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Application of a grey control system model to structural damage identification

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Abstract
Development of a health monitoring system for construction has become an important task for structural damage identification. Since the building structure has many uncertain factors, the method of using modal analysis to identify the structural damage is of low accuracy. A method applying a grey system to the structural damage identification is presented. The grey system for the frequency variance rate and the stiffness change is established with the first-order single-argument grey system (GM(1, 1)). GM(1, 1) prediction is used to reflect the global function of the structural dynamic fingerprints and find the relation of the frequency variance rate and the stiffness change. Vibration tests for frame structures were carried out, with many cases considered, including single-damage and multi-damage ones with different degrees and locations. The results show that for shear buildings, the damage degree and location can be determined by using the grey system and measuring the frequency change.

1. Introduction

Grey system theory which is now successfully applied to the fields of engineering control, economic management, ecosystem, agricultural system etc was first presented by Deng in 1982.

In the field of structural damage identification, a structure can be looked upon as a system. Structural vibration is the output of the system, and the structural state can be deduced though vibration monitoring and analysis. In this system, the relations of the factors affecting the structural vibration and the relations between these factors, vibration parameters, energy and frequency are not mathematically definite, so a structure could be looked upon as a grey system. The process of structural identification by vibration monitoring is a process of grey system 'whitenization', and can be studied using grey system theory.

There have been some applications of grey system theory to structural damage identification (Geng *et al* 1997, Liu and Xu 1992) over the years, but these studies were only able to identify damage locations. In this paper, a method of application of a grey control system model to shear structure damage identification is carried out. The method uses only frequency measurement to establish the control system model, and carries out grey control of the prediction to find the optimum sequence, so that structural damage can be identified with higher accuracy.

2. Expression relating the variation of structural frequency to the variation of stiffness

For a shear building, the stiffness variation of the ith floor only causes the variations of k_{ii}, $k_{i,(i-1)}$, $k_{i-1,i}$ and $k_{i-1,i-1}$ which are adjacent to i in the stiffness matrix. If the stiffness reduction of the ith floor is given by Δk_i, two elements $k_{i,i}$ and $k_{i-1,i-1}$ of the stiffness matrices are reduced by Δk_i, while $k_{i,i-1}$ and $k_{i-1,i}$ are increased by Δk_i. The variance of the frequency is expressed by

$$\Delta\omega_r = \left(\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial\omega_r}{\partial k_{ij}}\Delta k_{ij} + \frac{1}{2!}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial^2\omega_r}{\partial k_{ij}^2}\Delta k_{ij}^2 + \cdots + \frac{1}{n!}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial^n\omega_r}{\partial k_{ij}^n}\Delta k_{ij}^n + \cdots\right) \quad (1)$$

where ω expresses frequency and N is the number of the model's storey.

To avoid nonlinear equations, the Taylor's series may be truncated to first order:

$$\Delta\omega_r/\omega_r = \left(\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial\omega_r}{\partial k_{ij}}\Delta k_{ij}\right)\Big/\omega_r = \frac{\Delta k_i}{2\omega_r^2}\cdot\left(\Phi_{ir}^2 + \Phi_{i-1,r}^2 - 2\Phi_{ir}\Phi_{i-1,r}\right) \quad (2)$$

where Φ expresses the eigenvector.

The truncated second-order Taylor's series is of higher accuracy:

$$\Delta\omega_r/\omega_r = \left(\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial\omega_r}{\partial k_{ij}}\Delta k_{ij} + \frac{1}{2!}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial^2\omega_r}{\partial k_{ij}^2}\Delta k_{ij}^2\right)\Big/\omega_r$$

$$= \frac{\Delta k_i}{2\omega_r^2}\cdot\left(\Phi_{ir}^2 + \Phi_{i-1,r}^2 - 2\Phi_{ir}\Phi_{i-1,r}\right)$$

$$+ \frac{\Delta k_i^2}{2\omega_r^2}\left[-\frac{1}{4\omega_r^2}\Phi_{ir}^4 - \frac{2}{\omega_r^2}\Phi_{ir}^2\Phi_{i-1,r}^2 - \frac{1}{4\omega_r^2}\Phi_{i-1,r}\right.$$

$$+ \Phi_{i,r}\sum_{\substack{s=1\\(s\neq r)}}^{N}\frac{1}{\omega_r^2-\omega_s^2}\Phi_{is}\Phi_{ir}\Phi_{is}$$

$$+ \Phi_{i-1,r}\sum_{\substack{s=1\\(s\neq r)}}^{N}\frac{1}{\omega_r^2-\omega_s^2}\Phi_{i-1,s}\Phi_{i-1,r}\Phi_{i-1,s}$$

$$+ 2\Phi_{i-1,r}\sum_{\substack{s=1\\(s\neq r)}}^{N}\frac{1}{\omega_r^2-\omega_s^2}\left(\Phi_{is}\Phi_{i-1,r} + \Phi_{i-1,s}\Phi_{ir}\right)\Phi_{is}$$

$$\left. + 2\phi_{ir}\sum_{\substack{s=1\\(s\neq r)}}^{N}\frac{1}{\omega_r^2-\omega_s^2}\left(\Phi_{i-1,s}\Phi_{ir} + \Phi_{is}\Phi_{i-1,r}\right)\Phi_{i-1,s}\right] \tag{3}$$

where the value of the last four terms in the square brackets is equal to zero when $s = t$.

It can be seen that equation (3) is a nonlinear equation which is challenging to solve.

So equation (1) is expressed as

$$\{\Delta\omega/\omega\} = ([S_1] + [S_2]\cdot\{\Delta k\} + [S_3]\cdot\left\{\Delta k^2\right\}\cdots + [S_n]\cdot\left\{\Delta k^{n-1}\right\} + \cdots)\{\Delta k\} \tag{4}$$

where $[S_1]$ is the coefficient of Δk_{ij}'s first-order term, while $[S_n]$ is the coefficient of Δk_{ij}'s nth-order term.

Let

$$[S] = [S_1] + [S_2]\cdot\{\Delta k\} + [S_3]\cdot\left\{\Delta k^2\right\}\cdots + [S_n]\cdot\left\{\Delta k^{n-1}\right\} + \cdots. \tag{5}$$

Then, equation (4) can be given as

$$\{\Delta\omega/\omega\} = [S]\cdot\{\Delta k\}\,. \tag{6}$$

3. Grey system model of frequency variance ratio and stiffness variance

3.1. Grey system model

Let state vector $X = \{x_1, x_2, \ldots, x_n\}$ express the frequency variance rate, while output vector $Y = \{y_1, y_2, \ldots, y_m\}$ is the stiffness variance; then the grey system model of the frequency variance rate and stiffness variance is written as

$$\begin{aligned}\dot{X} &= A(\otimes)X + B(\otimes)\\ Y &= C(\otimes)X\end{aligned} \tag{7}$$

where $A(\otimes) \in G^{n\times n}, B(\otimes) \in G^{n\times 1}, C(\otimes) \in G^{m\times n}$ and $B(\otimes)$ is the parameter matrix of the grey action. From comparison with equation (5), it can be seen that $(S^{\mathrm{T}})^{-1} = C(\otimes)$.

3.2. State equation determination

As regards the frequency variance sequence $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}$, a first-order single-variable grey model GM(1, 1) (Liu and Xu 1992) can be established which is a first-order single-variable grey prediction model, and the grey differential equation (Deng 1989) is given by

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{8}$$

where a, b are undetermined parameters, $Z^{(1)}$ is the mean generation with consecutive neighbours of $X^{(1)}$, $X^{(1)}$ is the first-order accumulating generation (1-AGO) of $X^{(0)}$, which can be written as

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1); \qquad k = 2, 3, \ldots, n \tag{9}$$

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i); \qquad k = 1, 2, \ldots, n. \tag{10}$$

The parameter column is

$$[a, b]^{\mathrm{T}} = \left(B^{\mathrm{T}}B\right)^{-1}B^{\mathrm{T}}Y \tag{11}$$

where

$$Y = \begin{bmatrix} x^{(0)}(2)\\ x^{(0)}(3)\\ \vdots\\ x^{(0)}(n)\end{bmatrix}, \qquad B = \begin{bmatrix} -z^{(1)}(2) & 1\\ -z^{(1)}(3) & 1\\ \vdots & \vdots\\ -z^{(1)}(n) & 1\end{bmatrix}.$$

Then the 'whitenization' equation (image equation) of the grey differential equation $x^{(0)}(k) + az^{(1)}(k) = b$ can be given as

$$\frac{\mathrm{d}x^{(1)}}{\mathrm{d}t} + ax^{(1)} = b. \tag{12}$$

The response sequence of the GM(1, 1) grey differential equation $x^{(0)}(k) + az^{(1)}(k) = b$ is

$$\hat{x}^{(1)}(k) = \left[x^{(0)}(1) - b/a\right]\mathrm{e}^{-ak} + b/a \qquad (k = 2, 3, \ldots, n) \tag{13}$$

where ˆ denotes the estimated value.

The reduction procedure is given by

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1). \tag{14}$$

The procedure can be duplicated or a residual model can be established as GM(1, 1) (Geng *et al* 1997) if the accuracy of the GM(1, 1) model is not satisfactory.

3.3. Output equation determination

From analysis of equations (5) and (3), $S = S_1 + \delta$ can be obtained and the elements in S_1 are coherent, so the elements in S_1 may be arranged one by one as a column vector to establish the first-order single-variable grey prediction model, $S_1^{(0)} \to \hat{S}_1^{(0)}$—that is to say, let $S_1^{(0)}$ be the first-order single-variable process according to the procedures from equations (8) to (14); then $S = \hat{S}_1^{(0)}$. Finally, $C(\otimes) = (S^{\mathrm{T}})^{-1}$, so $C(\otimes)$'s 'whitenization' value may be obtained.

Table 1. Parameters of the four-storey frame structural model.

Component	Cross section	Area of section (m^2)	Inertia moment of section (m^4)	Elastic ratio ($N\ m^{-2}$)	Density ($kg\ m^{-3}$)
Beam	T-section	6×10^{-4}	8.50×10^{-8}	3×10^9	1.2×10^3
Column	Rectangular	7×10^{-4}	5.8333×10^{-9}	3×10^9	1.2×10^3

Table 2. Damage in four-storey structure.

Mode no.	Test no.	Inertia moment of section before damage (m^4)	Inertia moment of section after damage (m^4)	Stiffness decrease of storey (%)	Affix
1	d15	1.16667×10^{-8}	0.9998×10^{-8}	14.3	Damage in storey 2
2	d1510	1.16667×10^{-8}	0.5833×10^{-8}	21.43	Damage in storey 2
3	d15	1.16667×10^{-8}	0.9167×10^{-8}	50.0, 21.43	Damage in storey 2, 3
4	d20	1.16667×10^{-8}	0.833×10^{-8}	50.0, 28.6	Damage in storey 2, 3

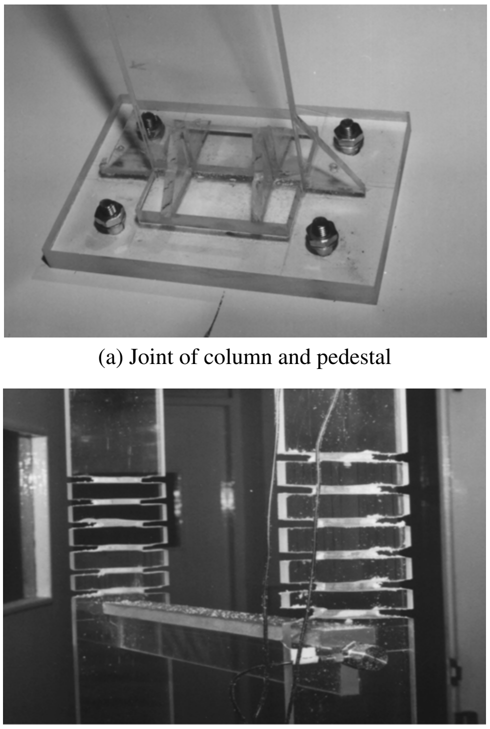
4. Vibration test study on the damage of the frame structure model

To prove the expression for the frequency variance, equation (7), vibration testing of the frame structural model (Xie 2001) is carried out. The model has four storeys with span length 330 mm and height of storey 350 mm. The column capital of the ground floor is connected to the baseplate by a haunched joint. Each beam with a T-section is spliced by two 30 mm × 10 mm rectangular beams. The section of each column is 70 mm × 10 mm. The parameters of the structural model are listed in table 1.

The model is made of organic glass, in which the joints of the beam and column are bonded by a special chemical solvent. To reduce the influence of vibration out of the plane, a single-pole single-span multistorey frame structure is constructed and the stiffness out of the plane of the columns is designed as far greater than that in the plane. The columns of the model structure are joined to a pedestal as shown in figure 1(a).

In order to simulate the reduction of structural stiffness, the columns of the model are slotted. To decrease the loss of structural mass, many thin notches, between which there is a rather short distance, are made in necessary places. So, the contribution from the part between notches to the stiffness can be ignored but the variance of the structural mass is quite small, so it can be thought that local stiffness is decreased but structural mass is unchanged. The case of simulated damage is shown in figure 1(b).

First, the simulation of a single damage location is proposed: the left column of the second floor is slotted, then the right column of the second floor is slotted to make the damage extend. Second, to simulate the double damage on the second floor and third floor, when the stiffness of the second floor is decreased by 50%, the third floor is to be slotted. An illustration of an operating mode is shown in table 2, where mode 1 and mode 2 are single-damage simulation examples while mode 3 and mode 4 are multi-damage examples. Acceleration sensors were installed in all storeys to obtain four acceleration components. The modal frequencies were calculated from four-dimensional time histories of the vibration experiment by applying an eigensystem realization algorithm (ERA) (Juang *et al* 1985). The frequency variances of different modes are shown in table 3.



(a) Joint of column and pedestal

(b) Simulation of column's damage

Figure 1. Test of the frame structure model.

Using the expression for the frequency variance, equation (7), with the measured frequency, the identification result for each damage mode is given in figure 2.

In figure 2, 'first-order' denotes the calculated stiffness reduction by application of equation (2); 'grey system' means the stiffness reduction calculated by application of the grey control system model; and 'actuality' is the actual stiffness reduction.

The identification of damage on the undamaged storeys arises from calculation error and experiment error; it is quite

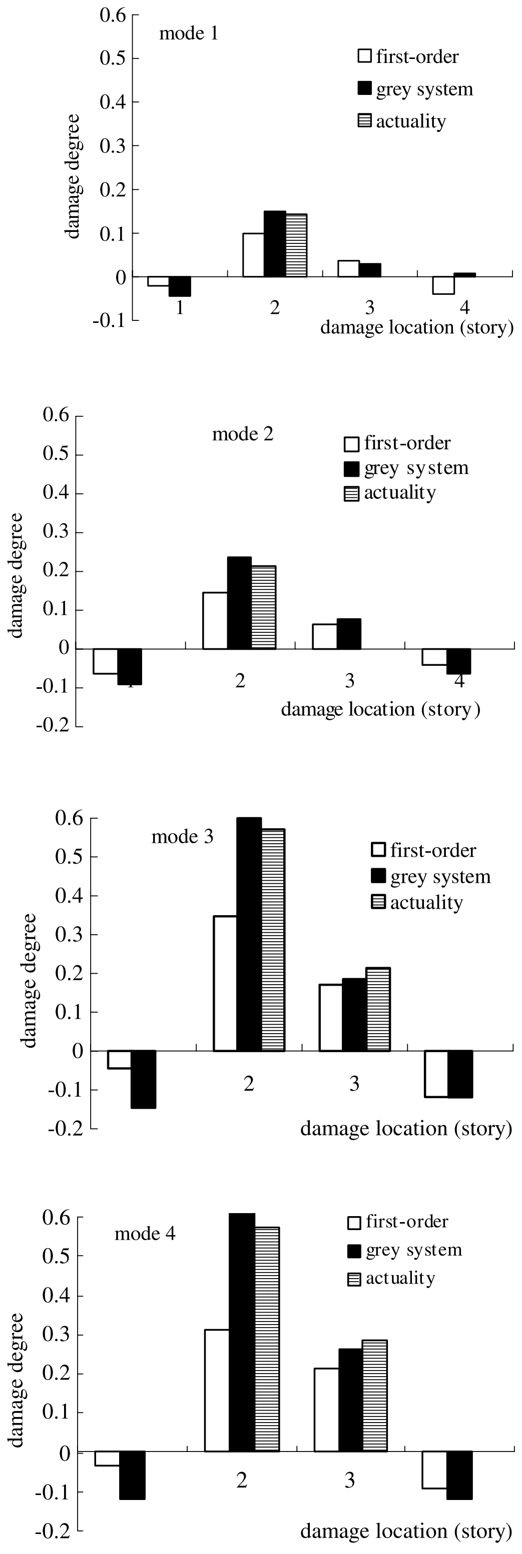


Figure 2. Comparison of actuality and identifications.

small relative to the identification of damage in the damaged storeys. And the stiffness variance may not be negative, so negative identification damage could be seen as zero; that is to say, the floor is undamaged.

From the comparative results for mode 1 and mode 2 which are single-damage cases in figure 2, it is seen that, by

Table 3. Measurement of frequency.

Mode no.	ω_1	(%)	ω_2	(%)	ω_3	(%)	ω_4	(%)
Before damage	8.179		24.69		40.1		51.09	
1	8.057	1.49	24.69	0	39.86	0.61	50.48	1.19
2	8.026	1.87	24.69	0	39.70	0.99	50.02	2.08
3	7.690	5.97	24.38	1.24	39.02	2.70	48.46	5.14
4	7.690	5.97	24.09	2.41	38.99	2.78	48.13	5.79

measurement of the structural frequency and application of a grey control system model, the identification of the damage degree and damage location may be achieved. The results from application of the grey control system model are better than ones based on first-order sensitivity and of higher security.

For example, in the condition of mode 1, the accuracy of identification of the damage degree by application of the grey control system model is higher by 25.87% than that achieved using equation (2), and the identification error is only 4.92%.

From the comparative results for mode 3 and mode 4 which are multi-damage cases in figure 2, it is seen that, by measurement of the structural frequency and application of the grey control system model, damage location may be identified accurately. Especially for the first case of damage (on the second floor), the accuracy of identification is greatly increased, and the results of higher security, which indicates its superiority. For subsequent damage (on the third floor), the accuracy of identification is somewhat increased, and the more the damage degree of subsequent damage is increased, the more apparent the superiority of the accuracy is. For example, in the condition of mode 4, for the first damage case, the identification accuracy of the damage degree in application of the grey control system model is higher by 38.10% than that achieved using equation (2), and the identification error is only 7.23%, while for the subsequent damage, the identification accuracy of the damage degree for application of grey control system model is increased by 17.33% over that achieved using equation (2), and the identification error is only 7.62%.

5. Conclusions

(1) From experiments and analyses it can be seen that the structural damage location and degree may be determined by measurements of the frequency variance for shear buildings.
(2) The damage identification accuracy is increased along with the damage degree increasing for not only single-damage cases but also multi-damage cases. For multi-damage cases especially, the superiority is apparent.
(3) For multi-damage cases, there is much uncertain complication, that is to say the degree of greyness in the grey system is greater, while application of a grey system could decrease randomness and reduce or even erase some disturbing factors. In reality, disturbances such as various noises are inevitable and the structure itself has many disturbing factors—that is to say, the structure is a grey system—so structural damage identification by application of a grey system model is very promising.
(4) This research focused solely on simple shear structures where all natural frequencies can be obtained. Further

research could investigate how to determine the damage location and degree for more complex structures.

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