	Jaya	I-Jaya
Population size *NP*	100	100	100	100
Maximum iterations G_m	500	500	500	500
Mutation operator	0.05	0.5		
Crossover operator	0.95	0.8		
Total evaluations	50,000	50,000	50,000	50,000
Termination threshold	10^{-10}	10^{-10}	10^{-10}	10^{-10}

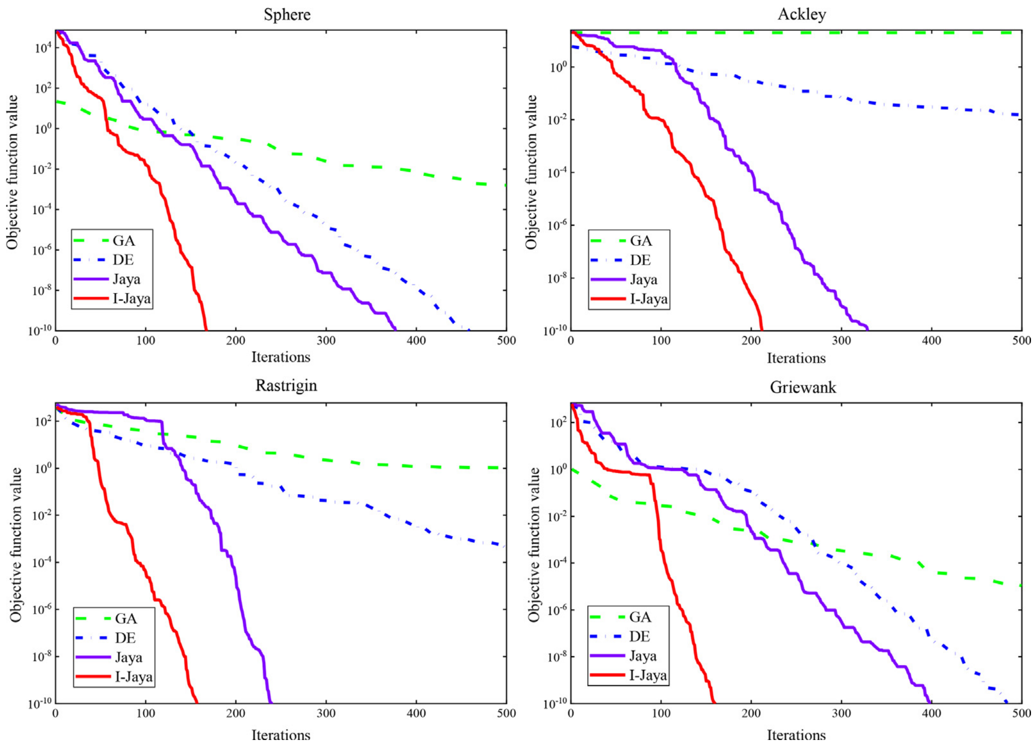

Fig. 4. The convergence process of the mentioned four benchmark functions.

Rastrigin functions since the identified value is far away from termination threshold 10^{-10} when the maximum number of iterations is reached. Jaya algorithm has competitive or sometimes better performance than representative heuristic algorithms GA and DE. It is noted that Jaya algorithm is free from algorithm-specific parameters, which is beneficial to use for the beginner. Furthermore, the proposed I-Jaya algorithm achieves faster convergence speed than other three methods with 168, 212, 157, 160 iterations only, which clearly proves the effectiveness of three improvement mechanisms, namely, fuzzy clustering competitive learning, experience learning and Cauchy mutation.

As a metaheuristic method, it is necessary for I-Jaya algorithm to conduct a study of diversity. Population diversity is measured by [33]

$$Diversity = \sum_{i=1}^{Dim} \sqrt{\frac{1}{NP} \sum_{i=1}^{NP} \left(X_{i,j} - \bar{X}_j \right)^2} \tag{25}$$

where $\bar{X}_j$ stands for the mean value of dimension j in the whole population.

Table 3
Number of iterations for GA, DE, Jaya, I-Jaya.

Number	Number of iterations			
	GA	DE	Jaya	I-Jaya
F1	500	461	377	168
F2	500	500	329	212
F3	500	500	240	157
F4	500	485	398	160

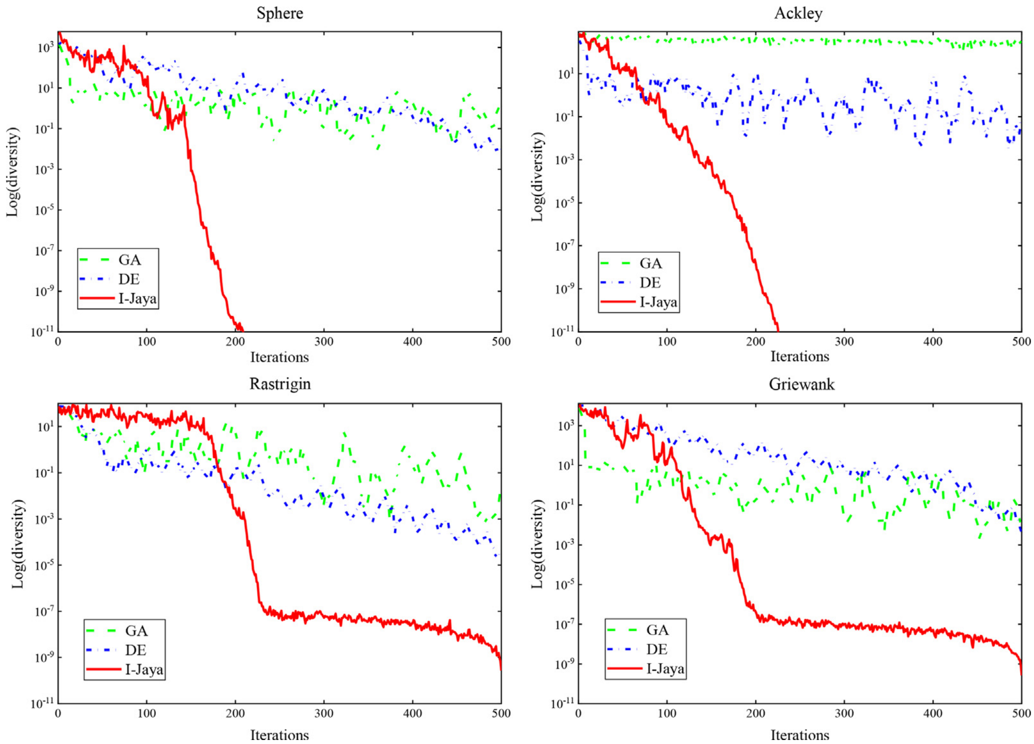

Fig. 5. Results of diversity analysis on classical benchmark functions.

The evolutions of population diversity of GA, DE and I-Jaya for benchmark functions including Sphere, Ackley, Rastrigin, Griewank, are illustrated in Fig. 5. The axis x and axis y correspond to the number of iterations and the diversity, respectively. Setting of parameters are as follows: population size $NP = 30$, Maximum iterations $G_m = 500$. By Fig. 5, it is observed that before 200 iterations, the diversity values are relatively large, which stands for that the individuals in the population scatter in the search domain and concentrate on the exploration search. After 200 iterations, high diversity values of GA and DE indicate that the local search phase is not reasonably conducted. Instead, the diversity value of I-Jaya algorithm is significantly small, which means the individuals gradually gather together and focus on the exploitation search.

4.1.2. CEC benchmark tests

CEC2005 functions are considered as an effective tool to test the performance of heuristic algorithm. Six representative functions, F1, F4 (unimodal functions), F10, F12 (multimodal biased functions), F13, F14 (expanded functions), as listed in Table 4 are employed. In order to verify the effectiveness of I-Jaya algorithm, the results from some recent advanced algorithms are compared. These benchmark functions in CEC2005 are implemented on dimensions $D = 10$ and $D = 50$ for a maximum of $1000 \times$ D function evaluations, and statistical solutions based on 30 independent runs are used. For dimension = 10, the optimization results by balanced teaching-learning-based optimization (BTLBO) algorithm [34], bare-bone Gaussian tree seed algorithm (BGTSA) [35], clustering based tree seeds algorithm (C-TSA) [33], gray prediction evolution algorithm based on the even difference (GPEAed) [36] and I-Jaya algorithm are listed in Table 5. For dimension = 50, BGTSA [35], C-TSA [33], Modified ABC [13], IDRCEA [37] and I-Jaya algorithm are listed in Table 6. There are some differences in the compared algorithms because it is difficult to find several algorithms that calculate CEC2005 benchmark functions with dimensions 10 and 50 simultaneously. It can be noted from Table 5 and Table 6 that no algorithm can outperform on all benchmark functions while I-Jaya algorithm can achieve more favorable results than other heuristic algorithms in most cases.

Table 4
Information of the CEC2005 benchmarks functions.

Number	Function's name	Range	Global value	Type
F1	Shift Sphere function	[−100, 100]	−450	Unimodal function
F4	Shift Schewfel's problem 1.2 with noise in fitness	[−100, 100]	−450	
F10	Shift Rotated Rastrigin's Function	[−5, 5]	−330	Multimodal biased function
F12	Schwefel's Problem 2.13	[-π, π]	−460	
F13	Expanded Extended Griewank's plus Rosenbrock's Function	[−3, 1]	−130	Expanded function
F14	Expanded Rotated Extended Scaffe's	[−100, 100]	−300	

Table 5
Performance of BGTSA, C-TSA, IDRCEA, GPEAed, I-Jaya on CEC2005 benchmarks (10D).

Num	BGTSA		C-TSA		IDRCEA		GPEAed		I-Jaya	
	Mean	std	Mean	std	Mean	std	Mean	std	Mean	std
F1	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	2.48E-06	1.17E-05	**0.00E+00**	**0.00E+00**
F4	2.62E-14	8.31E-14	**2.75E-21**	**1.05E-20**	4.96E+02	4.15E+02	4.19E+01	1.63E+02	1.67E-15	2.96E-16
F10	1.57E+00	9.42E-01	9.88E+00	2.83E+00	1.27E+01	6.59E+00	1.27E+01	7.14E+00	**1.46E+00**	**8.49E-01**
F12	1.69E+03	2.24E+02	1.92E+00	**6.73E-01**	2.34E+02	5.13E+02	3.56E+04	1.60E+04	**1.25E+00**	3.14E+01
F13	1.33E+00	6.66E-01	**7.21E-01**	**8.83E-02**	1.48E+00	7.38E-01	1.37E+00	9.39E-01	1.46E+00	4.58E-01
F14	1.57E+00	1.94E-01	3.04E+00	3.56E-01	3.96E+00	3.26E-01	3.68+00	3.00E-01	**1.32E+00**	**1.75E-01**

Note: the bold value stands for the best identified value.

Table 6
Performance of BGTSA, C-TSA, Modified ABC, IDRCEA, I-Jaya on CEC2005 benchmarks (50D).

Num	BGTSA		C-TSA		Modified ABC		IDRCEA		I-Jaya	
	Mean	std	Mean	std	Mean	std	Mean	std	Mean	std
F1	1.97E-15	6.02E-15	**1.55E-27**	**8.11E-28**	8.15E-16	1.07E-16	3.64E-04	2.84E-04	2.46E-16	5.36E-18
F4	2.77E+03	1.73E+03	**3.02E-02**	**8.81E-02**	6.59E+03	1.26E+03	8.52E+04	1.56E+04	4.20E+02	1.72E+01
F10	2.89E+01	**1.32E+01**	2.06E+02	4.15E+01	2.99E+02	8.42E+01	1.12E+02	2.90E+01	**2.45E+01**	1.48E+01
F12	2.09E+05	1.35E+05	7.83E+04	7.58E+03	1.32E+04	1.08E+04	1.42E+05	5.40E+04	**1.07E+04**	**2.51E+03**
F13	**4.06E+00**	3.17E-01	7.71E+00	6.29E-01	1.32E+00	**1.17E-01**	3.42E+01	7.99E+00	4.46E+00	2.78E+01
F14	1.31E+01	2.21E-01	2.30E+01	2.00E-01	2.26E+01	4.01E-01	2.37E+01	2.54E-01	**1.11E+01**	**1.78E-01**

Note: the bold value stands for the best identified value.

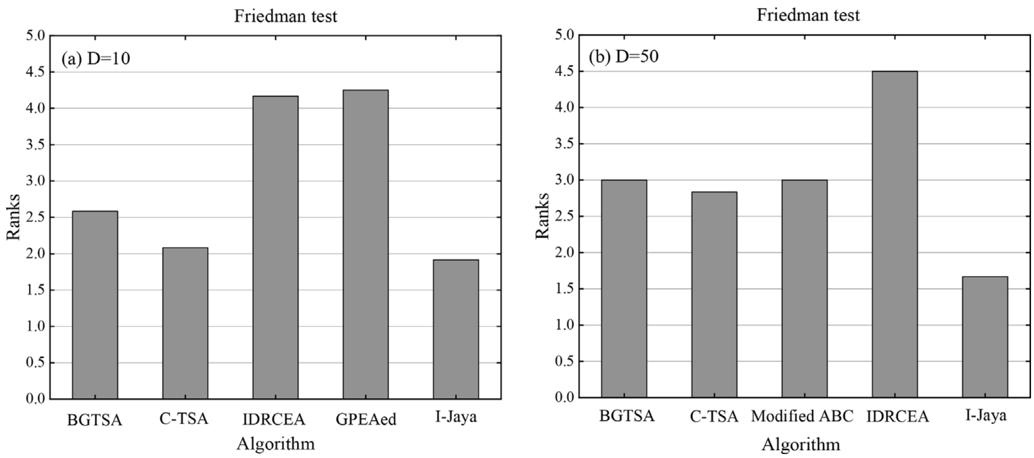

Fig. 6. Friedman rank test for the CEC2005 benchmarks: (a) $D = 10$; (b) $D = 50$.

With the purpose of evaluating the performance of the proposed I-Jaya algorithm from a statistical point of view, the non-parametric Friedman test [38] on the CEC2005 benchmarks is carried out. In this test, the mean errors of objective function values obtained by abovementioned algorithms are employed as the input. Frist of all, the Friedman test finds the rank of optimization algorithms for the individual problems. Then, the average rank is calculated to get the final rank of each algorithm for the considered problems. Fig. 6 presents the mean ranks of different heuristic algorithms for the CEC2005 mathematical benchmarks. It is clearly observed that the proposed I-Jaya algorithm obtains the best rank among these heuristic algorithms with the least score.

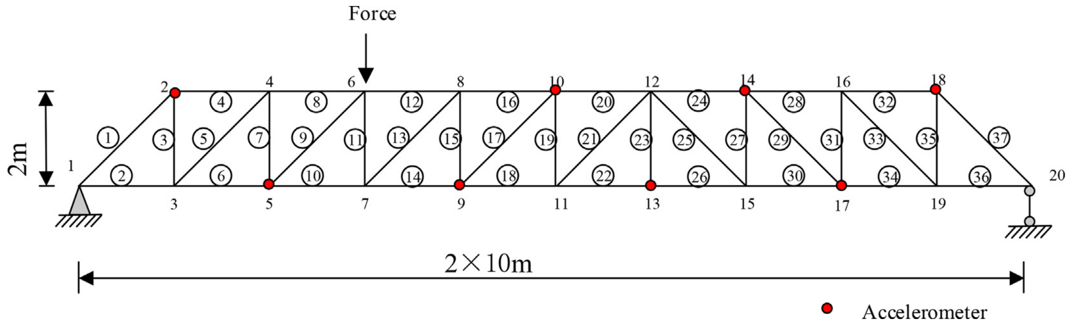

Fig. 7. Numerical model of 37-bar truss structure.

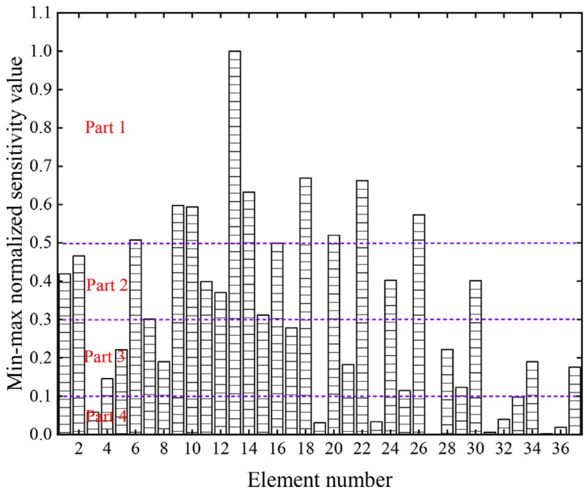

Fig. 8. The min-max normalization of sensitivity value for truss structure.

4.2. Identification with hybrid global-local method

As presented in Fig. 7, a relatively complex truss structure in Ref. [21] is employed as the numerical example to verify the performance of the proposed global-local hybrid strategy. In this numerical simulation, the assumption of priori known mass parameter is used because of the assertion that mass can be estimated accurately enough from the structural drawing. A broad initial search space limits for unknown parameters is defined as half to double of their true values. The simply-supported truss structure has 20 nodes and 37 elements in total, and its boundary condition is considered as a pin support and roller support at node 1 and 20, respectively. The Young's modulus, density and cross -sectional area of each bar are 2.1×10^{11} N/m^2, 7.8×10^3 kg/m^3, 0.0016 m^2, respectively. The structure is excited by an ambient excitation with magnitude of 200 N acting at nodes 6 in vertical direction. Accelerometers are installed at eight different locations, as highlighted in Fig. 7, to record dynamic responses with sampling rate of 2000 samples/s and time duration of 2 s along the vertical direction.

The sensitivity analysis is initially conducted to determine the order of reducing search space limits of unknown parameters. Sensitivity value of eight recorded measurements in 2 s with respect to structural parameters are computed. Fig. 8 presents the amplitude and distribution of response sensitivity with respect to element stiffness after the min-max normalization. As listed in Table 7, all elements of simply-supported truss structure are relatively evenly divided into four different Parts according to their similar sensitivity values. The search limits of the most influential parameters in Part 1 will be reduced at the 6th run, and these parameters are prone to be faster identified due to their standard deviations have small dispersion. Subsequently, the search range of unknown parameters within more Parts are gradually reduced. In this way, the less decisive parameters to be explored are assigned a greater region of search space, which can alleviate the possibility of wrong convergence. The last time of search space reduction method is implemented at the 30th run to redefine the search limits of all structural parameters.

The parameter settings of the proposed IJASR method are given as follows: population size $NP = 20$, maximum generations$G_m = 30$; the number of total runs and evaluations are 30 and 18,000; the number of independent runs before search space reduction is R_r=6; the number of solutions to be discarded R_w=1; the weighted mean value and standard deviation are calculated with $R_r - R_w$=5 solutions; width of window λ=4. After 30 runs, the final identified best value,

Table 7
The order of reduced search space limits according to response sensitivity.

Part	Domain of min-max normalized sensitivity	Parameters to be identified	Number of run to reduce limits	Reduce search limits for Part
1	[0.5, 1]	β, K_6, K_9, K_{10}, K_{13}, K_{14}, K_{18}, K_{20}, K_{22}, K_{26}	6	1
2	[0.3, 0.5)	K_1, K_2, K_7, K_{11}, K_{12}, K_{15}, K_{16}, K_{24}, K_{30}	12	1, 2
3	[0.1, 0.3)	α, K_4, K_5, K_8, K_{17}, K_{21}, K_{28}, K_{29}, K_{34}, K_{37}	18	1, 2, 3
4	[0, 0.1)	K_3, K_{19}, K_{23}, K_{25}, K_{27}, K_{31}, K_{32}, K_{33}, K_{35}, K_{36}	24, 30	1, 2, 3, 4

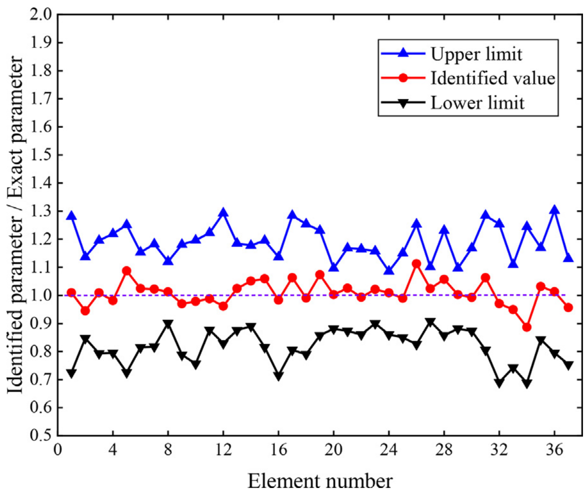

Fig. 9. Identified stiffness and its reduced search space limits with IJASR method.

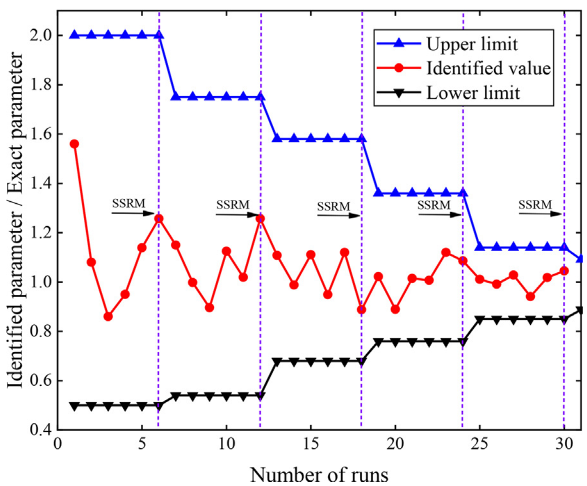

Fig. 10. Identified stiffness and its search space of element 20.

upper and lower search space limits are shown in Fig. 9. It is observed that the identified best solution successfully approaches to the neighborhood of exact value. In addition, compared with the initial large search range [0.5, 2.0], the mean lower and upper search space limits are refined to a more reasonable domain of [0.82, 1.19] by implementing the search space reduction method five times. The underlying reason why the proposed IJASR method can acquire favorable results is further elaborated. Taking element 20 as an example, Fig. 10 illustrates the identification process of K_{20} and its upper and lower search limits with the number of runs. The better 5 solutions out of total 6 independent runs are chosen to determine the search space of next run. The search space limits are gradually refined from initial [0.5, 2.0] to [0.54, 1.75], [0.68, 1.58], [0.76, 1.36], [0.85, 1.14] and [0.89, 1.09], which corresponds to each implementation of search space reduction. It is noted that good identification results and superior search space limits are achieved using the proposed IJASR method.

Table 8
Comparison of computational efficiency.

Methods	Objective function value	Total evaluations	Computational time (s)
IBOA [21]	305	40,000	17,642.01
Ham-IBOA [21]	686	20,000	9903.55
IJASR-LM	901	23,301	2418.90
IJASR-SQP	998	22,203	2361.54
IJASR-NM	970	28,000	2998.25

Table 9
Comparison of computational accuracy (%).

Methods	0% noise		5% noise		10% noise	
	Mean error	Max error	Mean error	Max error	Mean error	Max error
IBOA [21]	2.43	6.38	3.21	9.68	5.46	12.15
Ham-IBOA [21]	0.86	1.62	1.89	4.87	3.02	6.42
IJASR-LM	0.38	1.50	1.82	5.42	3.23	6.84
IJASR-SQP	0.14	0.86	0.42	1.95	0.89	2.42
IJASR-NM	0.21	1.05	0.69	2.34	1.54	4.02

Then, local mathematical methods including gradient based LM method, SQP method, and non-gradient based NM method are employed to further fine-tune the quality of solution starting from the best solution obtained by the proposed IJASR method. The predefined number of iterations for LM, SQP, NM methods are 70, 100, 10,000, respectively. Only IBOA and Ham-IBOA obtained in Ref. [21] are compared with the proposed global-local hybrid strategy, namely, IJASR-LM, IJASR-SQP, IJASR-NM. As listed in Table 8, the identification efficiency and total computational time are investigated. It is found that hybrid global-local identification methods present more favorable performance than IBOA and Ham-IBOA. Besides, the gradient based local optimizers LM and SQP are more efficient than non-gradient based NM method. Especially, IJASR-SQP method achieves the most satisfactory identification results with maximum objective function value and minimum computational time.

In addition, Gaussian white noise is considered as follows

$$\ddot{u}_{mea} = \ddot{u}_{clean} + Nl \times N_{noise} RMS(\ddot{u}_{clean}) \tag{26}$$

where Nl stand for the given noise level; N_{noise} represents the randomly generated noise vector with Gaussian zero mean and unit standard deviation; $RMS(\ddot{u}_{clean})$ denotes the root-mean-square value of the clean measurement $\ddot{u}_{clean}$. Three levels of noise 0%, 5% and 10% are used to consider the adverse effect of measurement noise on the computational accuracy. The identified errors under three levels of noise are shown in Table 9. It is noted that IBOA obtains unsatisfactory performance in consideration of more than 6% false identification for the noise-free case. Ham-IBOA and IJASR-LM are able to accurately identify structural parameters without noise but more than 6% maximum errors are observed when 10% noise case is considered. In contrast, the proposed hybrid global-local methods, IJASR-SQP, IJASR-NM, can provide more pleasant identification results with slight mean error. More specifically, the identified mean and maximum errors of IJASR-SQP are only 0.89% and 2.42% even for 10% noise case, which indicates the superiority of IJASR-SQP method.

In summary, it can be concluded that hybrid global-local identification strategy, roughly approaching to neighborhood of optimal solution by proposed IJASR method and then taking the identified best solution as initial point in the subsequent local search stage, provides a feasible approach to identify unknown parameters for large-scale and complex structures. Taking the identification accuracy and computational efficiency into consideration, the proposed IJASR-SQP method can provide more accurate identification result with the least computational times than IBOA, Ham-IBOA, IJASR-LM, IJASR-NM.

5. Experimental study

Structural health monitoring has achieved significant progress through various theoretical and experimental researches, whereas these works are basically based on different engineering structures and experimental conditions, rendering it quite difficult to evaluate the performance of different identification methods. To deal with this problem, a well-known benchmark structure was established as a platform to consistently assess new proposed structural parameter identification methodologies before real-life applications. Herein, the steel grid benchmark structure is employed to further validate the effectiveness of proposed hybrid global-local identification methods.

As presented in Fig. 11(a) [39], the physical model of benchmark structure has a two-span continuous beam across the middle supports, and its total length and width are 5.49 m and 1.83 m. The sections for girders, beams and columns are S3 × 5.7, S3 × 5.7 and W12 × 26, respectively. The Young's modulus and mass density of steel grid benchmark structure are 2.1×10^{11} N/m^2 and 7850 kg/m^3. Fig. 11(b) shows the finite element model has 14 nodes and 19 elements. Hinge connections, allowing rotation in a certain direction and restraining the other degrees of freedom, are considered as the boundary condition of support location at nodes 1, 4, 7, 8, 11, 14 [40]. The structure is excited by an ambient force acting

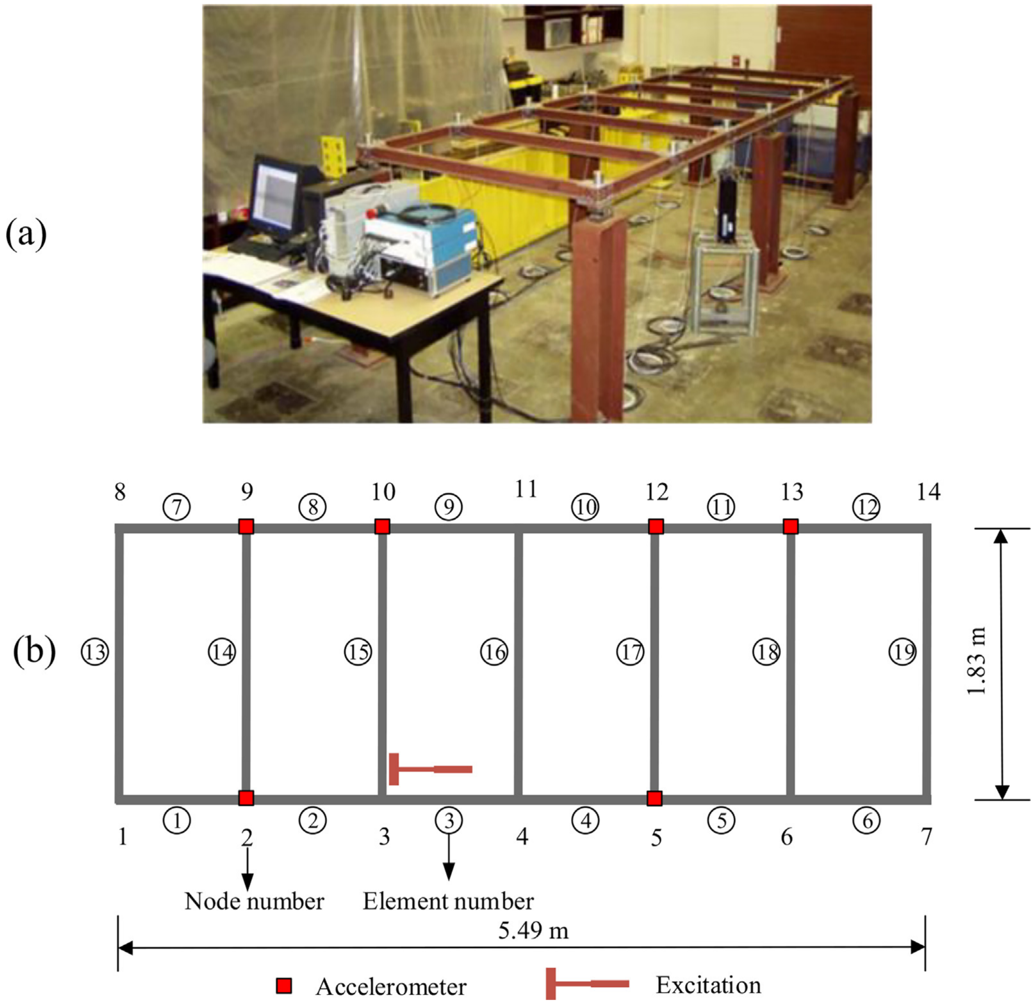

Fig. 11. The steel grid benchmark structure: (a) laboratory model; (b) numerical model.

Table 10
The order of reduced search space limits according to sensitivity value.

Part	Domain of min-max normalized sensitivity	Parameters to be identified	Number of run to reduce limits	Reduce search limits for Part
1	[0.6, 1]	β, K_1, K_2, K_7, K_8	6	1
2	[0.2, 0.6)	α, K_3, K_4, K_{10}, K_{14}, K_{15}	12	1, 2
3	[0.05, 0.2)	K_5, K_9, K_{11}, K_{13}, K_{16}, K_{17}	18	1, 2, 3
4	[0, 0.05)	K_6, K_{12}, K_{18}, K_{19}	24, 30	1, 2, 3, 4

at node 3 in vertical direction. Six accelerometers are instructed at nodes 2, 5, 9, 10, 12, 13 to record dynamic responses for 600 s with sampling frequency of 400 Hz along the vertical direction. More details about this structure can be found in Burkett [41].

Initially, sensitivity analysis is carried out to determine the sequences of reducing search limits of unknown parameters by calculating sensitivity value of six recorded measurements in 2 s with respect to structural parameters. The amplitude and distribution of min-max normalized sensitivity with respect to element stiffness for steel grid benchmark structure is displayed in Fig. 12. As listed in Table 10, all elements are relatively evenly divided into four different Parts according to its similar domain of sensitivity [0.6, 1], [0.2, 0.6), [0.05, 0.2), [0, 0.05). Only search range of elements within Part 1 will be reduced at the 6th run to alleviate the possibility of over-reducing search space limits. Subsequently, the search limits of unknown parameters within more Parts are gradually reduced in the order presented in Table 10.

In this study, parameter settings of the proposed IJASR method are listed as follows: population size $NP = 20$, maximum generationsG_m= 40, total runs and evaluations are 30 and 24,000; R_r=6, R_w=1, width of window λ=4. After 30 runs, the

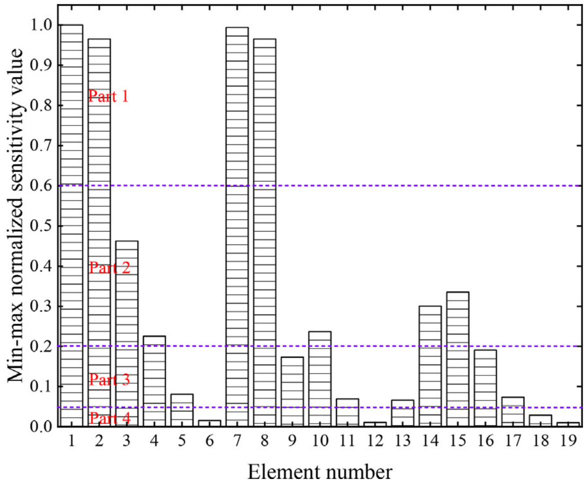

Fig. 12. The min-max normalized sensitivity value for steel grid benchmark structure.

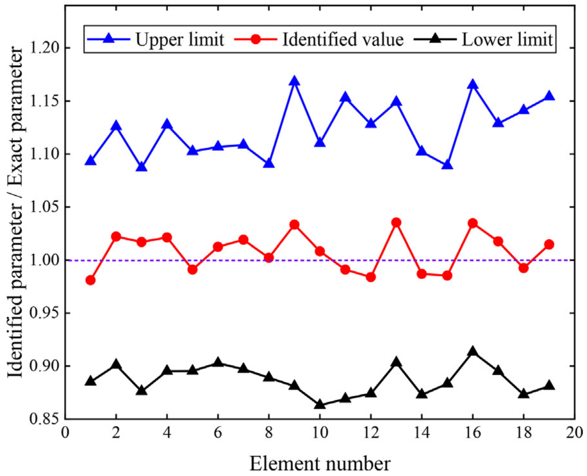

Fig. 13. The identification best value and reduced search space limits for grid benchmark structure.

Table 11
Identification stiffness errors for the steel grid benchmark structure under three noise levels (%).

Methods	0% noise		5% noise		10% noise	
	Mean error	Max error	Mean error	Max error	Mean error	Max error
IBOA [21]	2.03	5.48	2.92	8.79	3.82	11.87
Ham-IBOA [21]	0.63	1.28	1.52	4.11	2.14	6.05
IJASR-LM	0.32	1.08	1.87	5.78	3.37	11.87
IJASR-SQP	0.13	0.38	1.40	3.11	2.84	6.45
IJASR-NM	0.30	1.05	1.92	5.26	3.22	10.72
IJASR-I-Jaya	0.69	1.59	2.44	6.69	4.68	15.92

identified value of unknown parameters and its reduced search space limits are shown in Fig. 13. It is found that the accurate results could be acquired with the maximum error of less than 3.5%, and average lower and upper search space limits are [0.887, 1.123]. The identified best values are regarded as the initial point for LM, SQP and NM methods in the subsequent local search stage.

The number of iterations for LM, SQP, NM and I-Jaya methods are predefined as 60, 50, 5000, 100, respectively. IBOA and Ham-IBOA from Ref. [21] are utilized to compare the performance of the proposed hybrid identification methods. Table 11 provides the identification errors for the steel grid benchmark structure under 0%, 5% and 10% noise levels. In the noise free case, IJASR-SQP obtains the most satisfactory results with small maximum and mean errors of 0.38% and 0.13% among six different methods. When contaminated with 10% noise, better performance is obtained by Ham-IBOA and IJASR-SQP than other four methods, with less than 6.5% and 2.9% of maximum and mean errors. In addition, the computational efficiency is

Table 12
Comparison of computational efficiency for four identification methods (noise free).

Methods	Objective function value	Total evaluations	Computational time (s)
IBOA [21]	472	20,000	10,182.06
Ham-IBOA [21]	709	10,000	5122.74
IJASR-LM	983.46	26,602	2620.96
IJASR-SQP	999.81	26,344	2581.87
IJASR-NM	988.46	29,000	2886.49
IJASR-I-Jaya	970.57	30,000	2995.75

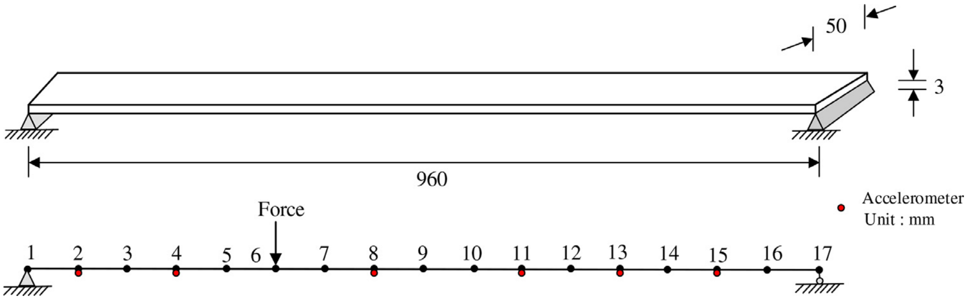

Fig. 14. Numerical model of 16-element simply supported beam.

also studied for IBOA, Ham-IBOA, IJASR-LM, IJASR-SQP, IJASR-NM and IJASR-I-Jaya, as listed in Table 12. It is obviously noticed that global methods, i.e., IBOA and Ham-IBOA takes more computation time than global-local hybrid methods. For two stage methods, the convergence speed of IJASR-I-Jaya is lower than IJASR-LM, IJASR-SQP, IJASR-NM since an initial population is regenerated for the starting value as opposed to the local optimization methods LM, SQP and NM, where a single good initial point is directly utilized from the previous global search stage.

In brief, hybrid global-local methods based on IJASR and local mathematical optimizer can accurately identify the unknown structural parameters with limited noise-polluted measurements. In particular, the proposed IJASR-SQP method can provide better identification results with less computational time than IBOA, IJASR-LM, IJASR-NM, IJASR-I-Jaya. Thus, the performance of proposed global-local identification methods is effectively validated.

6. Application in structural damage identification

Structural damage identification is a fundamental issue of the structural health monitoring, which can be formulated as an optimization problem in which the objective function is defined as the difference between the measured responses and the estimated responses from the finite element model. The inverse identification could be solved by minimizing the objective function using the proposed hybrid global-local identification methods. In this section, the effectiveness of the proposed methods in structural damage identification problem is validated. A numerical model of simply-supported beam is presented in Fig. 14, and its length, width and height are 960 mm, 50 mm and 3 mm, respectively. There are 16 identical elements and 17 nodes in total, resulting in 60 mm length for each element. Euler-Bernoulli beam theory is adopted with negligible shear strain considering the large ratio of length to height. Intermediate nodes (2–16) have two degrees of freedoms of vertical translation and rotation, while only rotation is considered for the boundary nodes. The Young's modulus of steel material is $E = 2.1 \times 10^{11}$ N/m^2 and its density is $\rho = 7850$ kg/m^3. It is assumed that a random white Gaussian noise with zero mean and unit standard deviation is vertically applied at node 6, and six accelerometers at nodes 2, 4, 8, 11, 13 and 15, as highlighted in Fig. 14, are instrumented to record translational acceleration responses for 2 s with a sampling rate of 2000 samples/s.

Damage in the simply supported beam is simulated as reduction in flexural stiffness. The beam width in elements 8 and 12 is reduced to 26 mm and 38 mm from 50 mm. The damage extent of the i th element can be defined as

$$D_i = \frac{EI_{i\_u} - EI_{i\_d}}{EI_{i\_u}} \times 100\% \tag{27}$$

where D_i is the damage extent of element i; EI_{i_u} and EI_{i_d} are the flexural stiffness of the undamaged and damaged element i. The damage extent in element 8 and 12 to be identified are 48% and 24% due to the reduction of element width.

In order to determine the sequences of reducing search limits of unknown parameters, sensitivity analysis is carried out by calculating sensitivity value of recorded accelerations in 2 s with respect to each structural parameter. The min-max normalized sensitivity value for the simply supported beam structure is displayed in Fig. 15. As listed in Table 13, parameters to be identified are divided into three different Parts according to its similar domain of sensitivity [0.4, 1], [0.03, 0.4), [0,

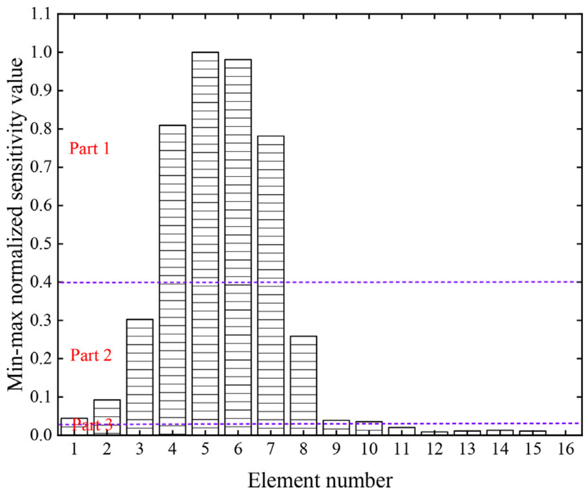

Fig. 15. The min-max normalized sensitivity value of simply supported beam structure.

Table 13
The order to reduce search space limits according to sensitivity value.

Part	Domain of min-max normalized sensitivity	Parameters to be identified	Number of run to reduce limits	Reduce search limits for Part
1	[0.4, 1]	α, β, K_4, K_5, K_6, K_7	6	1
2	[0.03, 0.4)	K_1, K_2, K_3, K_8, K_9, K_{10}	12	1, 2
3	[0, 0.03)	K_{11}, K_{12}, K_{13}, K_{14}, K_{15}, K_{16}	18	1, 2, 3

Table 14
Identification error with IJASR for different run numbers and noise levels (%).

Run number	0% noise		5% noise		10% noise	
	Mean error	Max error	Mean error	Max error	Mean error	Max error
1	15.67	40.74	18.42	41.75	20.48	40.76
7	10.24	32.82	11.86	34.62	13.03	33.74
13	6.45	21.54	7.75	22.40	8.61	23.46
19	2.48	11.41	3.43	13.72	4.09	14.29

0.03). Only search range of elements within Part 1 will be reduced at the 6th run to alleviate the possibility of over-reducing search space limits. Then, the search space limits of unknown parameters within Part 2 and Part 3 are gradually reduced in the order presented in Table 13.

The parameter settings of the proposed IJASR method are given as follows: population size NP = 20, maximum generationsG_m = 30, total runs = 19, R_r=6, R_w=1,R_r − R_w=5, λ=4. The identified results with the proposed IJASR method under 0%, 5% and 10% noise levels are listed in Table 14. As the level of noise increases, larger errors are observed. For the noise free case, in the 1st run, large identification errors are observed since candidate solutions are randomly generated in the initial broad domain, with mean and maximum errors of 15.67% and 40.74%. Subsequently, identification errors are roughly decreased in an iterative way owing to the gradually reduced search limits. In the 19th run, promising results of structural parameters are identified by the proposed IJASR method. However, local optimization methods are needed to further improve identification accuracy considering more than 11% maximum error for three levels of noise.

The identified best value using IJASR is taken as the initial point for LM, SQP and NM methods in the subsequent local search stage. The number of iterations for LM, SQP, NM are set as 50, 50, 5000, respectively. The identified damage extents for 0%, 5% and 10% noise cases are presented in Fig. 16. For IJASR-SQP method, with 0%, 5% and 10% noise contaminated measurements, the damage extents are 48.13%, 46.31% and 47.74% in element 8 and 23.73%, 23.96% and 23.85% in element 12. Similarly, the identified damage extents in elements 8 and 12 with IJASR-LM and IJASR-NM are also very close to the corresponding exact values of 48% and 24%. As shown in Table 15, the maximum identification error is less than 5% even for the 10% noise case, which indicate that the proposed global-local hybrid strategy can accurately locate and quantify structural damages even with noisy measurements.

By the numerical study on a simply supported beam structure, it is concluded that the proposed hybrid algorithms including IJASR-LM, IJASR-SQP, IJASR-NM, could be viewed as effective tools to solve the structural damage identification problem.

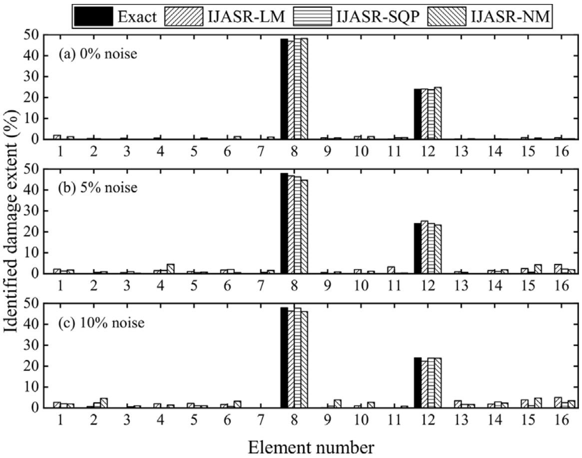

Fig. 16. Identification results for the simply-supported beam: (a) 0% noise; (b) 5% noise; (c) 10% noise.

Table 15
Mean and maximum identification errors for the simply-supported beam (%).

Methods	0% noise		5% noise		10% noise	
	Mean error	Max error	Mean error	Max error	Mean error	Max error
IJASR-LM	0.57	1.93	1.50	4.39	1.74	4.96
IJASR-SQP	0.17	0.73	0.86	2.21	1.01	2.90
IJASR-NM	0.61	1.34	1.52	4.46	2.17	4.73

7. Conclusions

In this paper, a novel hybrid identification method, combining the powerful global exploration capacity of population-based improved Jaya algorithm and strong local exploitation capacity of gradient or non-gradient based methods, for structural health monitoring is proposed and investigated. In one aspect, the original Jaya algorithm is improved by introducing fuzzy clustering competitive learning mechanism, experience learning mechanism and Cauchy mutation mechanism. Besides, with the purpose of reducing computational time on evaluating the candidates far away from the optimal solution, the adaptive search space reduction method is developed to reduce the search space limits of unknown parameters. In the other aspect, three local optimizers LM, SQP and NM are employed in local search stage to identify high-quality potential solution starting from the identified best estimate of IJASR method. In order to validate the feasibility and the effectiveness of the proposed methods, benchmark functions and 37-bar truss structure, as well as an experimental study of steel grid benchmark structure are adopted. Some interesting conclusions can be summarized as follows:

(1) Compared with GA, DE, Jaya, the proposed I-Jaya algorithm can achieve faster convergence speed in classical benchmark functions. The results of CEC benchmarks show improved performance owing to introducing fuzzy clustering competitive learning, experience learning and Cauchy mutation mechanisms.
(2) The numerical and experimental studies demonstrate that the proposed IJASR method can improve the identification accuracy and efficiency for more complex structures based on I-Jaya and adaptive search space reduction method by spending more computational resources on evaluation of the candidates close to promising search space limits.
(3) The hybrid global-local identification strategy, coarsely finding a favorable solution by IJASR method in the global search stage, and then taking it as initial point for local optimization methods in the local search stage, is proved feasible and effective to identify unknown parameters for relatively large-scale and complex structures.
(4) Taking the identification accuracy and computational efficiency into consideration, the proposed IJASR-SQP method can achieve more superior parameter identification results with less computational time than IJASR-LM, IJASR-NM, even with incomplete and 10% noise-polluted measurements.

Structural parameters are successfully identified with the proposed hybrid strategy, while optimal sensor placement and uncertainties, such as modeling error, time-varying environmental condition, boundary stiffness reduction etc., are not considered. These issues would be investigated on more complex and large-scale engineering structures in the future study.

Declaration of Competing Interest

The authors declare no conflict of interest.

Data availability

Data will be made available on request.

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