Transverse Isotropic Criterion Based on Generalized Nonlinear Strength

Zheng Wan[1]; Yangping Yao[2]; Liyu Xie[3]; and Chenchen Song[4]

Abstract: The geotechnical materials formed by natural deposition usually have transverse isotropic properties, and the transverse isotropic properties not only have a direct influence on the deformation of geotechnical materials but also have a nonnegligible influence on their strength characteristics. In order to reasonably consider the influence of transversal isotropic properties on strength properties, the soil's failure properties of complete isotropic and transversal isotropic soils were compared. At the same time, a hypothesis was adopted, that is, at the failure moment, the ratios of principal stresses corresponding to completely isotropic soil and transverse anisotropic soil had linear relationships to each other, and the ratio coefficient β was assumed to be a transverse isotropic parameter. The proposed parameter β was utilized to reflect the influence law of the original anisotropy on the shape of the failure curve on the deviatoric plane. The proposed parameters were employed to reflect the influence law of the original anisotropy on the shape of the failure curve on the deviatoric plane. For the relationship between the isotropic criterion and the transversal isotropic criterion, a kind of β transformation method is proposed, which can realize the transformation of isotropic stress space and transversal isotropic stress space. Based on the isotropic generalized nonlinear strength criterion (GNSC), the method of determining the transverse isotropic parameter β is given by making use of the condition that the shape parameters $\bar{\alpha}$ and α of GNSC in the isotropic stress space and the transverse isotropic stress space, respectively, are exactly the same. By comparing the strength test data under different Lode angles with the predicted results of the generalized nonlinear criterion of transverse anisotropy, the results showed that the proposed anisotropy criterion can simply and accurately reflect the failure law of the geotechnical materials under the true triaxial loading condition. **DOI: 10.1061/(ASCE)GM.1943-5622.0002084**. *© 2021 American Society of Civil Engineers.*

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Introduction

There are various kinds of engineering materials widely distributed in natural sites, all of which have different degrees of anisotropy, for example, wood, bamboo, rock, clay, sand, and so on. The profile of these materials will form regular textures, joints, or cracks due to the orientation of the internal components. These macroscopic directional joints usually have a significant influence on the loading results in space. In general, high shear strength will be formed when major principal stress is vertically applied to the joint surface, while low shear strength will be formed when major principal stress is parallel to the joint surface. Different from other cohesive materials, geotechnical materials are typical granular materials, which not only have obvious natural anisotropy properties but also have obvious stress-induced anisotropy. In other words, for isotropic soils, different generalized deviatoric stress strengths will be generated due to different intermediate principal stress coefficients. Casagrande and Carillo (1944) distinguished the original anisotropy and secondary anisotropy, defining the original anisotropy as "the intrinsic physical property of the material, and completely independent of the additional strain" and the secondary anisotropy as "the physical properties only related to the strain caused by additional stresses." For geomaterials, due to the deposition of gravity, the long axis of soil particles is usually parallel to the horizontal sedimentary surface at the mesoscopic level, while the direction of the short axis is perpendicular to the sedimentary surface. Moreover, due to the random distribution of the long axis of particles in the horizontal sedimentary plane, the characteristics of the soil material are transversely isotropic at the macro level. A large number of test results show that the shear strength of the loading direction parallel to the deposition plane is significantly smaller than that perpendicular to the deposition plane. However, the existing commonly employed strength criteria fail to take into account the effect of the original anisotropy on the strength reasonably. If the isotropic strength criteria are used in engineering design, the strength design value will be overestimated, which will cause great hidden trouble. To solve this problem, the effect of the original anisotropy property on shear strength needs to be taken into account, and it is a common practice to suggest a strength criterion taking into account the effect of original anisotropy. For the strength characteristics of transversely isotropic materials, the research on this aspect has been a focus and a difficult field till today. Generally, there are several methods for studying transverse isotropy of geomaterials at present, namely, (1) joint stress invariants method (Dafalias et al. 2004; Dafalias and Taiebat 2014; Dafalias 2016; Li and Dafalias 2002), (2) fabric tensor method (Kong et al. 2013; Oda 1972, 1981; Oda and Nakayama 1989; Oda et al. 1978, 1998; Ochiai and Lade 1983;

[1]Associate Researcher, Architectural Engineering College, North China Institute of Science and Technology, Hebei Langfang 065201, China (corresponding author). Email: zhengw111@126.com

[2]Professor, Dept. of Civil Engineering, Beihang Univ., Beijing 100191, China.

[3]Associate Professor, Dept. of Disaster Mitigation for Structure, Tongji Univ., Shanghai 200092, China.

[4]Engineer, China State Construction Engineering Corporation, China Construction Infrastructure Co., Ltd., Beijing 100013, China.

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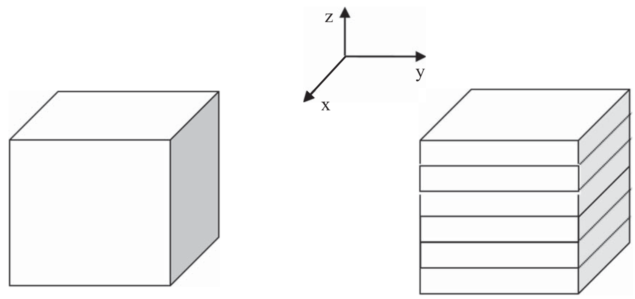
Lü et al. 2016); (3) classical strength criterion revising method (Pietruszczak et al. 1993; Lade and Musante 1978; Lade 2008; Lade et al. 2014; Matsuoka et al. 1980, 1999; Nakai and Matsuoka 1983; Nakai and Mihara 1984; Chowdhury and Nakai 1998; Mortara 2010; Liu and Indraratna 2011), (4) a modified method of the strength criterion based on large principal stress (Gao et al. 2010; Gao and Zhao 2012; Yang et al. 2016; Mirghasemi and Naeij 2015), and (5) a criterion revising method based on the physical significance of the failure surface (Yao et al. 2004; Yao and Zhou 2013; Tian and Yao 2018; Lu et al. 2017, 2019). Although these methods can be employed to consider some influence of original anisotropy on the strength criterion to a certain degree, they still have advantages and disadvantages. Dafalias et al. adopted the method of joint stress invariants, in which the transverse isotropic properties of soil materials were characterized by mesoscopic fabric tensors and the stress tensors and fabric tensors were formed into new joint stress invariants through some operation, so the constitutive model and strength criterion were established by using the joint stress invariants. Although the aforementioned method considers the effect of transverse isotropy on the stress–strain relationship to some extent, its essence is still a modification of the traditional stress. Moreover, this correction method still lacks theoretical basis and mechanism explanation. The construction method of structural stress joint invariants is still determined subjectively, and the correctness of the construction method is yet to be proved. The fabric tensor law is represented by Oda. Starting from the microscopic particles, some empirical formulas are obtained by analyzing the interactions between the particles and statistical means. The method tries to obtain a method to describe the macroscopic anisotropy through the interaction of mesoscopic particles. This method relies heavily on advanced test methods to determine the soil parameters that cannot be inferred empirically, so there are many objective limitations for its application. The classical strength criterion modification method is to introduce parameters reflecting anisotropy to modify common criteria in the past to obtain a uniform criterion reflecting anisotropy. However, the aforementioned introduction of anisotropy parameters is only used to determine the strength test results under a certain path and usually only has the description applicability of failure properties under some specific paths. For the failure conditions under various stress paths, the universality cannot be achieved. It is a common practice, although simple and direct, to use the angle between large principal stress and the deposited surface. However, in fact, under the plane strain condition, the strength value does not have a monotonic relationship with the aforementioned included angle but will form a "V"-shaped curve. Therefore, by taking the angle between the large principal stress and the composition of the sedimentary plane as the reference variable, the strength value is not unique. Based on the failure surface with physical meaning, the mechanism of material failure can be explained from the physical concept, which is helpful to understand the process of material failure and its formation reasons. However, some of the physical concepts proposed are strongly subjective conjectures, which relies heavily on the intuitive ability of the proposer, and therefore, the aforementioned way to propose concepts lacks rigorous proof. The generalized nonlinear strength criterion is a kind of strength criterion that is applicable to describe the wide applicability from metals to geotechnical materials. The linear interpolation form of the von Mises criterion and spatial mobilized plane (SMP) criterion was used in the deviatoric plane to describe all kinds of destruction curves from circular to curved triangles. In the meridian plane, the power function was used to describe the nonlinear failure properties such as the hydrostatic pressure effect on the meridian plane. However, the GNSC previously described apply only to completely isotropic materials. It cannot be directly applied to describe the failure characteristics of natural geotechnical materials. Therefore, it is a natural choice to extend the GNSC to a criterion that can describe the failure characteristics influenced by anisotropy. Based on the GNSC, this paper attempted to establish the transformation process of the aforementioned two spaces by using the hypothesis that the ratio of the two principal stresses in the isotropic stress space and that of the transverse isotropic stress space is a linear relationship. The aforementioned linear proportional coefficient β is used as a new parameter to reflect transverse isotropy. Then, it is assumed that the shape parameter $\bar{\alpha}$ of the GNSC in the isotropic space is equal to the shape parameter α in the transverse isotropic stress space to determine the new parameter β. The reflection of the criterion on anisotropy properties is further explained by a series of analyses on the failure characteristics of geotechnical materials.

Comparison of Transverse Isotropy versus Isotropy

For some fully isotropic properties of materials, for example, engineering materials such as metal, glass, the aforementioned isotropic material of a unit cell, the mechanical behavior of its different directions is exactly the same and the modulus of elasticity, strength, and deformation are exactly the same along the x-, y-, z-directions. What we learn from the aforementioned phenomena is that a fully isotropic material has the characteristic of having nothing to do with the loading direction, namely, the loading direction of independence. At the same time, there are kinds of transverse isotropic materials, such as sedimentary rocks, stratified distribution of clay or sand, wood, and other materials, that are typical transverse isotropic materials. The properties are the same when the loading direction is employed within a plane; meanwhile, the properties have different characteristics when the loading direction is employed along the vertical direction of the deposition plane. Elastic modulus values are of major difference between the texture direction and those perpendicular to the texture direction. The same is true for compressive strength. For stratified clay, the modulus and strength are usually the same in the horizontal direction, while the modulus and strength are usually higher in the vertical direction perpendicular to the deposition surface. For orthotropic and completely anisotropic materials, the deformation of orthotropic materials in the direction of mutual vertical loading is noncoupled; that is, the normal stress only causes the normal strain but not the shear strain, while the shear stress only causes the shear strain but not the normal strain. While completely anisotropic materials have the characteristics related to the loading direction, deformation in the vertical loading direction has coupling characteristics, that is, the normal stress causes both the normal strain and the shear strain, while the shear stress produces both the shear strain and the normal strain. They have the coupling property of the loading direction. In this paper, only transverse isotropy was considered. Transverse isotropy is a special case of three-dimensional orthogonal anisotropy with identical physical properties in two directions, while complete isotropy is a special case of three-dimensional orthogonal anisotropy with identical properties in three directions. In order to compare the differences between completely isotropic and transversely isotropic stresses, the constitutive equation differences of elastic materials under the influence of the aforementioned two characteristics are analyzed, and their strength descriptions are used as analogical analysis.

As can be seen from Fig. 1, for pure elastic materials, Fig. 1(a) is a completely isotropic material, while Fig. 1(b) is a transversely isotropic material. According to the elastic theory, the elastic



(a) (b)

Fig. 1. (a) Isotropic material and (b) transverse isotropic material.

constitutive relation of completely isotropic elastic materials can be expressed as follows:

$$\begin{bmatrix}\sigma_{11}\\\sigma_{22}\\\sigma_{33}\\\sigma_{12}\\\sigma_{23}\\\sigma_{31}\end{bmatrix}=\begin{bmatrix}c_{11}&c_{12}&c_{12}&0&0&0\\c_{12}&c_{11}&c_{12}&0&0&0\\c_{12}&c_{12}&c_{11}&0&0&0\\0&0&0&c_{44}&0&0\\0&0&0&0&c_{44}&0\\0&0&0&0&0&c_{44}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\\varepsilon_{22}\\\varepsilon_{33}\\2\varepsilon_{12}\\2\varepsilon_{23}\\2\varepsilon_{31}\end{bmatrix}\qquad(1)$$

For transversely isotropic elastic materials, the elastic constitutive relation can be expressed as follows:

$$\begin{bmatrix}\sigma_{11}\\\sigma_{22}\\\sigma_{33}\\\sigma_{12}\\\sigma_{23}\\\sigma_{31}\end{bmatrix}=\begin{bmatrix}d_{11}&d_{12}&d_{13}&0&0&0\\d_{12}&d_{11}&d_{13}&0&0&0\\d_{13}&d_{13}&d_{33}&0&0&0\\0&0&0&d_{44}&0&0\\0&0&0&0&d_{66}&0\\0&0&0&0&0&d_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\\varepsilon_{22}\\\varepsilon_{33}\\2\varepsilon_{12}\\2\varepsilon_{23}\\2\varepsilon_{31}\end{bmatrix}\qquad(2)$$

Suppose the following relation exists:

$$\begin{cases}d_{33}=k_1d_{11}\\d_{13}=k_2d_{12}\\d_{44}=k_3d_{66}\end{cases}\qquad(3)$$

Obviously, the relationship between isotropic and transversely isotropic elastic stiffness matrix can be established, which can be expressed as follows:

$$\begin{bmatrix}d_{11}&d_{12}&d_{13}&0&0&0\\d_{12}&d_{11}&d_{13}&0&0&0\\d_{13}&d_{13}&d_{33}&0&0&0\\0&0&0&d_{44}&0&0\\0&0&0&0&d_{66}&0\\0&0&0&0&0&d_{66}\end{bmatrix}=\begin{bmatrix}d_{11}&d_{12}&d_{12}&0&0&0\\d_{12}&d_{11}&d_{12}&0&0&0\\d_{12}&d_{12}&d_{11}&0&0&0\\0&0&0&d_{66}&0&0\\0&0&0&0&d_{66}&0\\0&0&0&0&0&d_{66}\end{bmatrix}+\begin{bmatrix}0&0&(k_2-1)d_{12}&0&0&0\\0&0&(k_2-1)d_{12}&0&0&0\\(k_2-1)d_{12}&(k_2-1)d_{12}&(k_1-1)d_{11}&0&0&0\\0&0&0&(k_3-1)d_{66}&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\qquad(4)$$

Obviously, isotropic elastic modulus can be expressed by transverse isotropic modulus and can be expressed by Eq. (4):

$$\begin{bmatrix}d_{11}&d_{12}&d_{12}&0&0&0\\d_{12}&d_{11}&d_{12}&0&0&0\\d_{12}&d_{12}&d_{11}&0&0&0\\0&0&0&d_{66}&0&0\\0&0&0&0&d_{66}&0\\0&0&0&0&0&d_{66}\end{bmatrix}=\begin{bmatrix}d_{11}&d_{12}&d_{13}&0&0&0\\d_{12}&d_{11}&d_{13}&0&0&0\\d_{13}&d_{13}&d_{33}&0&0&0\\0&0&0&d_{44}&0&0\\0&0&0&0&d_{66}&0\\0&0&0&0&0&d_{66}\end{bmatrix}-\begin{bmatrix}0&0&(k_2-1)d_{12}&0&0&0\\0&0&(k_2-1)d_{12}&0&0&0\\(k_2-1)d_{12}&(k_2-1)d_{12}&(k_1-1)d_{11}&0&0&0\\0&0&0&(k_3-1)d_{66}&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\qquad(5)$$

Assuming that the transformed stress space is a completely isotropic stress state, the isotropic stress space can be expressed by the transformed stress space:

$$\begin{bmatrix}\tilde{d}_{11}&\tilde{d}_{12}&\tilde{d}_{12}&0&0&0\\\tilde{d}_{12}&\tilde{d}_{11}&\tilde{d}_{12}&0&0&0\\\tilde{d}_{12}&\tilde{d}_{12}&\tilde{d}_{11}&0&0&0\\0&0&0&\tilde{d}_{66}&0&0\\0&0&0&0&\tilde{d}_{66}&0\\0&0&0&0&0&\tilde{d}_{66}\end{bmatrix}=\begin{bmatrix}d_{11}&d_{12}&d_{13}&0&0&0\\d_{12}&d_{11}&d_{13}&0&0&0\\d_{13}&d_{13}&d_{33}&0&0&0\\0&0&0&d_{44}&0&0\\0&0&0&0&d_{66}&0\\0&0&0&0&0&d_{66}\end{bmatrix}-\begin{bmatrix}0&0&(k_2-1)d_{12}&0&0&0\\0&0&(k_2-1)d_{12}&0&0&0\\(k_2-1)d_{12}&(k_2-1)d_{12}&(k_1-1)d_{11}&0&0&0\\0&0&0&(k_3-1)d_{66}&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\qquad(6)$$

According to Eq. (6), a transformation relation of the elastic modulus matrix of transverse isotropic stress space to that of transition stress space is obtained.

Obviously, the transverse isotropic elastic stiffness matrix can be expressed by the completely isotropic elastic stiffness matrix. Parameters K1, K2 and K3 can be determined by experiments. The aforementioned is the relationship between the moduli of transverse isotropy and complete isotropy. According to the aforementioned ideas, it is obvious that the stress at the moment of failure is similar to that of transverse isotropic and complete isotropic materials.

The strength criterion is a set of mathematical expressions to describe the stress state points at the moment of material failure. For generally completely isotropic materials, its general analytical expression can be expressed as follows:

$$f(\sigma_{ij},\xi_i)=0\qquad(7)$$

where σ_{ij} = stress tensor; and ξ_i = failure constant determined by tests.

According to the completely isotropic material in Fig. 1, the uniaxial compressive strength of the material unit is equal along the three directions, that is, $f_x=f_y=f_z$, while for the transversely isotropic material, the compressive strength of the three directions has the following relationship:

$$f_x = f_y \neq f_z \tag{8}$$

From the aforementioned equation, it can be seen that the compressive strength along the z-direction is different from the strength along the x- or y-direction. Therefore, the strength ratio of the two can be set as a constant K, and the relationship between the two can be determined:

$$f_z = kf_x = kf_y \tag{9}$$

If there is a transformed stress space, which belongs to a completely isotropic stress space, it is obvious that the compressive strength in all directions is exactly the same. Then, it can be known that in the transformed stress space, $\tilde{f}_x = \tilde{f}_y = \tilde{f}_z$, in which the simplest transformation formula of the transverse isotropic stress space to the completely isotropic stress space can be given:

$$\begin{cases} \tilde{f}_x = kf_x \\ \tilde{f}_y = kf_y \\ \tilde{f}_z = f_z \end{cases} \tag{10}$$

Obviously, the transverse isotropic parameter K can be determined through two compressive strength experiments on transverse isotropic materials in the x- and z-directions, and then the transformed stress space can be obtained through the aforementioned transformation relations. The transformed stress space corresponds to a completely isotropic stress space. Therefore, the existing relatively mature and classical isotropic strength criterion formulas, such as the typical Mohr–Coulomb criterion, can be adopted in the transformed stress space:

$$\tilde{f}_x = \tilde{c} + \tilde{\sigma} \tan \tilde{\varphi} \tag{11}$$

Then, the transformation formula that takes the transversely isotropic properties into account is combined with the Mohr–Coulomb criterion formula in the transformed stress space, and finally the Mohr–Coulomb strength criterion that takes the transversely isotropic properties into account can be obtained. The idea of this paper is to find the transformation relation from the transversely isotropic stress space to the transition stress space and then combine with the existing GNSC based on the isotropic stress space and finally obtain the generalized nonlinear strength criterion that can reflect transversely isotropic properties.

Generalized Nonlinear Strength Criterion

Based on the experimental law of friction materials and previous research results, the GNSC (Yao et al. 2004) is proposed for different secondary anisotropic materials. It uses an expression to uniformly describe the nonlinear strength characteristics of the material on the deviatoric plane and the meridian plane. There are four material parameters, all of which have definite physical meaning and can be determined by simple experiments. The failure function of the GNSC in the deviatoric plane is a linear interpolation of the spatial mobilized plane (SMP) criterion and the von Mises criterion.

1. The failure curve of the von Mises criterion on the deviatoric plane is circular. The expression can be written as follows:

$$q_M^* = \sqrt{I_1^2 - 3I_2} \tag{12}$$

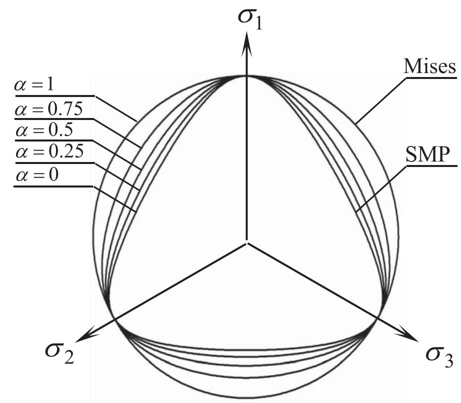


Fig. 2. Strength curve of the GNST in the deviatoric plane.

2. The failure curve of the SMP criterion on the deviatoric plane can be expressed as follows:

$$q_s^* = \frac{2I_1}{3\sqrt{(I_1I_2 - I_3)/(I_1I_2 - 9I_3)} - 1} \tag{13}$$

3. The expression of the GNSC on the deviatoric plane is as follows (Yao et al. 2004):

$$q_\alpha^* = \alpha q_M^* + (1 - \alpha) q_S^* \tag{14}$$

In the expression, $I_1 = \sigma_1 + \sigma_2 + \sigma_3$; $I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$; $I_3 = \sigma_1\sigma_2\sigma_3$; and α = material constant.

α is the material parameter of the GNSC criterion, and different strength criteria are employed through the change of α. As shown in Fig. 2, $\alpha = 0$ represents the SMP criterion, and $\alpha = 1$ represents the von Mises criterion. In fact, parameter α represents the shape factor in the deviatoric plane of the GNSC (Yao et al. 2004).

If used as a yield criterion, it is made of two parts: On the deviatoric plane, the yield curve is interpolated by the SMP criterion yield curve and the von Mises criterion yield curve, while the shape of the GNSC yield curve is a curved triangle between the aforementioned two criterion curves. On the meridian plane, the GNSC expression is a power function form, and the open curve yield surface is adopted to reflect the shear yielding property of geomaterial.

Based on this, GNSC reflects the failure characteristics of materials under generalized deviatoric stress. The shape on the deviatoric plane reflects the weight of cohesion and friction in the final failure stress state. If cohesion is dominant, it reflects the cohesive materials represented by metallic materials. On the contrary, if friction is dominant, it reflects the discrete frictional material represented by sand material.

On the meridian plane, the expression of the GNSC can be written as an open power function:

$$\bar{q}_\alpha^* = M_f \bar{p} \tag{15}$$

Thereinto, the mean stress in the transition space is expressed as follows (Yao et al. 2004):

$$\bar{p} = p_r \left(\frac{p + \sigma_0}{p_r} \right)^n \tag{16}$$

where p and σ_0 = effective mean stress and true triaxial extension strength, respectively; and p_r is a reference mean stress representing under this stress value. The uniqueness of the secant slope of the failure curve on the meridian surface that can be guaranteed by the ratio expression of Eq. (16), so that the left and right hand sides of Eq. (5) have the same dimension. For granular materials, p_r is usually employed as a standard atmospheric pressure value.

On the deviatoric plane, the generalized deviatoric stress can be expressed as follows:

$$\bar{q}_\alpha^* = \alpha\sqrt{\bar{I}_1^2 - 3\bar{I}_2} + \frac{2(1-\alpha)\bar{I}_1}{3\sqrt{(\bar{I}_1\bar{I}_2 - \bar{I}_3)/(\bar{I}_1\bar{I}_2 - 9\bar{I}_3)} - 1} \tag{17}$$

Thereinto, the stress tensor in transition space is expressed as follows (Yao et al. 2004):

$$\left.\begin{aligned} \bar{I}_1 &= \bar{\sigma}_1 + \bar{\sigma}_2 + \bar{\sigma}_3 \\ \bar{I}_2 &= \bar{\sigma}_1\bar{\sigma}_2 + \bar{\sigma}_2\bar{\sigma}_3 + \bar{\sigma}_3\bar{\sigma}_1 \\ \bar{I}_3 &= \bar{\sigma}_1\bar{\sigma}_2\bar{\sigma}_3 \end{aligned}\right\} \tag{18}$$

$$\bar{\sigma}_i = \sigma_i + \left[p_{\mathrm{r}}\left(\frac{p+\sigma_0}{p_{\mathrm{r}}}\right)^n - p\right] \quad (n \in [0, 1], i = 1, 2, 3) \tag{19}$$

The stress space corresponding to stress tensor with a bar is a transition stress space. The transition stress space and the ordinary stress space only correspond to the transformation of the mean stress, and the strength line in the power function curve of the ordinary stress space in the meridian surface, through the transformation of the stress, is expressed in the form of the linear form in the transition stress space. The failure surfaces are shown in Fig. 3. Fig. 3(a) shows a failure curve in the ordinary stress space, and Fig. 3(b) shows a failure curve in the transition stress space for the GNSC (Yao et al. 2004). Obviously, the bending part of the failure curve in the ordinary stress space is transformed to the straight line in the transition stress space.

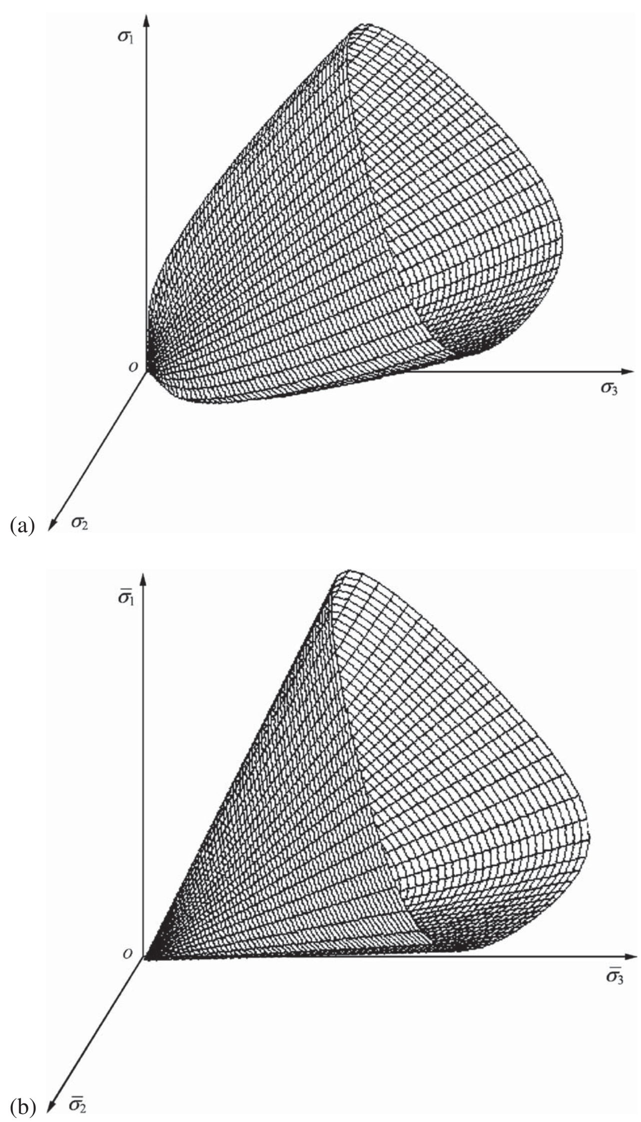


Fig. 3. (a) Yield surface in ordinary stress space; and (b) yield surface in transitional stress space.

Physical Meaning and Determination Method of Parameters

1. Meaning and determination of α

On the deviatoric plane, the expression of the yield criterion is the same as the expression of the strength criterion, so in Eq. (17), α = weighted value of the triangle between the von Mises circle and the SMP curve in the deviatoric plane. As shown in Fig. 4, the variation of α from 1 to 0 represents the yield shape between the von Mises circle and the SMP curved triangle for the GNSC (Yao et al. 2004), which describes the failure yield characteristics of a series of materials, such as metal, concrete, noncohesive soil, and so on. On the same deviatoric plane, if the ratio between the triaxial extension deviatoric stress q_e and the triaxial compression deviatoric stress q_c is r, which can be used to represents α, there is the following relationship:

$$\alpha = \frac{r(3+M^*) - 3}{r^2M^*} \tag{20}$$

If triaxial compression and triaxial extension tests are performed, the internal friction angle under the aforementioned two paths can be obtained, and Eq. (20) can be expressed by using the two friction angles as follows:

$$\alpha = \frac{3(3+\sin\varphi_e)(\sin\varphi_e - \sin\varphi_c)}{2\sin^2\varphi_e(3-\sin\varphi_c)} \tag{21}$$

where $\sin\varphi_e$ and $\sin\varphi_c$ = sine values of the internal friction angle of triaxial extension and triaxial compression, respectively.

When $r = 1$ and $\alpha = 1$, the proposed criterion reduces to the von Mises strength criterion; when $r = 3/(3+M_f)$ and $\alpha = 0$ the

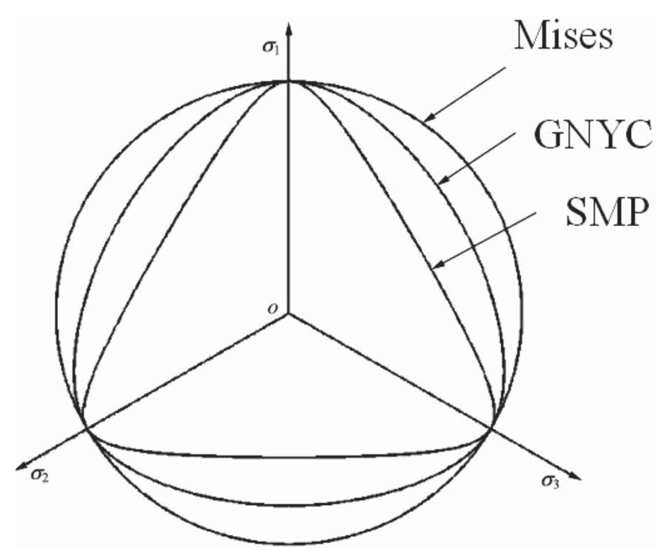


Fig. 4. Yield curves on the deviatoric plane.

proposed criterion reduces to the SMP strength criterion. Therefore, parameter α can be determined by triaxial compression and triaxial extension value.

2. Meaning and determination of M_f

In the original GNSC, M_f represents the secant slope of the strength curve under reference stress p_r. When the GNYC is used as the yield criterion, as shown in Fig. 5, the physical meaning of M_f remains unchanged.

3. Meaning and determination of n

n is the power parameter of the power function, which indicates the bending degree of the failure curve on the meridian surface. As shown in Fig. 5, when $n = 0$, the failure curve degenerates into a straight line independent of the hydrostatic pressure and parallel to the hydrostatic pressure axis. When $n = 1$, the power function curve becomes a diagonal line passing through the origin; when $0 < n < 1$, the power function curve is an open curve between the two straight lines aforementioned for the GNSC (Yao et al. 2004).

Combining Eqs. (15) and (16), and the following expression can be obtained:

$$\ln \frac{\bar{q}_\alpha^*}{p_r} = n \ln \frac{p + \sigma_0}{p_r} + \ln M_f \tag{22}$$

The results of the triaxial compression test data can be arranged in the double log coordinate system. Obviously, it can be shown as a linear-type fitting straight line of which the slope is n, while the intercept value is ln M_f.

4. Meaning and determination of σ_0

σ_0 is the left intersection point value of the strength curve and the hydrostatic pressure axis. The physical meaning of it indicates that the material can bear the extension load within a certain limit. This parameter can reflect the cohesion of the material (not cohesive force). The failure curves that are influenced by parameter σ_0 are shown in Fig. 6. The physical meaning of σ_0 is the extension strength for the GNSC (Yao et al. 2004).

Parameter σ_0 is the true triaxial extension strength of the material, and it is difficult to achieve the general true triaxial extension strength in real tests. For noncohesive soil, it is 0. For cohesive materials, such as concrete, according to the research results of Pietruszczak et al. (2002), we can use the following formula to obtain the value of σ_0:

$$\sigma_0 = f_{ttt} = 0.9 f_t = 0.09 f_c \tag{23}$$

where f_{ttt} = true triaxial extension strength; f_t = biaxial extension strength; and f_c = uniaxial compression strength.

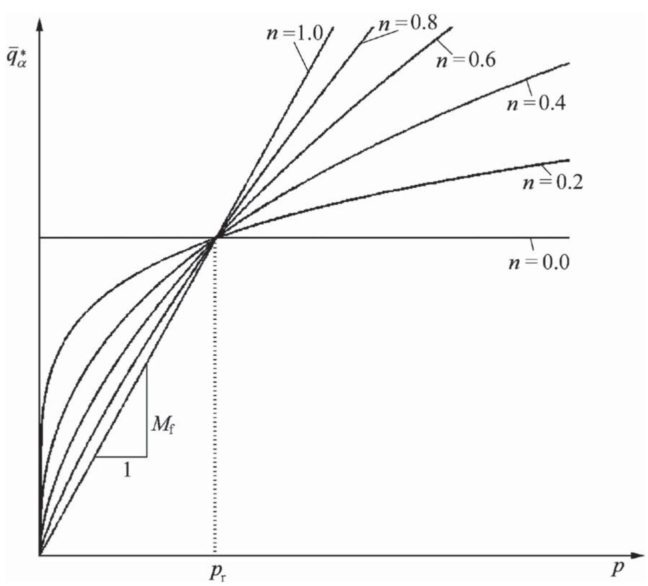


Fig. 5. Strength curves on the meridian plane.

Anisotropic Transformation Stress Method for Materials

It is assumed that the soil is orthotropic, that is, isotropic in the sedimentary plane. Only the vertical direction is different from that of the sedimentary surface, which is anisotropic. Finally, the orthotropic soil is transformed into an equivalent isotropic soil by a linear transformation. As shown in Fig. 7, the major principal stresses of the anisotropic soil are $\bar{\sigma}_X$, $\bar{\sigma}_Y$, $\bar{\sigma}_Z$ respectively, and the corresponding equivalent isotropic soil's principal stresses are σ_X, σ_Y, σ_Z, respectively. The major principal stress of the equivalent isotropic soil is coaxial with the principal stress of the anisotropic soil. The X–Y plane is parallel to the sedimentary surface, while the Z-axis is perpendicular to the sedimentary surface. β_1 and β_2 are the anisotropy parameters corresponding to X- and Y-directions respectively. It is assumed that the relationship between anisotropic soil and equivalent isotropic soil is shown as follows:

$$\frac{\sigma_Z}{\sigma_X} = \beta_1 \frac{\bar{\sigma}_Z}{\bar{\sigma}_X}, \quad \frac{\sigma_Z}{\sigma_Y} = \beta_2 \frac{\bar{\sigma}_Z}{\bar{\sigma}_Y} \tag{24}$$

For the original anisotropic soil, the $\beta_1 = \beta_2 = \beta$ can be obtained because of the axisymmetric characteristics in the direction parallel to the sedimentary surface. In this way, we can express $\bar{\sigma}_X$, $\bar{\sigma}_Y$, $\bar{\sigma}_Z$ by using σ_X, σ_Y, σ_Z, which is given as follows:

$$\left.\begin{aligned} \bar{\sigma}_X &= (\bar{\sigma}_Z/\sigma_Z)\cdot(\beta\sigma_X) \\ \bar{\sigma}_Y &= (\bar{\sigma}_Z/\sigma_Z)\cdot(\beta\sigma_Y) \\ \bar{\sigma}_Z &= (\bar{\sigma}_Z/\sigma_Z)\cdot\sigma_Z \end{aligned}\right\} \tag{25}$$

It is assumed that $p = \bar{p}$, so

$$\sigma_X + \sigma_Y + \sigma_Z = (\bar{\sigma}_Z/\sigma_Z)\cdot(\beta\sigma_X + \beta\sigma_Y + \sigma_Z) \tag{26}$$

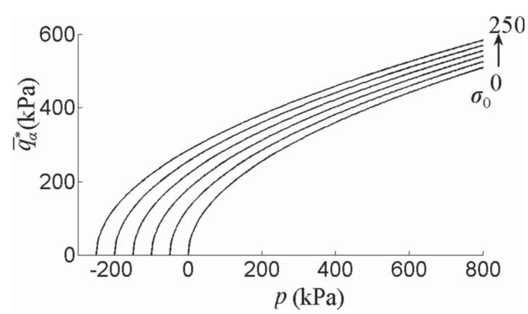


Fig. 6. Influence of cohesion on failure curves on the meridian plane.

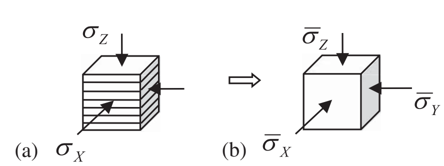


Fig. 7. Transformed stress method for (a) anisotropic soil; and (b) isotropic soil.

$$\frac{\bar{\sigma}_Z}{\sigma_Z} = \frac{\sigma_X + \sigma_Y + \sigma_Z}{\beta(\sigma_X + \sigma_Y) + \sigma_Z} \tag{27}$$

According to Eq. (27), the three orthogonal direction stresses in the equivalent isotropic space can be expressed by the three orthogonal stresses in the general anisotropic space. According to anisotropic parameter β, the normal stresses of equivalent isotropic soil $\bar{\sigma}_X$, $\bar{\sigma}_Y$, $\bar{\sigma}_Z$ can be expressed as follows:

$$\left.\begin{aligned} \bar{\sigma}_X &= \frac{\sigma_X + \sigma_Y + \sigma_Z}{\beta(\sigma_X + \sigma_Y) + \sigma_Z} \cdot (\beta\sigma_X) \\ \bar{\sigma}_Y &= \frac{\sigma_X + \sigma_Y + \sigma_Z}{\beta(\sigma_X + \sigma_Y) + \sigma_Z} \cdot (\beta\sigma_Y) \\ \bar{\sigma}_Z &= \frac{\sigma_X + \sigma_Y + \sigma_Z}{\beta(\sigma_X + \sigma_Y) + \sigma_Z} \cdot \sigma_Z \end{aligned}\right\} \tag{28}$$

In the equivalent isotropic stress space, there is also a generalized nonlinear strength criterion that reflects the failure characteristics of fully isotropic stress. The GNSC can be expressed as follows:

$$\bar{q}_\alpha^* = \alpha\sqrt{\bar{I}_1^2 - 3\bar{I}_2} + \frac{2(1-\alpha)\bar{I}_1}{3\sqrt{(\bar{I}_1\bar{I}_2 - \bar{I}_3)/(\bar{I}_1\bar{I}_2 - 9\bar{I}_3)} - 1} \tag{29}$$

This transformation is named as β transformation. Through Eq. (28), the principal stress mapping relationship between the transversely isotropic stress space and the fully isotropic stress space can be established. It can be seen that in the transversely isotropic stress space the failure surface can be completely transformed into the isotropic failure surface by using β transformation, and the corresponding anisotropic soil can be treated as equivalent isotropic soil. Through this transformation, the anisotropy property can be linked with the various isotropic strength criteria to better simulate the behavior of the soil, and the determination of parameter β in this transformation is simple. Only the results of conventional triaxial compression and conventional extension tests are needed.

Parameter Determination Method for Generalized Nonlinear Strength Criterion Considering Anisotropy

A method for determining the anisotropy parameter β of transversely isotropic soils is introduced in the following. The idea is displayed as follows. Using the one-to-one mapping relation with the aforementioned transformation formula between the transversely isotropic failure curve and the completely isotropic failure curve mentioned in the previous section, β transformation can be employed for the failure curve in the general stress space, then the failure curves obtained in the transformed stress space should be exactly the same as those in the complete isotropic stress space. The failure curve of isotropic stress space coincides with that of the transformed stress space; it is obvious that the shape parameter $\bar{\alpha}$ that determines the shape of the curve in the deviatoric plane should be completely equal to shape parameter α of the failure curve in the isotropic stress space. Through the aforementioned principle of shape parameter equation $\bar{\alpha} = \alpha$, the corresponding anisotropic parameter β can be solved.

As shown in Fig. 8, point c represents the triaxial compression test point, point i represents the triaxial extension test point of isotropy, and point a represents the triaxial extension test point of anisotropy. From the stress state, it can be seen that the triaxial compression point c and the triaxial extension point i should be on the same isotropic strength curve, which is called α line. While the triaxial compression point c and the triaxial extension point a are not on the same isotropic strength curve due to the influence of the original anisotropy, they need to be transformed to eliminate the influence of original anisotropy, so that the aforementioned three points a, c and i should be displayed on one isotropic strength curve. The stress ratios under three-dimensional stress states are as listed as follows:

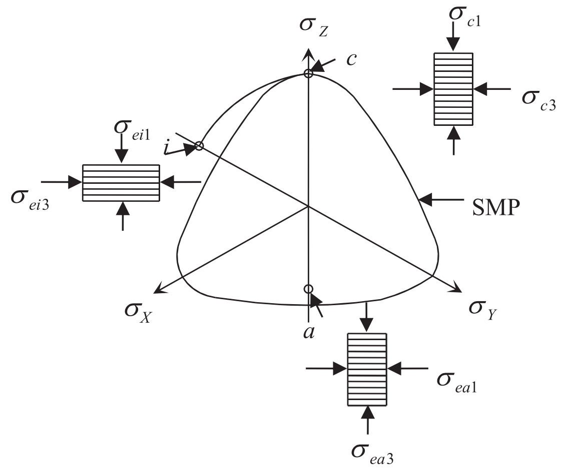


Fig. 8. Illustration of the strength theory in the real stress space.

$$R_c = \frac{\sigma_{c1}}{\sigma_{c3}},\ R_{ea} = \frac{\sigma_{ea1}}{\sigma_{ea3}},\ R_{ei} = \frac{\sigma_{ei1}}{\sigma_{ei3}} \tag{30}$$

The corresponding internal friction angles are φ_c, φ_{ea}, φ_{ei}, and the representation method in the transformation stress space are as follows.

$$\sin\bar{\varphi}_c = \frac{\sigma_{c1} - \beta\sigma_{c3}}{\sigma_{c1} + \beta\sigma_{c3}} = \frac{R_c - \beta}{R_c + \beta} \tag{31}$$

$$\sin\bar{\varphi}_{ea} = \frac{\beta\sigma_{ea1} - \sigma_{ea3}}{\beta\sigma_{ea1} + \sigma_{ea3}} = \frac{\beta R_{ea} - 1}{\beta R_{ea} + 1} \tag{32}$$

According to the stress state shown in Fig. 8, the shape parameters are obtained according to Eq. (21)

$$\alpha = \frac{3(3 + \sin\varphi_{ei})(\sin\varphi_{ei} - \sin\varphi_c)}{2\sin^2\varphi_{ei}(3 - \sin\varphi_c)} \tag{33}$$

Triaxial compression point c and triaxial extension point a are affected by original anisotropy. On an anisotropic strength curve, the anisotropic strength curve is transformed into an equivalent isotropic strength curve by transformation, and shape parameter $\bar{\alpha}$ should be equal to shape parameter α after transformation.

Therefore, the stresses of triaxial compression point c and triaxial extension point a are transformed according to the Eq. (28) and $\bar{\alpha}$ can be obtained as follows:

$$\bar{\alpha} = \frac{3(3 + \sin\bar{\varphi}_{ea})(\sin\bar{\varphi}_{ea} - \sin\bar{\varphi}_c)}{2\sin^2\bar{\varphi}_{ea}(3 - \sin\bar{\varphi}_c)} \tag{34}$$

It is assumed that $\bar{\alpha} = \alpha$, and the formula can be simplified by substituting (31), (32)–(34).

$$\beta^3 + d\beta^2 + e\beta + f = 0 \tag{35}$$

It can be obtained as follows:

$$\beta = \lambda - \frac{d}{3} \tag{36}$$

Thereinto,

By substituting Eq. (36) into Eq. (35), the following simplified cubic equation with one variable can be obtained.

$$\lambda^3 + \eta_1 \lambda + \eta_2 = 0 \tag{37}$$

$$\eta_1 = e - \frac{d^2}{3}, \eta_2 = \frac{2}{27}d^3 - \frac{de}{3} + f \tag{38}$$

$$d = \frac{\alpha R_c - (4\alpha/R_{ea}) - (3/R_{ea})}{2\alpha - 6} \tag{39}$$

$$e = \frac{(3R_c/R_{ea}) + (\alpha/R_{ea}^2) - (\alpha R_c/R_{ea})}{\alpha - 3} \tag{40}$$

$$f = \frac{(\alpha R_c/R_{ea}^2) + (3R_c/R_{ea}^2)}{2\alpha - 6} \tag{41}$$

According to the range of the ratio of major principal stress to minor principal stress, the value range of the discriminant can be obtained as follows:

$$\Delta = \left(\frac{\eta_2}{2}\right)^2 + \left(\frac{\eta_1}{3}\right)^3 < 0 \tag{42}$$

Moreover, $\eta_1 < 0$. It can be seen that Eq. (37) has three unequal real roots.

The three real roots are expressed as follows:

$$\lambda_1 = 2\sqrt[3]{\rho}\cos\omega \tag{43}$$

$$\lambda_2 = 2\sqrt[3]{\rho}\cos(\omega + 120^\circ) \tag{44}$$

$$\lambda_3 = 2\sqrt[3]{\rho}\cos(\omega + 240^\circ) \tag{45}$$

$$\rho = \sqrt{-\left(\frac{\eta_1}{3}\right)^3} \tag{46}$$

$$\omega = \frac{1}{3}\arccos\left(-\frac{\eta_2}{2\rho}\right) \tag{47}$$

Substituting the three roots of λ into Eq. (36), the expression of the anisotropic parameter β, in turn, can be obtained as follows:

$$\beta_1 = 2\sqrt[3]{\rho}\cos\omega - \frac{d}{3} \tag{48}$$

$$\beta_2 = 2\sqrt[3]{\rho}\cos(\omega + 120^\circ) - \frac{d}{3} \tag{49}$$

$$\beta_3 = 2\sqrt[3]{\rho}\cos(\omega + 240^\circ) - \frac{d}{3} \tag{50}$$

according to the three roots' value range to determine the final reasonable value.

It is obvious that the transformed stress space is equivalent to the true stress anisotropy space when β is equal to 1, and the strength criterion curves of the former are exactly the same. Also when $\beta > 1$, the triaxial extension deviatoric stress value of the anisotropic generalized nonlinear strength criterion (AGNSC) curve in the corresponding real stress space is smaller than that of the isotropic GNSC curve. Generally speaking, for the aforementioned three root choices, you can make the right solution according to $\beta > 1$. When $\alpha = 0$, the GNSC isotropic criterion is reduced to the SMP isotropic criterion. At this point, the new parameter β, which is derived from Eq. (35), can be expressed as follows:

$$\beta_{\mathrm{SMP}} = \sqrt{\frac{R_c}{R_{ea}}} \tag{51}$$

Eqs. (36)–(51) are the formulas for solving anisotropic parameters based on the GNSC transformation.

In Fig. 8, in consideration of actual test determination, generally, point c corresponding to the result of the conventional triaxial extension strength is obtained with the major principal stress direction perpendicular to the sedimentary surface of the soil. Similarly, due to the need to ensure isotropy, point i was obtained by carrying out the conventional triaxial extension test results of isotropic soil, corresponding to the test results of isotropic strength criterion. For point a, due to the influence of the original anisotropy, the loading condition corresponding to the conventional triaxial extension strength is obtained with the major principal stress direction parallel to the sedimentary surface of the soil. Through the aforementioned three sets of experiments, the parameter β reflecting transversely isotropy can be obtained.

AGNSC Property Analysis

To facilitate the display of the transverse isotropic GNSC on the impact of the failure curve, the failure curves should be displayed. It defines the direction of major principal stress 1 corresponding to the principal axis of material *I*, as shown in Fig. 9(a). The material deposition surface is perpendicular to the *z*-axis direction, as shown in Fig. 9(b). The material deposition surface is perpendicular to the direction of principal axis *x*, and the material deposition surface is perpendicular to the direction of principal axis *Y*, as shown in Fig. 9(c). The intermediate principal stress 2 corresponds to the direction of the material spindle II and the small principal stress 3 corresponds to the direction of the material spindle III. At this point, the solid line on the deviatoric plane corresponds to AGNSC, the dashed line corresponds to completely isotropic GNSC, and the point line corresponds to von Mises criteria.

The principal stress ratio for σ_1 versus σ_3 can be expressed by R_{ea} with the triaxial extension loading path corresponding to the transverse isotropic material, and the principal stress ratio for σ_1 versus σ_3 can be expressed by R_{ei} with the triaxial extension loading path corresponding to the isotropic material. Apparently because of the influence of the anisotropy, causing $R_{ea} < R_{ei}$, an anisotropic state parameter $s = R_{ea}/R_{ei}$ can be proposed to describe the anisotropic degree. When the anisotropic state parameter $s = 1$, the transverse isotropy is completely degraded into fully isotropy. At this time parameters $\beta = 1$. When $s < 1$, it would result in $\beta > 1$. Considering the relationship between S versus β with different shape factors α, it can be seen from Fig. 10 that there are two rules. (1) With an increase of the shape factor α, the domain of the anisotropy degree state parameter S gradually increases. When $\alpha = 0.1$, $0.97 < s < 1$. When $\alpha = 0.9$, $0.75 < s < 1$. (2) For a certain shape factor, with an increase of the value of S, the value of parameter β gradually decreases, showing a monotonically decreasing relationship. When $S = 1$, parameter $\beta = 1$. The smaller the value of S, the more significant the anisotropy of the material, and the greater the corresponding value β, which is in line with the law of anisotropy's influence on the strength characteristics of the material.

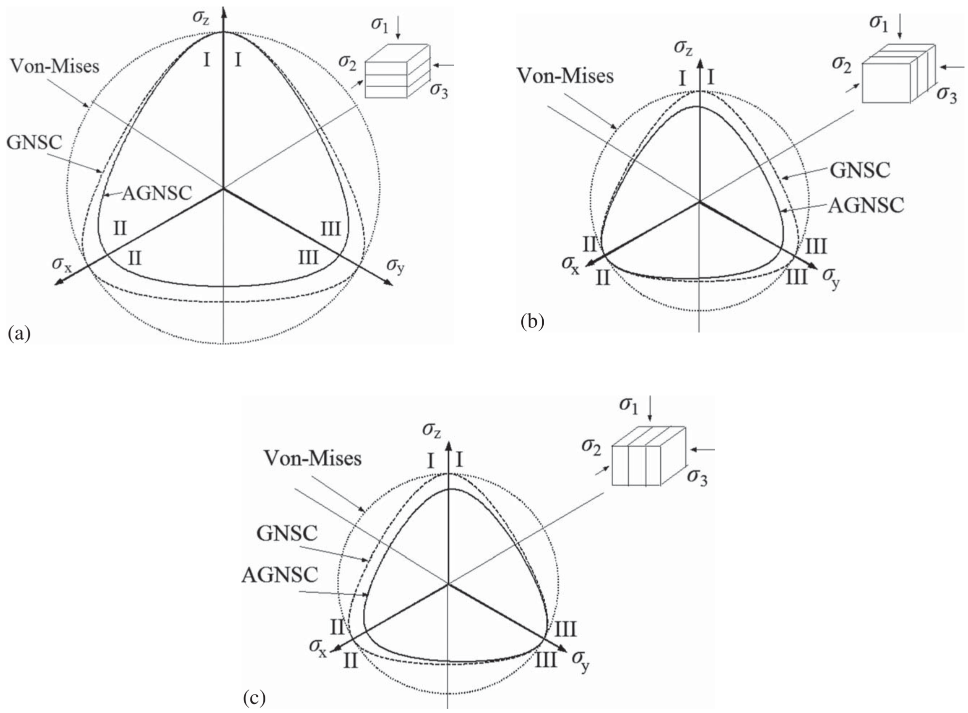


Fig. 9. (a) Failure surface in the deviatoric plane when the depositional plane is perpendicular to the z-axis; (b) failure surface in the deviatoric plane when the depositional plane is perpendicular to the x-axis; and (c) failure surface in the deviatoric plane when the depositional plane is perpendicular to the y-axis.

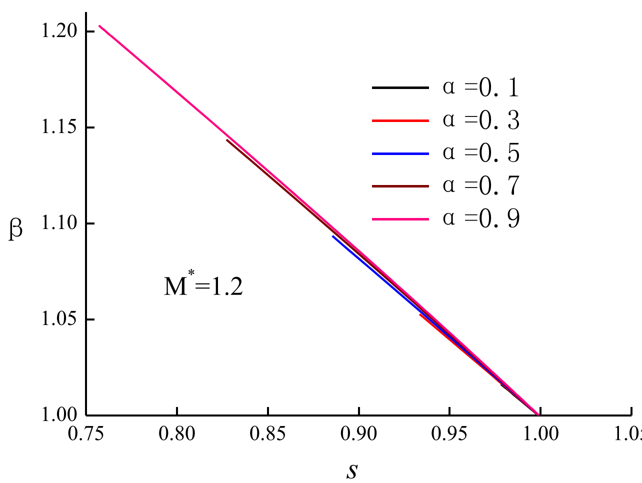


Fig. 10. Relationship of β versus s with different values of α.

Verification of Anisotropic Generalized Nonlinear Strength Criterion

Lade et al. conducted a detailed study on the failure characteristics of San Francisco Bay Mud clay under true triaxial loading, and obtained the failure point data of San Francisco Bay Mud clay under different stress Lode angles. Samples employed in this study were obtained from a trench excavation at a site located about 1 mi (1.6 km) south of the San Francisco International Airport, California. The trench exposed the upper sedimentary deposits of San Francisco Bay. The soils are widely known as San Francisco Bay Mud. Cylindrical block samples with both diameter and height of foot of 30.5 cm were taken at a depth of about 6.5 m. The natural clay at the depth sampled exhibits a total unit weight of about 1.44 g/cm^3 and a water content of about 98.5%. The corresponding material parameters are listed in Table 1.

Table 1. Material parameters

Parameters	p_r (MPa)	σ_0 (MPa)	n	M_f	α	β
Grundite clay	0.092	0.01	0.97	1.23	0.149	1.11
San Francisco bay clay	0.001	0.01	1	1.49	0.55	1.15
Cambria sand	0.001	0.01	1	1.62	0.61	1.08
Trachyte	100	1.4	0.74	2.2	0.83	1.05
Concrete	93.2	1.52	0.94	2.2	0.82	1.04

In Fig. 11, the discrete points correspond to the failure point data of San Francisco Bay Mud clay under conventional triaxial compression. After collating in the $\ln \bar{q}_\alpha^*$ and $\ln[(p+\sigma_0)/p_r]$ coordinate system, the result basically conforms to the linear rule. According to Eq. (22), which is a linear equation, and parameters n, p_r, 0, M_f on the meridian plane can be calibrated successively.

Lade et al. carried out a detailed study on the failure characteristics of San Francisco Bay Mud clay under true triaxial loading conditions and obtained the failure data of San Francisco Bay Mud clay under different stress Lode angles. The experimental data in Fig. 12 are the corresponding test results.

The original test data points are shown in Fig. 12 in the ordinary stress space, that is, the triaxial test of different Lode angles. Thus, $\alpha = 0.149$, $\beta = 1.11$, and $\beta_{\mathrm{SMP}} = 1.086$ can be obtained, and the values of β and β_{SMP} are substituted into Eq. (27) to transform into the equivalent isotropy on the deviatoric plane.

The failure curve in Fig. 12 is the model curve represented by the SMP criterion. Because the SMP criterion is a failure criterion based on the assumption with the existence of the spatial mobilized plane, the material is a typical frictional one, which is suitable for describing the failure property of discrete accumulation materials. According to Fig. 12, as the contribution of the cohesive force to the resistance of failure cannot be considered in the SMP criterion,

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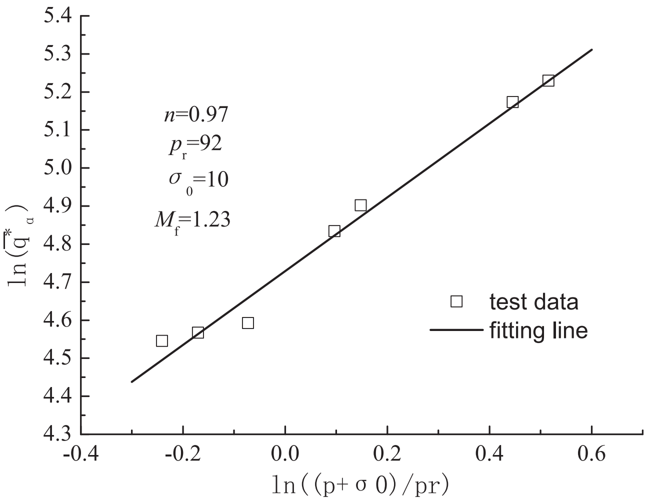

Fig. 11. Calibration of meridional surface parameters under triaxial compression in the transitional space.

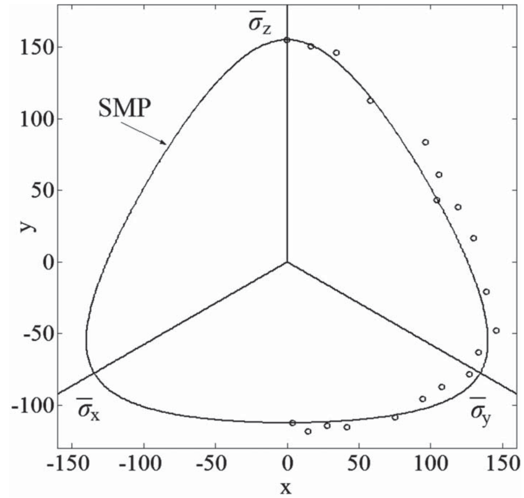

Fig. 13. Illustration of the experimental data and SMP strength theory in the transformed stress space.

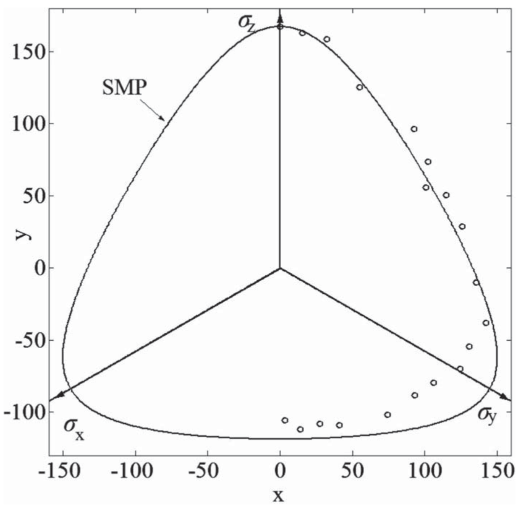

Fig. 12. Illustration of the original data and SMP strength theory in the ordinary stress space.

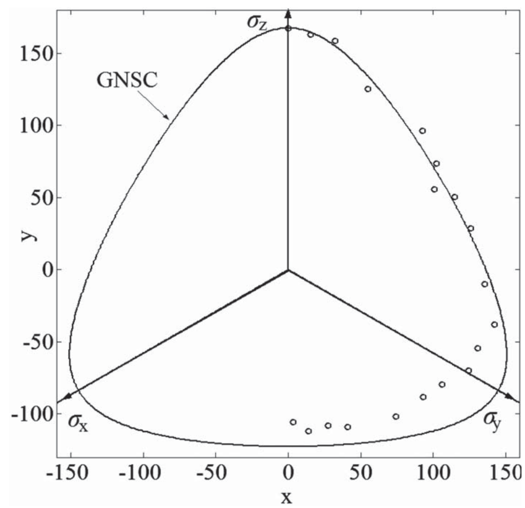

Fig. 14. Illustration of the original data and $\alpha = 0.149$ strength theory in the ordinary stress space.

the failure curves at the stress Lode angle near the conventional triaxial extension path, which corresponding curve radius is smaller, indicating that the corresponding generalized deviatoric stress strength is smaller.

Fig. 13 shows the comparison between test data and SMP criterion in transformation stress space corresponding to the general principal stress space in Fig. 12. It can be seen from Fig. 13 that although the SMP criterion revised by anisotropic parameters can be adopted to reflect the effect of transversely isotropy on the deviatoric plane with different stress Lodes angles, the contribution of the SMP criterion to the cohesive force on failure curve with different Lodes angles cannot be considered, the difference in the stress ratio of the stress-induced anisotropy cannot be fully reflected.

Comparison between the prediction results and the experimental data using the fully isotropic GNSC is displayed in Fig. 14. It can be seen from Fig. 14 that although the GNSC can be adopted to describe the contribution ratio of cohesion and friction to strength stress ratio, the GNSC is a linear interpolation analytic formula that is based on the linear interpolation formula of the fully isotropic generalized von Mises criterion and SMP criterion. Therefore, the GNSC is also the complete isotropic strength criterion that cannot take into account the influence of the geomaterial deposition surface on the strength value under different stress Lodes angles. As shown in Fig. 14, the highest point corresponding to the failure curve is the result of conventional triaxial compression which can be reflected better. Because of the loading condition corresponding to the direction of major principal stress perpendicular to the direction of the deposition surface, using the triaxial compression strength at the vertical condition to extrapolate the strength values corresponding to the other Lodes angles, larger prediction results are obtained, especially for the point at the bottom of the curve, which corresponds to the normal triaxial extension loading result. The curve corresponding to the major principal stress is perpendicular to the deposition surface. However, the test point is the direction of major principal stress parallel to the direction of the sedimentary surface. Therefore, the strength value approaching to the Lodes angle of the conventional triaxial extension path is overestimated.

Comparison between prediction and test results by using the curve of AGNSC and the aforementioned comparison results are shown in the transformation stress space in Fig. 15. It can be seen from the figure that the anisotropic generalized nonlinear

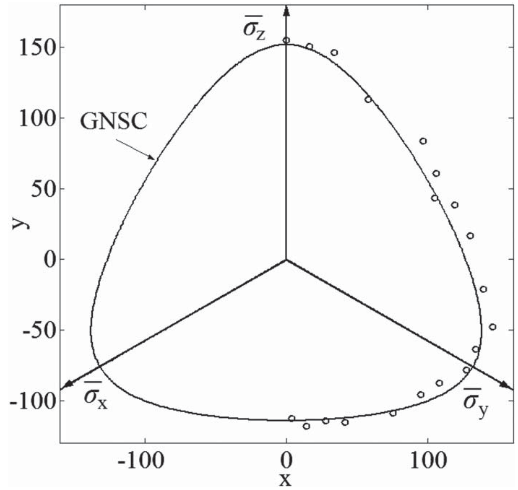

Fig. 15. Illustration of the experimental data and $\alpha = 0.149$ strength theory in the transformed stress space.

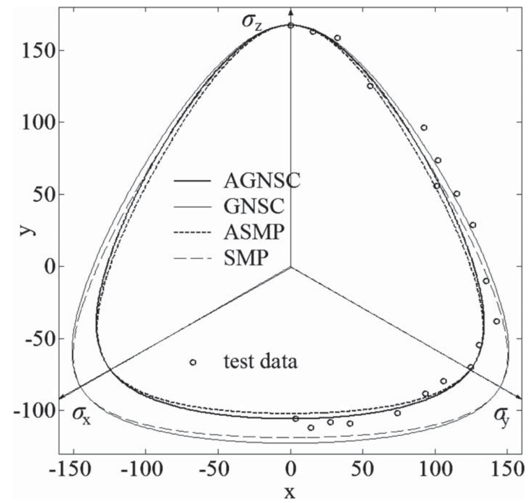

Fig. 16. Illustration of the original data and strength theory in the ordinary stress space.

strength criterion (AGNSC) can be employed to better reflect the influence of the deposition surface on the strength with different stress Lodes angles due to the use of anisotropic parameter β that reflects the transverse isotropy.

It can be seen from Figs. 12–15 that the SMP criterion cannot be used to reflect the stress-induced anisotropy and the original anisotropy of the soil well. The generalized nonlinear strength criterion can be adopted to reflect the stress-induced anisotropy of the soil well, but it cannot consider the influence of the original anisotropy on the strength. By using the anisotropic transformation method, the AGNSC can agree well with the experimental results in the transformed stress space.

Comparison with the test results and the prediction by using the fully isotropic SMP criterion, the GNSC, the anisotropic SMP (ASMP) criterion and the anisotropic GNSC is shown in Fig. 16. It can be seen from the figure that for the fully isotropic criterion, such as the SMP criterion and the GNSC, the effect of the original anisotropy on the strength values under different Lodes angles cannot be considered, so the stress path loading results under nontriaxial compression are generally larger. As for the ASMP criterion, since stress-induced anisotropy cannot be considered more reasonably, which means the difference of the loading results under the different stress Lodes angles cannot be well considered, the conventional triaxial extension results of the major principal stress perpendicular to the sedimentary surface are underestimated, while the triaxial extension results of the major principal stress parallel to the sedimentary surface are overestimated. For the AGNSC, on the one hand, the effect of stress-induced anisotropy is considered, which is reflected by parameter α. On the other hand, the influence of the original anisotropy on the strength value can be considered, which is reflected by parameter β. In summary, the proposed AGNSC curve in Fig. 16 is in good agreement with the experimental results.

In order to quantitatively assess AGNSC of superiority, Fig. 16 shows that we can use the distance mean square error of the discrete point distance criterion curve as the evaluation standard. The difference between the test point radius r_t and the criterion pole radius r_p divided by the test point radius r_t is the distance difference ratio. The square sum and square root of the distance difference ratios of a total of 20 discrete points can be used to obtain the mean variance data of the four criterion curves, which can be used to judge the degree of proximity between the criterion curve and the test data:

$$E_{\text{AGNSC}} = \sqrt{\frac{\sum_{i=1}^{20}\left((r_t - r_{\text{pAG}})/r_t\right)^2}{20}} = 0.0429 \tag{52}$$

$$E_{\text{GNSC}} = \sqrt{\frac{\sum_{i=1}^{20}\left((r_t - r_{\text{pG}})/r_t\right)^2}{20}} = 0.0757 \tag{53}$$

$$E_{\text{ASMP}} = \sqrt{\frac{\sum_{i=1}^{20}\left((r_t - r_{\text{pAS}})/r_t\right)^2}{20}} = 0.0579 \tag{54}$$

$$E_{\text{SMP}} = \sqrt{\frac{\sum_{i=1}^{20}\left((r_t - r_{\text{pS}})/r_t\right)^2}{20}} = 0.0816 \tag{55}$$

Obviously, the smaller the mean variance, the closer the proposed criterion curve to the measured data. Thus, the aforementioned quantitative evaluation results show that the AGNSC is the best, followed by the ASMP criterion, the GNSC criterion, and finally the SMP criterion. This is consistent with the degree of proximity between the predicted curve and the discrete points in Fig. 16.

Therefore, for the reflection of stress-induced anisotropy, the generalized nonlinear strength criterion is superior to the SMP criterion. The generalized nonlinear strength criterion can reasonably consider the proportion distribution of cohesive force and friction force contributed to the strength value, so it can be employed to describe the failure law of geotechnical materials with certain cohesion. With regard to the influence of the original anisotropy on the shape of the strength curve on the deviatoric plane, it can be determined by a certain proportion of the two results. One is the triaxial extension results of major principal stress perpendicular to the sedimentary surface and parallel to the sedimentary surface, the other is the triaxial compressive strength values of major principal stress perpendicular to the sedimentary surface. It can be seen that for the soil, the generalized strength shape parameter α should be determined first, and then, according to the influence weight of the original anisotropy on the strength value to determine parameter β, make it ultimately reflect the influence of stress-induced anisotropy and original anisotropy according to the failure behaviors.

As shown in Fig. 17, the discrete points in the figure are the true triaxial test results obtained on San Francisco Clay under a mean stress of 167 kPa (Kirkgard and Lade 1993), while the solid lines

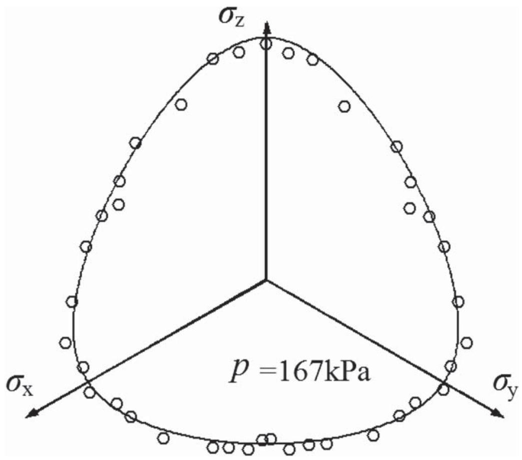

Fig. 17. Comparison the test data of San Francisco clay and predicted result by using the proposed strength criterion in the ordinary stress space.

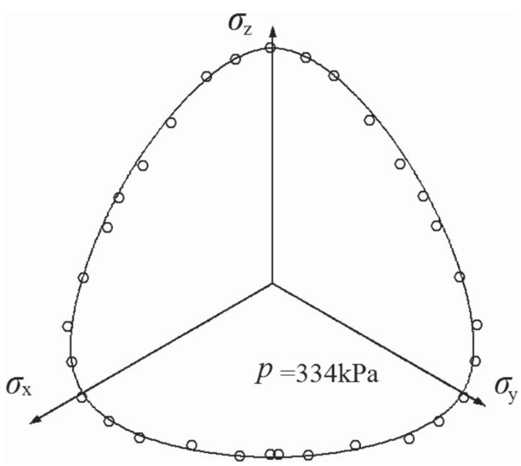

Fig. 18. Comparison the test data of Cambria sand and predicted result by using the proposed strength criterion in the ordinary stress space.

represent the results predicted by the proposed transversely isotropic GNSC criterion. It can be seen from the figure that the proposed criterion can well describe the influence of transversely isotropic effect on shear strength on the deviatoric plane. The shear strength obtained in the direction perpendicular to the deposition surface is obviously greater than that in the horizontal direction.

In Fig. 18, the discrete points are the true triaxial test results of the Cambria sand on the deviatoric plane (Ochiai and Lade 1983), while the solid lines are the prediction results using the proposed criteria. It can be seen from the comparison results that the proposed criterion can well describe the influence of transversely isotropic factors on the shear strength with different stresses Lode angles. The shear strength is the largest in the *z*-axis direction perpendicular to the sand deposition surface.

In order to explore the applicable scope of the proposed criterion, the failure test data of trachyte with a certain cohesion was selected as the prediction object (Mogi 1971). The discrete points in Fig. 19 are the true triaxial test results of the trachyte, and the mean stress is 167 MPa. The solid line is the prediction result of the proposed criterion, and the comparison results show that the proposed criterion can also be applied to the prediction of the true triaxial failure result of materials with a certain degree of cohesion.

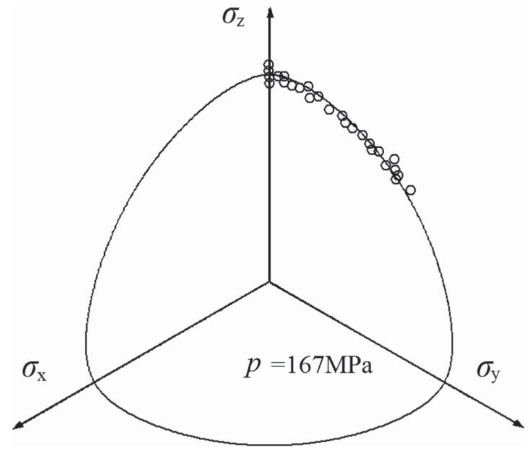

Fig. 19. Comparison the test data of Trachyte and predicted result by using the proposed strength criterion in the ordinary stress space.

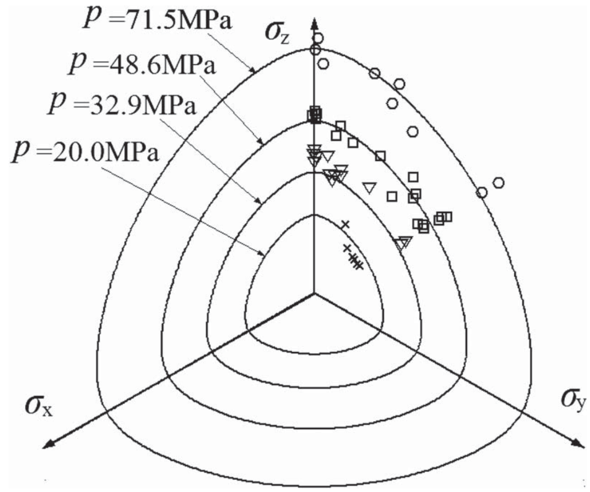

Fig. 20. Comparison the test data of concrete and predicted result by using the proposed strength criterion in the ordinary stress space.

Fig. 20 shows the comparison results of concrete loading tests and predictions (Launay and Gachon 1970). Among them, the four curves corresponding to different mean stresses; when the mean stress is gradually increased, the corresponding prediction results of failure curve become flatten, and when the value of mean stress is small, the corresponding failure curve tends to a sharp curve triangle form. The aforementioned properties show that the proposed criterion can be adopted to describe the effect of hydrostatic pressure on failure curve shape. In addition, there is also a certain degree of anisotropy in concrete, and its influence on the failure curve cannot be ignored. Except that the predicted shear strength is smaller than the test value when the mean stress is 32.9 MPa, the predicted results under other mean stresses are consistent with the experimental results. According to the preliminary analysis, the test results of the mean stress with 32.9 MPa indicate that the large difference in shear strength under different stress Lode angles may be caused by the more significant anisotropy properties.

Conclusions

Based on the existing GNSC, this paper expands the original generalized nonlinear strength criterion. It can only be applied to the description of isotropic geomaterial before, but now it can also consider the anisotropic failure behavior for geotechnical materials reasonably. The proposed criterion has the following characteristics:

1. Parameter α can be used in the generalized nonlinear strength criterion to reflect the stress ratio strength of stress-induced anisotropy under true triaxial loading paths, which can be determined by conventional triaxial compression and extension path.
2. By assuming that the geomaterial is transversely isotropic, for the loading mode perpendicular to the sedimentary surface and parallel to the sedimentary surface, it is assumed that the ratio of the major principal stress strength values of the aforementioned two loading conditions is β, and β is the distribution weight coefficient of the original anisotropy for the ratio of the major principal stress strength, which can be determined by the triaxial compression extension path perpendicular to the sedimentary surface and parallel to the sedimentary surface.
3. In this paper, a simple nonlinear strength criterion is proposed. Stress-induced anisotropy and original anisotropy can be considered reasonably, and the criterion can also be applied to practical geotechnical engineering simply and conveniently. The application scope of proposed criterion is relatively broad. It can not only be used to describe the failure property of clay and sand, but also be used to describe the failure property of some cohesive materials, such as rock and concrete.

Data Availability Statement

All data, models, and code generated or used during the study appear in the published article.

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