SELF ADAPTIVE PARTICLE FILTER FOR STRUCTURAL SYSTEM IDENTIFICATION

H.S. Tang[1], S.T. Xue[1,2], C.X. Fan[3] and X.N. Yang[1]

[1] *Research Institute of Structural Engineering and Disaster Reduction, Tongji University, Shanghai, China*
[2] *Dept. of Architecture, School of Science and Engineering, Kinki University, Osaka, Japan*
[3] *Dept. of Civil Engineering, Suzhou University of Science and Technology,Suzhou, China*
Email: thstj@mail.tongji.edu.cn, xue@arch.kindai.ac.jp

ABSTRACT :

A self adaptive particle filter method for structural system identification is presented in this paper. Such an adaptive technique that uses statistical methods to adapt the number of particles at each iteration. This method improves the efficiency of state estimation by adapting the size of sample sets during the estimation process through KLD-Sampling. Within this adaptation process the number of samples is increased if the state uncertainty is high and decreased if the density distribution is focused on a small part of the state space. Simulation results of system identification for tracking the dynamic parameter changes are presented to demonstrate the effectiveness of the proposed method.

KEYWORDS: Self adaptive particle filter, system identification, KLD-Sampling

1. INTRODUCTION

In the field of civil engineering, real-time structural identification of dynamic system subjected to the earthquake motion has been focused on the accurate prediction as well as structural health monitoring and damage assessment. A widely adopted approach to addressing this problem is the Kalman filter over the past years (Shinozuka et al. 1982), which is optimal in the cases when the data are modeled as a Gaussian state-space model. When nonlinearities have to be tackled, Taylor series expansion around the operating point is adopted, which leads to the extended Kalman filter (EKF) method (Yang et al. 2005). Taylor series approximation leading to the EKF makes gross simplification of the probabilistic specification of the model. It has been shown in the literature that in many situations the EKF, due to the implemented approximations, can diverge in the tracking of the unknowns and in general can provide poor performance lead to estimation divergence. In recent, many alternative approaches to overcome the deficiencies of the EKF and the limitations of linear assumption and Gaussian characteristics of noise in Kalman filter, have developed including the Gaussian sum filters (Ito and Xiong 2000) and the unscented Kalman filter (van der Merwe et al. 2000; Tang and Sato 2005; Wu et al. 2007).

A recently developed filtering technique, called particle filter (PF) (also called Monte-Carlo filter, bootstrap filter, condensation, etc.) was proposed by Gordon et.al. (1993) and Kitagawa (1996). It is a useful tool to perform dynamic state estimation via Bayesian inference. It provides great efficiency and extreme flexibility to approximate any functional nonlinearity. Particle filters have recently been applied with great success to a variety of state estimation problems, such as radar tracking (Herman 2002), human motion tracking (McKenna 2007) and parameter identification (Li et al. 2004). This success is mostly due to their simplicity and their ability to represent arbitrary, nonlinear and non-Gaussian state space models. The increased representational power, however, comes at the cost of higher computational complexity. Most existing approaches to particle filters use a fixed number of samples during the entire state estimation process. This can be highly inefficient, since the complexity of the probability densities can vary drastically over time.

In this paper we introduce self adaptive real-time particle filters that greatly increase the performance of particle filters under limited computational resources. Our approach improves the efficiency of state estimation by adapting the size of sample sets during the estimation process through KLD-Sampling. Examples of structural parameters identification show that our approach yields drastic improvements over particle filters with fixed sample set sizes.

2. PARTICLE FILTERS FOR BAYESIAN ESTIMATION

In a general discrete-time stochastic system model, the evolution of the state sequence $\{\boldsymbol{x}_k, k \in \mathrm{N}\}$ of the system is given by

$$\boldsymbol{x}_k = \boldsymbol{f}(\boldsymbol{x}_{k-1}, \boldsymbol{v}_{k-1}) \tag{2.1}$$

where $\boldsymbol{f} : \mathrm{R}^{n_x} \times \mathrm{R}^{n_v} \to \mathrm{R}^{n_x}$ is a possibly nonlinear function of the state $\boldsymbol{x}_{k-1}$, $\{\boldsymbol{v}_{k-1}, k \in \mathrm{N}\}$ is an i.i.d. process noise sequence, n_x, n_v are dimensions of the state and process noise vectors, respectively, and N is the set of natural numbers. The objective of system is to recursively estimate form measurement

$$\boldsymbol{z}_k = \boldsymbol{h}(\boldsymbol{x}_k, \boldsymbol{n}_k) \tag{2.2}$$

where $\boldsymbol{h} : \mathrm{R}^{n_x} \times \mathrm{R}^{n_n} \to \mathrm{R}^{n_z}$ is a possibly nonlinear function, $\{\boldsymbol{n}_k, k \in \mathrm{N}\}$ is an i.i.d. measurement noise sequence, and $\boldsymbol{n}_z, \boldsymbol{n}_n$ are dimensions of the measurement and measurement noise vectors, respectively.

The main idea in Bayesian estimation is to compute the posterior density $\boldsymbol{x}_k$ based on the set of available measurements $\mathbf{z}_{1:k} = \{\mathbf{z}_i\}_{i=1}^{k}$ up to time *k*. The sequential Monte Carlo methods simulate this distribution with a finite number of samples, or “particles”. The more particles, the better the approximation gets. Because of the

concept of particles, these filters are often referred to as particle filters.

The particle filter approximates $p(\boldsymbol{x}_k \mid \boldsymbol{z}_k)$ with a set of N random samples $\{\boldsymbol{x}_k^i\}_{i=1}^N$, where each particle $\boldsymbol{x}_k^i$ is assigned a weight w_k^i. The weight of each particle should in some way reflect the probability that the properties of this particle are the correct ones. Those particles with the highest weights are propagated in time and natural selection is performed. It should be noted that the number of particles, N, has to be chosen large enough to accurately represent the underlying probability density function. At each iteration the operation of the particle filter can be seen as an importance sampling process. The sampling and weighting steps of the particle filter correspond to the basic steps of an importance sampling process. In this case, the samples are drawn from an importance function that corresponds to the dynamic prior $p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1})$ (Doucet et al. 2000). Using this importance function, a generic algorithm of sampling importance resampling (SIR) particle filter using transition prior density as proposed distribution is given as follows.

For time steps k = 0, 1, 2, …

1. Initialization: for i=1,…, N, sample $\boldsymbol{x}_0^i \sim p(\boldsymbol{x}_0)$, set $w_0^i = 1/N$.
2. Importance sampling: for i=1,…, N, draw samples $\boldsymbol{x}_k^i \sim p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}^i)$.
3. Weight update: calculate the importance wights $w_k^i = p(\boldsymbol{z}_k \mid \boldsymbol{x}_k^i)$ for i=1,…,N.
4. Normalize the importance weights: $\tilde{w}_k^i = \dfrac{w_k^i}{\sum_{j=1}^N w_k^j}$.
5. Resampling: generate N new particles $\boldsymbol{x}_k^j$ (j=1,…, N) from the set $\{\boldsymbol{x}_k^i\}_{i=1}^N$ according to the importance weights $\tilde{w}_k^i$.
6. Repeat steps 2 to 5.

Recursively calculating the above six steps generates the basic particles in system identification, which may estimate the system states and parameters on-the-fly.

The accuracy and robustness of particle filters are closely related with the dimension of particle state and the size of particle sets. Unluckily, the computational complexity is proportional to the size of particle sets and increases exponentially with the dimension of particles state. Consequently, the focused improvement to particle filters is seeking for more efficient trade-off between accuracy, simplification and robustness of the particle filtering algorithms.

3. THE SELF ADAPTIVE PARTICLE FILTERS (SAPF)

The selection of the number of particles is a key factor in the efficiency and accuracy of the particle filter. The computational load and the convergence of the filter depend on this number. Most applications select a fixed number of particles in advance. Unfortunately, the use of a fixed number of particles is often inefficient. The dynamics of most processes usually produces great variability in the complexity of the posterior distribution such as unexpected measurement loss and physical parameter changes in the dynamic system. As a consequence, the initial estimation of the number of particles can be much larger than the real number of particles needed to perform a good estimation of the posterior distribution or, worse, at some point, the selected number of particles can be too small causing the filter to diverge.

Obviously, the number of samples has a significant influence on the computational burden of the system. While a higher number of samples might be needed to represent the belief during early stages of the tracking process, a smaller number of samples should be sufficient once the estimate has converged to a compact distribution about the true values.

In order to improve the efficiency of the generic particle filter further, we propose KLD-Sampling (Fox 2001) based in the theory of statistics that can be used to adaptively estimate the number of particles to represent the target posterior distribution without adding a significant load to the normal operation of the filter. At each cycle of the particle filter, this technique estimates the number of particles that, with a certain level of

confidence, limits the maximum error in the approximation.

The approach used here was first introduced by Fox (2001) and is called KLD-Sampling. One can make use of Kullback-Leibler (KL) distance to update the necessary particle size in a certain error bound between the sample-based maximum likelihood estimate and the current approximation of the true posterior. Therefore, KLD sampling achieves improvement of real time performance while maintaining the estimate accuracy of particle filter in system identification.

KLD-Sampling is based on the assumption that the true posterior can be represented by a discrete piecewise constant distribution consisting of a set of multidimensional bins. To derive this bound, we assume that the true distribution is given by a discrete, multinomial distribution – depicted by a number of bins *k*. Since the error between the filter-estimate and the true distribution is measured by the Kullback-Leibler distance. The KL-distance describes the distance of two probability densities (*p* and *q*) as a kind of matching ratio of the distributions:

$$K(p,q) = \sum_x p(x) \log \frac{p(x)}{q(x)} \tag{3.1}$$

Suppose that *n* samples are drawn from a discrete distribution with *k* different bins. Let the vector $\bar{X}$ = (X_1, . . . ,X_k) denote the number of samples drawn from each bin. $\bar{X}$ is distributed according to a multinomial distribution, i.e. $\bar{X}$ ~Multinomial$_k$(n, $\bar{p}$), where $\bar{p}$ = p_1, …, p_k specifies the true probability of each bin.

The maximum likelihood (ML)estimate of $\bar{p}$ using the *n* samples is given by $\hat{p} = n^{-1}\bar{X}$. Furthermore, the likelihood ratio statistic λ_n for testing $\bar{p}$ is

$$\log \lambda_n = \sum_{j=1}^{k} X_j \log(\frac{\hat{p}_j}{p_j}) \tag{3.2}$$

Since X_j is identical to $n\hat{p}_i$, we get

$$\log \lambda_n = n\sum_{j=1}^{k} \hat{p}_j \log(\frac{\hat{p}_j}{p_j}) \tag{3.3}$$

From Eqn.3.1 and Eqn.3.3, we can see that the likelihood ratio statistic is *n* times the KL-distance between the MLE and the true distribution:

$$\log \lambda_n = nK(\hat{p}, p) \tag{3.4}$$

It can be shown that the likelihood ratio converges to a chi-square distribution with *k*-1 degrees of freedom:

$$2\log \lambda_n \rightarrow_d \chi^2_{k-1} \quad (n \rightarrow \infty) \tag{3.5}$$

Now let $P_{\bar{p}} = (K(\hat{\bar{p}}, \bar{p}) \le \varepsilon)$ denote the probability that the KL-distance between the true distribution and the sample-based MLE is less than or equal to ε (under the assumption that $\bar{p}$ is the true distribution). The relationship between this probability and the number of samples can be derived as follows:

$$P_p(K(\hat{p}, p) \le \varepsilon) = P_p(2nK(\hat{p}, p) \le 2n\varepsilon) \tag{3.6}$$

$$\cong P_p(\chi^2_{k-1} \le 2n\varepsilon) \tag{3.7}$$

Eqn.3.7 follows from Eqn.3.4 and the convergence result stated in Eqn.3.5. The quantiles of the chi-square distribution are given by

$$P_p(\chi^2_{k-1} \le \chi^2_{k-1,1-\delta}) = 1-\delta \tag{3.8}$$

If we choose *n* such that $2n\varepsilon$ is equal to $\chi^2_{k-1,1-\delta}$, we can combine Eqn.3.7 and Eqn.3.8 and get

$$P_p(K(\hat{p}, p) \le \varepsilon) = 1-\delta \tag{3.9}$$

Now we have a clear relationship between the number of samples and the resulting approximation quality. To summarize, if we choose the number of samples *n* as

$$n = \frac{1}{2\varepsilon}\chi^2_{k-1,1-\delta} \tag{3.10}$$

then we can guarantee that with probability 1-δ, the KL-distance between the MLE and the true distribution is less than ε. In order to determine n according to Eqn.3.10, we need to compute the quantiles of the chi-square distribution. A good approximation is given by the Wilson-Hilferty transformation (Johnson et al. 1994), which yields

$$n = \frac{1}{2\varepsilon}\chi^2_{k-1,1-\delta} \cong \frac{k-1}{2\varepsilon}\left(1 - \frac{2}{9(k-1)} + \sqrt{\frac{2}{9(k-1)}}z_{1-\delta}\right)^3 \tag{3.11}$$

where $z_{1-\delta}$ is the upper 1-δ quantile of the standard normal distribution. The values of $z_{1-\delta}$ for typical values of δ are readily available in standard statistical tables.

In the KLD-Sampling process the bins k are estimated by the number of grid cells that are at least occupied by one particle. To approximate the multinomial distribution, a fixed, three-dimensional grid is used. During the prediction step of the particle filter the algorithm checks if the newly generated particle falls into an empty cell of the grid or not. If the grid cell is empty, the number of bins k is incremented and the cell is marked as not-empty. The determination of k can be done incrementally by checking for each generated sample whether it falls into an empty bin or not. After each sample, the Eqn.3.11 is used to update the number n of samples required for current estimate of k. This process is repeated till no more empty grid cells are filled and k stops increasing. Consequently n stabilizes. The grid is reset after every filter update.

4. NUMERICAL EXAMPLES

In order to validate the self adaptive particle filter identification method, numerical simulations of a 3-DOF system as shown in Fig.2 are carried out. The equation of motion is given by

$$\boldsymbol{m}\ddot{\boldsymbol{u}} + \boldsymbol{c}\dot{\boldsymbol{u}} + \boldsymbol{k}\boldsymbol{u} = -\boldsymbol{m}\ddot{\mathrm{g}} \tag{4.1}$$

where $\boldsymbol{m}$, $\boldsymbol{c}$ and $\boldsymbol{k}$ are the mass, damping and stiffness matrices respectively, $\boldsymbol{u}$ the relative displacement vector to the ground and $\ddot{g}$ the ground motion acceleration.

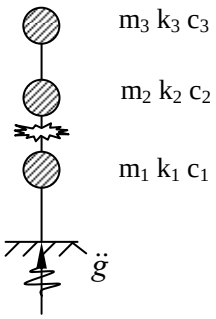

Figure 1 A 3-DOF structure system

Regarding the unknown parameters as state variables of the state vector $\boldsymbol{x}$, such as stiffness and damping, one can define an augmenting state transition as

$$\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \boldsymbol{v}_k \tag{4.2}$$

where $\boldsymbol{v}_k$ is the process noise.

When structural responses to the acceleration (or velocity, displacement) is available for the identified system, then the observation equation is

$$\boldsymbol{z}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{n}_k = -\boldsymbol{m}^{-1}\boldsymbol{c}_k\dot{\boldsymbol{u}}_k - \boldsymbol{m}^{-1}\boldsymbol{k}_k\boldsymbol{u}_k + \boldsymbol{n}_k \tag{4.3}$$

in which $\mathbf{z}_k$ is the observation, $\boldsymbol{H}_k$ is the observation matrix, and $\boldsymbol{n}_k$ is the observation noise.

The El Centro (NS, 1940) earthquake record with modified maximum amplitude of 25 cm/sec^2 was input excitation. The seismic responses of the system were simulated as observation data for identification. The sampling interval of the structural responses to be used for identification is 0.02s. The following parametric values are used in the simulation study: m_i = 12.553 kg, c_i=0.07 kN s/m, k_i =24.5 kN/m (i=1, 2, 3). For verifying the online non-stationary tracking ability of the proposed method, it is assumed that damage occurs at the second node, where the stiffness decreases from 24.5 to 19.6, and damping increases from 0.07 to 0.105 at t=15s. In this example, the parameter ε is 0.005, and 1-δ is 95%. The initial particle number is set as 10000.

Figures 2-4 show the results of tracking the stiffness and damping using the self adaptive particle filter. It can be seen from these figures that the SAPF successfully track the abrupt changes of the stiffness and damping.

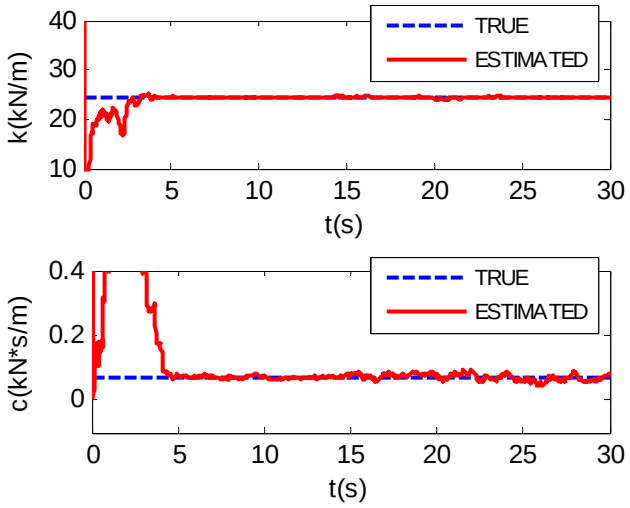

Figure 2 The estimated results of the 1st story

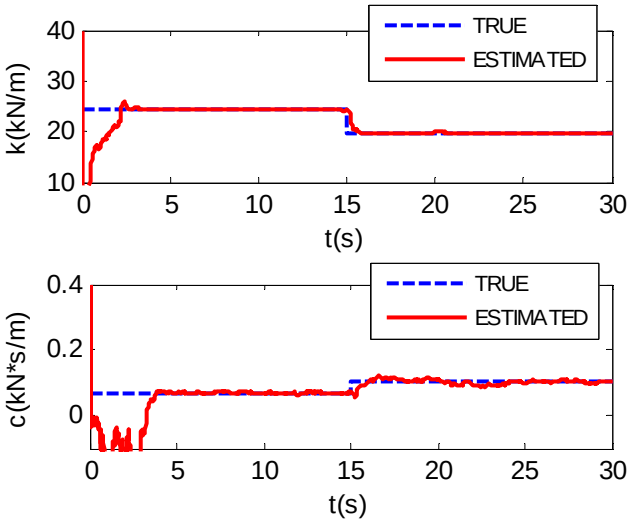

Figure 3 The estimated results of the 2nd story

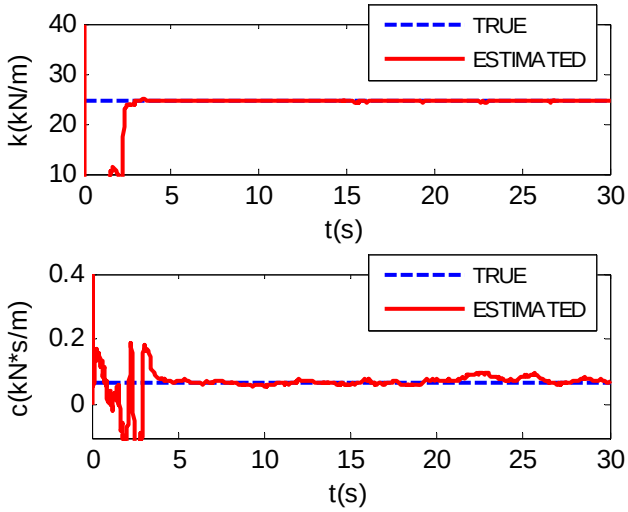

Figure 4 The estimated results of the 3rd story

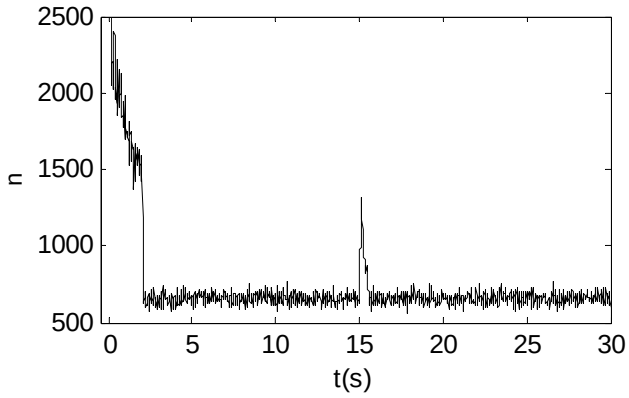

Figure 5 Typical evolution of number of samples for a tracking parameter sudden changes run

Figure 5 shows the sample set size during typical tracking sudden changes of the system parameters run using KLD-Sampling. From the figure 5, it shows that the particles number will adaptive turning with system parameter abrupt changes in real time through the KLD-Sampling. These figures indicate that the SAPF, even though based on several approximations, is able to accurately track the parameters' sudden changes using far smaller sample sets on average. Therefore, the SAPF method is suitable to solve the real time damage detection problems.

ACKNOWLEDGMENTS

This work was supported by the Nature Science Foundation of China No. 50708076 and SRF for ROCS, SEM of China.

CONCLUSIONS

A statistical method called self adaptive particle filtering method based KLD-Sampling to adapt the sample set sizes of particle filters during the structural system identification process is presented. The SAPF method is to bound the error introduced by the sample-based belief representation. At each iteration, the SAPF generates samples until their number is large enough to guarantee that the KL-distance between the maximum likelihood estimate and the underlying posterior does not exceed a pre-specified bound. SAPF method is most

advantageous when the complexity of the system parameters change drastically over time, as is the case, for example, in real time damage detection.

REFERENCES

Doucet, A., Godsill, S., Andrieu, C. (2000). On sequential Monte Carlo sampling methods for Bayesian filtering. *Statistics and Computing* **10:3**, 197-210

Fox, D. (2001). KLD-sampling: Adaptive Particle Filters. *Advanced in Neural Information Processing Systems* 14, NIPS-2001, MIT press.

Gordon, N. J., Salmond, D. J. and Smith, A. F. M. (1993). Novel approach to nonlinear/non-Gaussian Bayesian state estimation. *IEE Proceedings-F* **140:2**, 107–113.

Herman, S.M. (2002). *A Particle Filtering Approach to Joint Passive Radar Tracking and Target Classification*. PhD thesis, Univ. of Illinois at Urbana-Champaign.

Ito, K. and Xiong, K.(2000). Gaussian filters for nonlinear filtering problems. *IEEE Trans. Automat. Contr.***45:5**, 910-927, 2000.

Johnson, N., Kotz, S. and Balakrishnan, N. (1994). Continuous univariate distributions, vol. 1. John Wiley & Sons, New York.

Kitagawa, G. (1996). Monte Carlo filter and smoother for non-Gaussian nonlinear state space models. *Journal of Computational and Graphical Statistics* **5**, 1–25.

Li, S.J., Suzuki, Y. and Noori, M. (2004), Identification of hysteretic systems with slip using bootstrap filter. *Mechanical Systems and Signal Processing* **18**, 781-795.

McKenna, S. J., Nait-Charif, H. (2007). Tracking human motion using auxiliary particle filters and iterated likelihood weighting. *Image and Vision Computing* **25:6**, 852-862.

Shinozuka, M., Yun, C., Imai, H. (1982). Identification of linear structural dynamic systems. *ASCE J. Eng. Mech* .**108:6**, 1371–1390.

Tang, H, Sato, T. (2005). SVD-based Derivative-Free Kalman Filter for Nonlinear Hysteretic Structural System Identification. 9th International Conference on Structural Safety and Reliability (ICOSSAR'05), Rome, Italy.

van der Merwe, R., de Freitas, J. F. G., Doucet, A. and Wan, E.(2000). The unscented particle filter. Tech. Rep. CUED/FINFENG/TR 380, Cambridge Univ. Engineering Dept.

Wu, M. and Smyth, A. W. (2007). Application of the unscented Kalman filter for real-time nonlinear structural system identification. *Struct. Control Health Monit.* **14**, 971-990.

Yang, J., Lin, S., Huang, H., Zhou, L. (2005). An adaptive extended Kalman filter for structural damage identification. *Structural Control and Health Monitoring* **13:4**, 849–867.