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Evidential uncertainty quantification of dynamic response spectrum analysis

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Abstract. This study presents an evidential uncertainty quantification (UQ) approach for dynamic response spectrum analysis of a structural system with epistemic uncertainty. The present method is performed using an evidence theory to quantify the uncertainty present in the structure's parameters such as material properties. In order to alleviate the computational difficulties in the evidence theory based UQ analysis, a differential evolution (DE) based interval optimization for computing bounds method is developed. With comparison of probability theory and interval method, the computational efficiency and accuracy of this approach method are also investigated.

Introduction

The response spectrum method transforms the dynamic problem into static problem, making it easier to determine the dynamic response of the complex structures. Thus, response spectrum method has gained wide engineering acceptance in the world, which has been applied to the code for seismic design of buildings by many countries.

Uncertainties are often prevalent in practical engineering applications. Oberkampf[1] and Helton[2] categorize the uncertainties as either aleatory uncertainty or epistemic uncertainty according to the source of uncertainty. Aleatory uncertainty is due to inherent variability in a physical phenomenon, such as uncertain geometric parameters and operating conditions. Epistemic uncertainty is sometimes referred to as state of knowledge uncertainty, such as those due to unknown physical phenomena. In general, the uncertainty of the structure's geometric and/or material characteristics is not considered in the dynamic response spectrum analysis. However, due to assembly process and manufacturing tolerances, the structural geometric and/or material properties are usually uncertain, including aleatory uncertainty and epistemic uncertainty. The stiffness, mass, damping are expressed as functions of the above parameters. As a result, the structural dynamic response is also uncertain. Therefore, it is important to estimate the effect of these uncertainties on the structural dynamic response.

The probabilistic approach[3] is a very powerful tool in solving random uncertainty problems in engineering. In probabilistic models, the uncertain variables are usually described with a random quantity or a stochastic process. However, the traditional probability theory intended only for aleatory uncertainty is not capable of capturing epistemic uncertainty[4]. To investigate the epistemic uncertainty of structural response, a large number of nonprobabilistic methods have been developed, such as the fuzzy set theory[5], interval method[6,7], evidence theory[8].

Among these methods, evidence theory has great potential in uncertainty quantification which is more general than probability and possibility theories. Evidence theory was first proposed by Dempster [9] and extended by Shafer[10], which offers a framework for modeling both epistemic uncertainty and aleatory uncertainty through a more flexible representation of uncertainty. It uses plausibility and belief to measure the likelihood of event, without making additional assumptions. Evidence theory can formulate various basic probability assignment (BPA) structures, and it also can provide equivalent formulations to fuzzy sets and interval method, respectively. Evidence theory is widely used in artificial intelligence related fields, and recently it has been extended to conduct reliability analysis and design for engineering structures and mechanical systems[8,11,12,13,14].

In evidence theory, the uncertainty in a system is propagated through a discrete basic belief assignment (BPA) structure. Hence the resulting uncertainty in a system is usually quantified by many repetitive system simulations for all the possible propositions given by BPA structures of uncertain variables[11]. As a result, intensive computational cost might be inevitable in quantifying uncertainty using evidence theory[12]. In order to improve computational efficiency, we use the differential evolution algorithm[15] to calculate the uncertainty propagation.

In this work, an evidential uncertainty quantification method for dynamic response spectrum analysis of a structural system with epistemic uncertainty is developed. This method enhances the deterministic dynamic response spectrum analysis by including the presence of uncertainty at each step of the analysis procedure.

Fundamentals of evidence theory

Evidence theory[10] is an uncertainty reasoning and decision-making theory based on the frame of discernment. Any problem of likelihood takes some possible sets as given. The family of all the possible sets is defined as frame of discernment (FD), which is labeled as Θ. All the elements in Θ are mutually exclusive to each other. In evidence theory, the basic propagation of information is through Basic Probability Assignment (BPA). BPA expresses the degree of belief in a proposition. BPA is assigned by making use of a mapping function (m) in order to express our belief with a number in the unit interval [0,1]

$$m: 2^{\Theta} \rightarrow [0, 1] \tag{1}$$

m must satisfy the following three axioms:

$$m(\Phi) = 0, \quad \sum_{A \subseteq \Theta} m(A) = 1, m(A) > 0 \tag{2}$$

in which, set A is called focal element.

Belief function *Bel(A)* is defined as the total of BPAs for propositions which are included in proposition A fully. Plausibility function *Pl (A)* is defined as the total of BPAs for propositions whose intersection with proposition A is not an empty set.

$$\mathrm{Bel}(A) = \sum_{B \subseteq A} m(B) \tag{3}$$

$$Pl(A) = \sum_{B \cap A \neq \varnothing} m(B) \tag{4}$$

The belief function represents the degree of belief on that proposition A is true. The plausibility function represents the degree of belief on that proposition A is not false. Belief function and Plausibility function constitute the lower bound and upper bound of proposition A. The interval [*Bel(A), Pl(A)*] represents the belief degree of proposition A. Evidence from different sources can be aggregated by Dempster's combinational rule.

Deterministic response spectrum analysis

Response spectrum analysis (RSA) is a complex that assumes the multi-degree-of-freedom (MDOF) system as the single-degree-of-freedom (SDOF) system. Extracting the biggest response value corresponding to each mode of vibration, then coupling with suitable method, the RSA predicts the maximum response value. Generally, in the elastic dynamic response analysis of a (MDOF) system, mode-superposition response spectrum method is adopted. The seismic action of a MDOF system is decomposed into the maximum seismic action of n independent equivalent SDOF system based on modal analysis and modal orthogonality principle. Thus each SDOF response is obtained by response spectrum. Coupling these SDOF responses using suitable method, such as square root of the sum of the squares (SRSS) method, the target response can be obtained.

Under horizontal earthquake action, a MDOF vibration equation of elastic system is given as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{I\}\ddot{x}_g \tag{5}$$

where $[M]$, $[C]$, $[K]$, $\{x\}$, $\{I\}$ represent the global mass matrix, damping matrix, stiffness matrix, displacement matrix and unit matrix respectively. $\ddot{x}_g$ is the ground acceleration.

The response of a structure can be defined as a combination of many special shapes (modes) that in a vibrating string correspond to the "harmonics". The modal analysis gives the solution using SRSS method as following,

$$S = \sqrt{\sum_{j=1}^{n} S_j^{2}} \tag{6}$$

where S is the total response, S_j is the j[th] modal response.

Evidential uncertainty quantification of response spectrum analysis

The quantification of uncertainty framework is outlined by Fig. 1 and involves three necessary steps: uncertainty representation, propagation and measurement. For uncertainty representation, the UQ framework uses all possible obtained values of structural material constants provided by designer in constructing separate belief structures for material constants. Then, DE global optimization method is used for propagation of the represented uncertainty through RSA simulations of a MDOF structure using SRSS method in finite element analysis. Finally, observed evidence on simulation responses is used in determination of target propositions to estimate uncertainty measures.

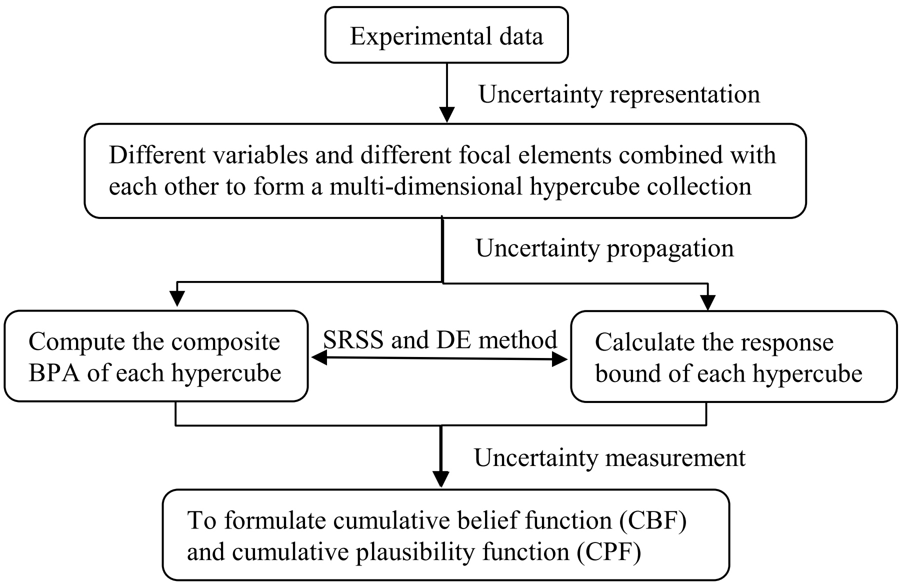

Fig.1. Three stages of UQ of response spectrum analysis

Example

A five-layer reinforced concrete frame structure is used to investigate on the condition of fortification intensity of 7 degrees (0.1g), Ⅱ class venue, and Tg= 0.35s. Structural damping ratio is 0.05. The mass($m1$~$m5$=m) and stiffness ($k1$~$k5$=k) of every layer are assumed as uncertainty parameters. The BPAs of k and m are shown in Table 1. SRSS method is adopted to solve the shear force V of frame column under horizontal earthquake action.

Table 1 BPA values for uncertain parameter

Variables		Focal elements	BPA
Source 1	k	[200000,240000][240000,260000][260000,300000]KN/m	0.25,0.5,0.25
	m	[2,2.4][2.3,2.6][2.5,2.8][2.6,3]*1.0e3./9.8ton	0.1,0.3,0.4,0.2
Source 2	k	[200000,240000][230000,270000][260000,300000]KN/m	0.3,0.5,0.2
	m	[2,2.4][2.3,2.7][2.6,3]*1.0e3./9.8ton	0.2,0.6,0.2

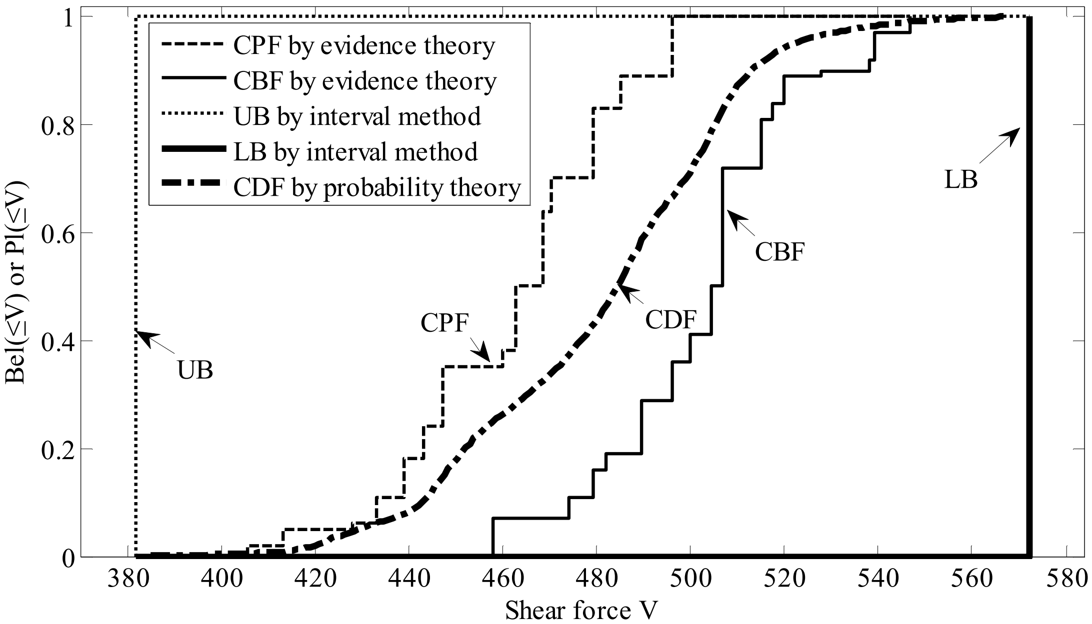

Fig.2 Shear force cumulative distribution of underlying column based on probability theory, interval method and evidence theory

Base shear force results based on probability theory, interval method and evidence theory are shown in Fig.2, where CPF, CBF and CDF denote the cumulative distribution function for plausibility, belief and probability respectively. For comparison with probability theory, the approximate PDFs of uncertain variables are obtained by the assumption that probability mass (BBA) in each interval is uniformly distributed. UB and LB denote the upper bound and the lower bound by interval method respectively. If there is enough sufficient information on the uncertainty parameters, probability theory can be adopted to get a unique probability distribution curve. If there is less information on uncertainty parameters, evidence theory should be adopted to describe the uncertainty. Curves of probability theory generally fall in the area between the CBF and CPF curve. So the probability theory can be seen as special cases of evidence theory.

Both evidence theory and interval method can deal with epistemic uncertainty. If there are only upper and lower bound of uncertain variables, evidence theory would be evolved into interval method.

Table 2 Comparison of the results based on probability theory, interval method and evidence theory

Results	Probability	Evidence theory	Interval method
Expectations of V(kN)	480.931	[462.010, 502.2184]	[381.558, 572.337]
Probability of V less than 480	44.47%	[16%,83%]	[0,100%]
95% reliability of V(kN)	521.9	[496.0,539.2]	[381.558, 572.337]

Table 2 shows some information of interest from the results calculated by probability theory, interval method and evidence theory. The results of the probability theory are single values, while the results of interval method and evidence theory are all interval values. Interval method can solve the problem under the condition of less knowledge, but its results are usually too conservative. The uncertain variables without exact probability distribution are suitable for evidence theory to handle, and it avoids the error caused by probability theory effectively, for example, the result calculated by probability theory shows that the probability is no more than 5% when the base shear force exceeds 521.9kN, while the result in the evidence theory is 539.2kN. These results indicated that evidence theory has good compatibility with probability theory and interval algorithm, so evidence theory has the potential to handle epistemic uncertainty quantization.

Conclusions

Evidence theory is a good method to deal with both aleatory and epistemic uncertainty, which is able to integrate the information coming from multiple sources and reflect different opinions synthetically. Evidence theory has a wider range of application than probability theory, interval method and fuzzy sets.

In this paper, an example of five-layer reinforced concrete frame is investigated based on evidence theory in response spectrum method, which has good reference value for engineering.

It should be pointed out that this work is concentrated on linear elastic response spectrum. In our future work, we intend to propose elastic-plastic response spectrum method based on evidence theory to analyze dynamic response with uncertainties.

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References

[1] W.L. Oberkampf, et al. Challenge problems: uncertainty in system response given uncertain parameters. *Reliability Engineering & System Safety* 85.1 (2004): 11-19.

[2] J.C. Helton. Uncertainty and sensitivity analysis in the presence of stochastic and subjective uncertainty. *Journal of Statistical Computation and Simulation* 57.1-4 (1997): 3-76.

[3] Wei Gao. "Random seismic response analysis of truss structures with uncertain parameters." *Engineering structures* 29.7 (2007): 1487-1498.

[4] K. Sentz, and S. Ferson. *Combination of evidence in Dempster-Shafer theory*. Vol. 4015. Albuquerque, New Mexico: Sandia National Laboratories, (2002).

[5] G.C. Marano, et al. A fuzzy random approach of stochastic seismic response spectrum analysis. *Engineering Structures* 32.12 (2010): 3879-3887.

[6] M. Modarreszadeh. *Dynamic analysis of structures with interval uncertainty*. Diss. Case Western Reserve University, (2005).

[7] Zhiping Qiu, and Ni Zao. Interval modal superposition method for impulsive response of structures with uncertain-but-bounded external loads. *Applied Mathematical Modelling* 35.3 (2011): 1538-1550.

[8] Y.C. Bai, et al. Evidence-theory-based structural static and dynamic response analysis under epistemic uncertainties. *Finite Elements in Analysis and Design* 68 (2013): 52-62.

[9] A.P. Dempster, Upper and lower probabilities induced by a multiplicand mapping, Annals of mathematical statistics, (1967), 38:325-339.

[10] G.A. Shafer. *Mathematical theory of evidence*. Vol. 1. Princeton: Princeton university press, (1976).

[11] H.R. Bae, R.V. Grandhi, and R.A. Canfield. An approximation approach for uncertainty quantification using evidence theory. *Reliability Engineering & System Safety* 86.3 (2004): 215-225.

[12] H.R. Bae, R.V. Grandhi, and R.A. Canfield. Canfield. Epistemic uncertainty quantification techniques including evidence theory for large-scale structures. *Computers & Structures* 82.13 (2004): 1101-1112.

[13] C. Jiang, et al. A novel evidence-theory-based reliability analysis method for structures with epistemic uncertainty. *Computers & Structures* 129 (2013): 1-12.

[14] Z.P. Mourelatos, and Jun Zhou. A design optimization method using evidence theory. *Journal of mechanical design* 128 (2006): 901.

[15] R. Storn, and K. Price. Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. *Journal of global optimization* 11.4 (1997): 341-359.

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