非対称剛性部材で支持された cable-bracing inerter system の最適設計

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振動制御　　イナータシステム　　ケーブルブレース
最適化設計　　パラメトリックスタディ

1. Introduction

Conventional structural control designs are performed by adjusting the stiffness of the structures and installing energy-dissipating devices, which means that two terms, the stiffness coefficient and the damping coefficient are modified in the equation of motion. Another method to protect the structure from undesired vibrations is to decrease earthquake ground motion inputs. The inerter is a two-terminal mechanical element that can adjust the structural inertia characteristic without introducing considerable physical mass[1]. By the property that the mass of the left side in the equation of motion for an SDOF system increases with the addition of the inertance, while the mass of the right side remains unchanged (as seen in Eq. (1)), the inerter can be used to reduce the vibration response produced by earthquakes, wind, etc.

The previous study[2] has already proposed the mechanical model of the cable-bracing inerter system (CBIS). A CBIS is composed of the combination of a parallel system with an inertial mass element and a damping element, and a spring element connecting to the parallel system in series contracting a tuned viscous mass damper (TVMD)-like system. It uses tension-only cables to convert translational movement into rotational movement between the structure and the inerter-based devices. However, cables can not transmit compression force. Combined with compression-resistant materials, a nonlinear cable brace model[3] was proposed to transmit compression force and improve the performance of tension-only cables. Thus, it has the characteristics of being able to bear both tensile and compressive forces. A series of parametric analyses indicates that the CBIS with nonlinear cable model can outperform a tuned viscous mass damper system having symmetric stiffness support member.

In this paper, we focus on the optimal design problem to minimize the magnitudes of structural displacement responses, in which the fixed-point method is used to obtain the initial parameters. Then, seismic responses of the CBIS-controlled system are evaluated in the time domain taking the non-linearity and the damping enhancement effect into account.

2. Equations of motion

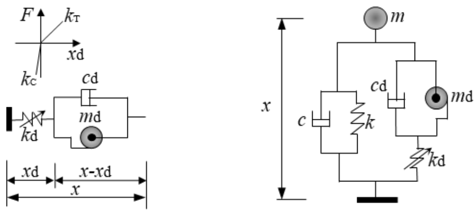
A nonlinear CBIS (shown in Fig.1a), is incorporated into an SDOF frame structure (shown in Fig.1b) with mass m, stiffness k, and damping coefficient c. With the ground displacement $\ddot{x}_g(t)$, the motion equation of this system is expressed as follows:

$$\begin{cases} m\ddot{x}(t)+c\dot{x}(t)+kx(t)+F=-m\ddot{x}_g(t) \\ F=k_\mathrm{d}x_\mathrm{d}(t)=m_\mathrm{d}(\ddot{x}_\mathrm{d}(t)-\ddot{x}(t))+c_\mathrm{d}(\dot{x}_\mathrm{d}(t)-\dot{x}(t)) \end{cases} \tag{1}$$

where $x(t)$ and $x_\mathrm{d}(t)$ are respectively the displacement of the primary structure relative to the ground and the deformation of the nonlinear cable brace. Superimposed dots indicate derivatives with respect to the time. We express the inertance of inerter element m_d and the damping coefficient of the inerter system c_d. F is the output force of the CBIS, and k_d is the equivalent stiffness of the nonlinear cable brace which is given by Eq. (2):

$$\begin{cases} k_\mathrm{d}=k_\mathrm{T}, x_\mathrm{d}>0 \\ k_\mathrm{d}=k_C, x_\mathrm{d}<0 \end{cases} \tag{2}$$

where k_T and k_C are the tension and compressive stiffnesses of the cable brace element, respectively (shown in Fig.1a).



(a)　　　　　　(b)

Fig. 1 Mechanical models of an SDOF structure with a nonlinear cable model

3. Optimum response control of an SDOF structure with a CBIS

In this section, a numerical optimization method, sequential quadratic programming (SQP), is used to design the inerter system parameters. First, the initial design parameters for the numerical optimization are determined according to the fixed-point method. The optimum angular frequency of the inerter system ω_r is obtained from the condition where the two fixed points[1] have the same ordinates on the resonance curves. The

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inertance-mass ratio μ is set to 0.1, and the optimum frequency ratio and optimum damping ratio can be obtained by:

$$\beta_{\mathrm{opt}} = \frac{\omega_{\mathrm{r}}}{\omega} = \frac{1}{\sqrt{1-\mu}} \tag{3}$$

$$\xi_{\mathrm{opt}} = \frac{1}{2}\sqrt{\frac{3\mu}{2-\mu}} \tag{4}$$

The inherent damping of the primary structure is ignored. The inerter system is tuned to the primary structure by setting the average of the compressive stiffness and the tension stiffness of cable braces equal to the supporting spring stiffness k_{d}:

$$k_{\mathrm{d}} = \frac{k_{\mathrm{C}} + k_{\mathrm{T}}}{2} = \beta_{\mathrm{opt}}^2 \mu k \tag{5}$$

The optimization design problem can be expressed mathematically to purse the optimal solutions:

$$\begin{cases} \text{find} & y = \{\xi,\ \mu,\ \kappa,\ \gamma,\} \\ \text{to minimize} & x_{\max}(y) \\ \text{subject to} & \kappa = \dfrac{2\mu}{(1+\gamma)(1-\mu)} \end{cases} \tag{6}$$

This procedure includes four key parameters[3], namely the inertance-mass ratio μ, supporting spring stiffness ratio κ, compression-tension stiffness ratio γ and damping ratio of an inerter system ξ. y is the design variable vector consisting of four parameters. The essential performance index is the maximum displacement responses $x_{\max}$ of the structure that need to be minimized. Using the SQP method, we employed an artificial earthquake BCJ-L2 as the input ground motion and obtained a set of optimal parameters listed in Table 1.

Table 1. Optimal Design

μ=0.1			
	κ	ξ	γ
Initial parameters	0.1111	0.1987	1.0000
Optimum parameters	0.1038	0.0147	1.1410

Then, time-history analyses based on Newmark's β method (β=1/4) were conducted to verify the accuracy of the SQP method (shown in Fig.2) and the vibration reduction effects are listed in Table 2.

Table 2. Responses of an SDOF structure (BCJ-L2)

	Displacement (m)		Acceleration (m/s^2)	
	Peak	RMS	Peak	RMS
Primary structure	0.2380	0.0579	10.5223	2.4534
With CBIS	0.1061	0.0298	7.0371	1.5974

The damping enhancement evaluation parameter α[4] is defined as Eq. (7) and is used to assess the energy dissipation capacity of the damping element in the inerter system. It this case, α is 2.40, which means the deformation of the damping element is amplified by the interactions among these elements and a better response mitigation effect is obtained.

$$\alpha = \frac{\mathrm{RMS}(x(t) - x_{\mathrm{d}}(t))}{\mathrm{RMS}(x(t))} \tag{7}$$

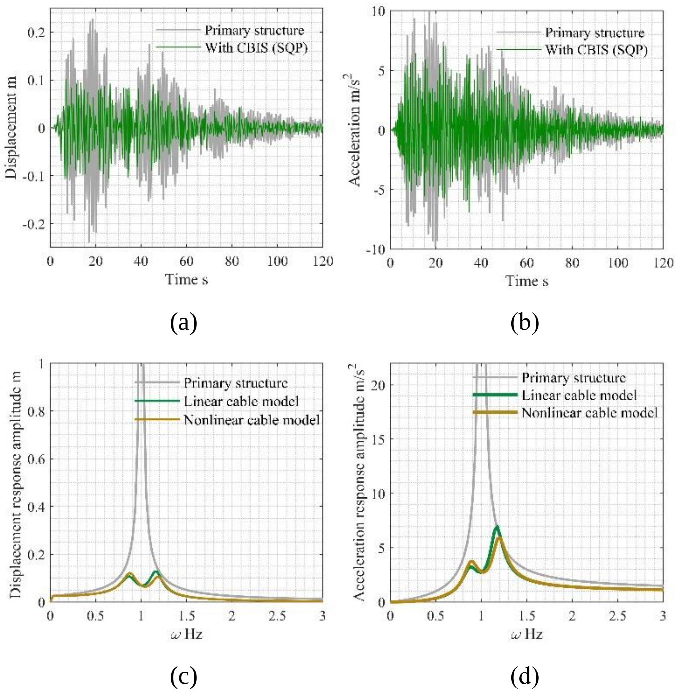

Fig. 2 Responses of an SDOF structure with and without a nonlinear CBIS (BCJ-L2)

4. Conclusions

In this paper, an optimum design problem to obtain a set of optimum parameters for a nonlinear CBIS that contains a supporting member having asymmetric spring stiffness in tension and compression is formulated. The transfer function of the optimized nonlinear CBIS indicates that it can outperform a TVMD system having symmetric stiffness spring. We conducted a time-history analysis to verify the optimum design derived using the SQP method and confirmed that the nonlinear CBIS efficiently mitigated the seismic response achieving damping enhancement effect.

References:

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