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A Stochastic Collocation Method for Uncertainty Analysis of Fatigue Damage Prognosis

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Abstract. This article proposes a stochastic collocation method to investigate the uncertainty quantification in fatigue damage prognosis where experimental data are limited and only interval bounds on uncertain parameters are given. The method derived from tensor-products or sparse grids consists in a Galerkin approximation in random space, requires the use of structured collocation point sets and naturally leads to the solution of uncoupled deterministic problems as in the Monte Carlo approach. The distribution of remaining useful life can be acquired by dividing each interval into several small parts and assuming the corresponding random variable obeys uniform distribution in the small range. Compared with Monte Carlo method and interval arithmetic, this approach is much more efficient, time-saving and gets more accurate predictions. An experimental investigation of fatigue life prediction of a metallic plate with a central crack is presented to demonstrate the efficiency and effectiveness of the proposed method.

Introduction

One of the most valuable research branches of Structural Health Monitoring (SHM) is fatigue damage prognosis, which mainly concerns the residual service life of a structure under the promise of giving the precise diagnosis of current damage state of the structure and the feature of the future load it will bear [1, 2]. Generally, this will depend mostly on the validity of the diagnostic method and the well understanding of the physical mechanism of damage process. However, accurately estimating the remaining useful life (RUL) for aging structural components under fatigue loading has been a challenge due to the complexity and uncertainty in service environments and multidisciplinary damage mechanisms. Therefore, developing a promising technique for tackling this challenge and predicting the fatigue life with a high degree of confidence is desired.

The increased attention for uncertainty quantification (UQ) methodologies originates from the experience that currently available methods are inadequate for application to fatigue damage prognosis problems. For example the classical Monte Carlo (MC) technique, which generates ensembles of random samples for the prescribed stochastic inputs and calculates the system outputs for each sample by utilizing deterministic solvers repeatedly [3]. Liu and Mahadevan [4] proposed a concept of equivalent initial flaw size (EIFS) and used MC simulation to predict the probabilistic fatigue life. Because MC method is based on performing a large number of random computations, is impractical for complex problems with quiet large number of variables which can already be computationally intensive in the deterministic sense.

Another method for dealing with uncertainty propagation is interval arithmetic [5], which is notable for its simplicity and speed. With only knowing the interval bounds on input variables, one can obtain the value range of the outputs by using this approach. Worden and Manson [6] utilized interval arithmetic to compute the possibility of damage prognosis in a thin plate with a mode I central through crack. Surace and Worden [7] extended this to a more complex case for cracking prognosis, and also achieved a good result. Although this method does not need to know the exact distribution characteristics of variables, but the estimation result is too conservative, in practical engineering applications will cause a large waste.

In reality, the situation where the experimental data are limited and still need to study the probability characteristic of the fatigue lifetime is regularly encountered. Recently, a stochastic collocation (SC) scheme has been introduced in which simulations are performed at specific collocation points in the stochastic space [8]. SC methods are attractive techniques for UQ due to their strong mathematical basis and ability to produce functional representations of stochastic variability. Using this technique, we can quantify the complete probabilistic variability of fields of interest as opposed to extracting only limited statistics. Zhao et al. [9] combined this approach with Bayesian method in fatigue crack prognosis of metallic material, in which the distributions of random parameters are given with some certain types of distribution such as normal distribution, which require plenty of measured data.

In this work, we use the concept of stochastic spaces for representing random parameters. The prognosis uses fracture mechanics-based fatigue crack growth modeling, along with quantification of various sources of uncertainty, including natural variability, data uncertainty. In this modeling, an isotropic plate with a central mode I through crack presented in [6] is used. The Paris-Erdogan (PE for short) law [10] based fracture mechanics is chosen to describe the crack propagation and the parameters of material $\lg C$ and m are considered as random variables. With comparison of Monte Carlo simulation, the computational efficiency and accuracy of this approach method are also investigated.

This paper is organized as follows: we give a brief description of the stochastic collocation technique in Sec. 2 and a computational model of crack propagation in a metallic plate is presented in Sec. 3. We provide details of an experimental investigation of metallic plate crack damage prognosis, including the comparisons between the SC and MC methods, in Sec. 4 and we follow it up with conclusions in Sec. 5.

Stochastic collocation methods

In this section, we provide some basic characteristics of the stochastic collocation technique. Stochastic collocation methods combine with the merits of MC simulation and interval arithmetic and can be divided into pre- and post-processing. The pre-processing is actually a high-dimensional interpolation process which is also known as a surrogate model, namely constructing multivariate interpolation polynomial through selecting appropriate interpolation points in the random space, similar to MC simulation [8, 11]. This is only demands the interval values of uncertain parameters, similar to interval arithmetic. The post-processing is aim to calculate the probability characteristic of fatigue lifetime based on the surrogate model, according to the distributions of uncertain parameters. A brief introduction will be given here, and more details refer to literature [8, 11 and 12].

SC simulation proposed in this article is sampled on a sparse grid constructed by Smolyak algorithm which is the linear combination of product formulas [13] and can be given by

$$\mathcal{A}(d+k,d)=\sum_{k+1\le|\mathbf{i}|\le k+d}(-1)^{d+k-|\mathbf{i}|}\cdot\begin{pmatrix} d-1 \\ d+k-|\mathbf{i}| \end{pmatrix}\left(Q_{i_1}\otimes\ldots\otimes Q_{i_d}\right) \tag{1}$$

where d>1 is the dimensional random space; $k \in N^0$ is the *level* of Smolyak construction and Q_i is the one-dimensional interpolation operator. In this paper the Lagrange interpolation polynomials are used as building blocks of Smolyak algorithm and One-dimensional nodal sets are nested. The interpolation is defined at the Chebyshev Gauss-Lobatto nodes which are given by

$$z_i^1 = 0, \text{ for } m_i = 1 \quad \text{and} \quad z_i^j = -\cos\frac{\pi\cdot(j-1)}{m_i - 1},\ j = 1,\ldots,m_i,\ \text{ for } m_i > 1 \tag{2}$$

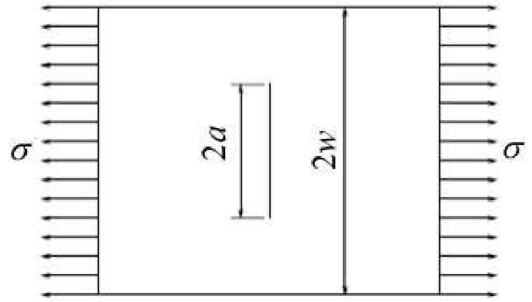

Fig. 1 A plate with central mode I crack under the constant cyclic-load

Computational model

A thin plate under a constant cyclic-load as shown in Fig.1, of which the material is supposed as Ti-6Al-4V Ru ((extra-low interstitials, (ELI))) as its Paris coefficient and exponent have been studied in [14] and [15]. In according with the experiment in [14], the geometrical dimensions of the plate and load are given as: width w of 200mm, thickness t of 2mm, stress range $\Delta\sigma$=40MPa (i.e. $\sigma_{\max}$=200MPa corresponding to maximum applied load of 80kN) and the load ratio R of 0.8.

The simplest law suitable for metallic material to describe the relationship between the crack growth rate *da/dn* and the stress intensity factor (SIF) range ΔK is the PE equation as follow:

$$\frac{da}{dn} = C(\Delta K)^m \tag{3}$$

where C is the Paris coefficient with units of $\mathrm{MPa}^{-m}\mathrm{m}^{-m/2}\mathrm{mm}$ and m is the Paris exponent with units of 1. The parameters can vary from sample to sample for them greatly depend on microstructure of material, and the effect of variability is the great impetus to this paper. The SIF range ΔK is given by

$$\Delta K = Y(\bar{a})\cdot\Delta\sigma\sqrt{\pi a} \tag{4}$$

where $\bar{a} = a/(2w)$ is a dimensionless parameter; $Y(\bar{a})$ is the parameter related to the size of geometry and for an infinite plate, the analytical solution is $Y(\bar{a}) = 1$, while other several forms of $Y(\bar{a})$ is given in [16] for finite plate as follow:

$$Y(\bar{a}) = 1 + 0.256 + 1.152\bar{a}^2 + 12.20\bar{a}^3 \tag{5}$$

$$Y(\bar{a}) = \sqrt{\sec(\pi\bar{a})} \tag{6}$$

$$Y(\bar{a}) = \frac{1}{\sqrt{1-(2\bar{a})^2}} \tag{7}$$

The remaining cycles N can be acquired by integrating the Eq. (3). When the half length of initial crack a_0 is determined by detection, the form of N is

$$N = \int_{a_0}^{a_c} \frac{1}{C(\Delta K)^m} da \tag{8}$$

Estimating the fatigue life using stochastic collocation method

This section is aim to analyzes the uncertainty quantification in the plate including infinite plate and other three expression of SIF described in Sec. 3 through the stochastic collocation method presented in Sec. 2, with using the limited experimental data (16 groups of test data [14]). Befor the implementation of this, it is neccsary to study on study on the effectiveness and efficiency of the method for estimating the probability of fatigue life of the plate.

According to the experimental data in [14], the interval bounds on Paris parameters $\lg C$ (a logarithm to base 10) and m are $(-15.0,-11.6)$ and $(3.7,6.2)$ respectively. Additionally, the fracture toughness is taken the minimum value of $K_{IC}=75\text{MPa}\sqrt{\text{m}}$ and the initial condition is $a_0=10\text{mm}$ (corresponding to the length of initial crack of 20mm).

Computational effectiveness and efficiency. In this section a fatigue-life estimation with two types of distribution (normal distribution of $\lg C\sim N(-12.3,4.1^2)$ and $m\sim N(7.3,4.1^2)$ [15], and uniform distribution [6]) for Paris parameters $\lg C$ and m are considered to investigate the effectiveness and efficiency of the SC method. In the post-processing of SC method, 2000 samples are used to calculate the probability density function (PDF) for the remaining useful life $\lg N$, and the standard solution of the PDF are provide by Monte Carlo simulation with the same samples.

In order to measure the computational accuracy of the SC method, a definition of cumulative error of PDF in the each sample set is given by

$$err_c=\sum_{i=1}^{Num_{sample}}\left|\text{SC\_}pdf(\lg N_i)-\text{MC\_}pdf(\lg N_i)\right| \tag{9}$$

Fig. 2 shows the curves of cumulative error of PDF for $\lg N$ of each SIF expression with the calculation of SC method under each interpolation level k. As the interpolation level k increase, the PDF produced by SC method approach the result of MC simulation, and the errors are approximately to zero at $k=7$.

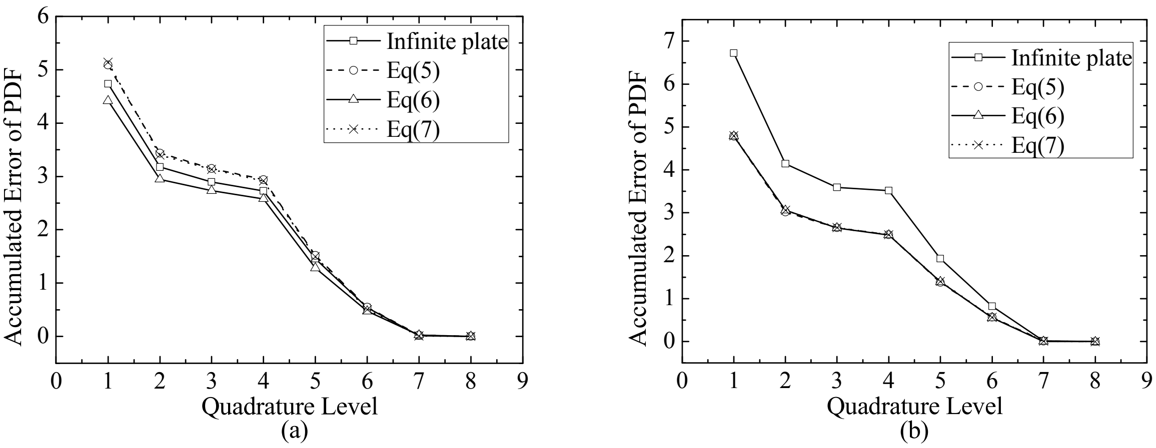

Fig. 2 Curves of cumulative error of PDF for $\lg N$ calculated by SC method at each interpolation level with comparison of MC simulation. (a) Normally distributed parameters; (b) Uniformly distributed parameters.

For the pre-processing (i.e. core process) of SC method will not be influenced by the variation of the distribution types of random parameters when the random field is presented, the calculation time of that will almost keep the same. The cost time of 7the-order SC method with 2000 samples is only 31s under the two distribution patterns of parameters, while the computation time spent by MC simulation with the same samples is 300s. Apparently, the SC method is more efficient than MC simulation.

Uncertainty quantification of a damaged plate using the stochastic collocation method. This section uses the 7th-order SC method to analyze the probability characteristics of fatigue life of a damaged plate in case of limited experimental data ([14]). The value ranges of random parameters $\lg C$ and m are divided into several intervals with providing the corresponding probabilities, as shown in Fig. 3 and each variable is assumed to obey uniform distribution in every small range. The PDF for fatigue life $\lg N$ of each SIF expression are extracted from 2000 samples with using Kernel Density Estimation, as shown in Fig. 4. The four curves produced by SC method are almost coincident and are similar to the probability density curve of normal distribution (the red solid line).

On the basis of PDF for fatigue life $\lg N$ of each SIF expression, the interval bounds of $\lg N$ with probability of 95% are shown in Table 1. The bounds of fatigue life $\lg N$ of infinite plate are (7.6113, 12.2463), while the corresponding bounds provided by interval arithmetic are (6.9858, 12.7817) given in [6]. It is clear that the lower bound calculated by SC method is higher, which illustrates that the probability of the minimum fatigue life estimated by the interval method is low and conservative.

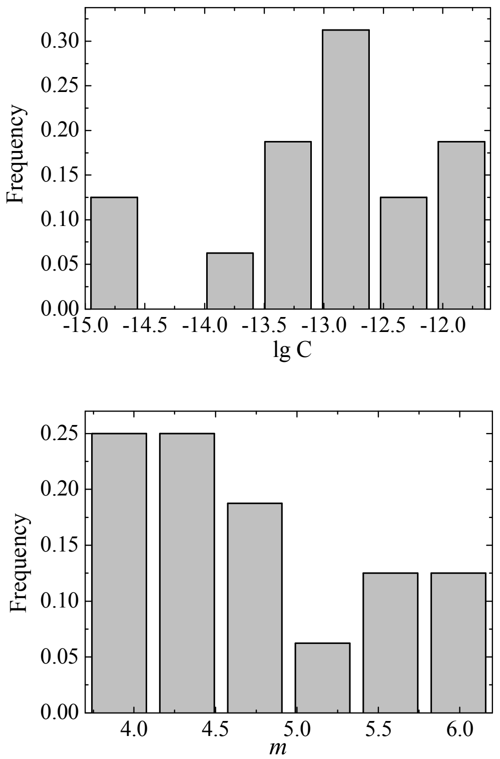

Fig. 3 The probability of random parameters

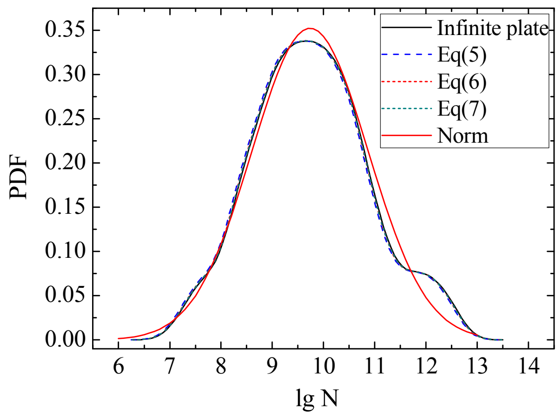

Fig. 4 Curves of PDF for fatigue life lg$N$ of each SIF expression

Table 1 The interval bounds of lg$N$ with probability of 95%

	Infinite plate	Eq(5)	Eq(6)	Eq(7)
Lower bound	7.6113	7.5660	7.5888	7.5888
Upper bound	12.2463	12.1938	12.2312	12.2390

Conclusions

In this paper, a stochastic collocation method is used in the fatigue damage prognosis of a plate under the constant loading with considering the material parameters $\lg C$ and m as random variables. The SC method includes a pre-processing based on sparse grid interpolation and a post-processing considering randomness of parameters. With comparison of Monte Carlo simulation, this method is more robust and time-saving. Changing the distribution patterns of uncertain parameters does not affect the computational efficiency and accuracy of this method.

An example based on limited experimental data is used to demonstrate the effectiveness of the proposed approach. The results demonstrate that the proposed UQ method can effectively tackle the model parameters' uncertainties and get more accurate RUL predictions based on the observed sparse data.

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