Nonlinear identification of base-isolated buildings by reverse path method

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Nonlinear Identification of Base-Isolated Buildings by Reverse Path Method

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Abstract: The performance of reverse path methods applied to identify the underlying linear model of base-isolated structures is investigated. The nonlinear rubber bearings are considered as nonlinear components attached to an underlying linear model. The advantage of reverse path formulation is that it can separate the linearity and nonlinearity of the structure, extract the nonlinearity and identify the underlying linear structure. The difficulty lies in selecting the nonlinearity function of the hysteretic force due to its multi-valued property and path-dependence. In the thesis, the hysteretic force is approximated by the polynomial series of displacement and velocity. The reverse path formulation is solved by Nonlinear Identification through Feedback of Output (NIFO) methods using least-square solution. Numerical simulation is carried out to investigate the identification performance.

Keywords: NIFO, reverse path method, nonlinear identification, hysteresis

1 Introduction

For the purpose of system identification, most structures are approximated by a linear model. And the modal parameters (frequency, modal shape, etc.) or physical parameters (stiffness and damping coefficient) describe the status of linear structures, the variance of these parameters indicates the damage to the structure. However, nonlinearity exists inevitably in a real world, and sometimes it behaves so strongly that the methods for linear structures fail to identify the system and detect the damage by linear approximation. For example, it is well observed that base-isolated buildings exhibit nonlinear and hysteretic properties under experiment situation. The dynamic parameters will vary with the intensity of excitation even if the system is intact, consequently the change of these parameters can not indicate the status of nonlinear systems.

In recent years, system identification of nonlinear dynamic structures has made significant progress. Kerschen et al. [1] investigated the past and recent developments in nonlinear system identification methods. Although the number of methods is large, there are no method to identify any type of nonlinearity.

Rice & Fitzpatrick [2], and Bendat [3] proposed a reverse path identification method for single-input/single-output nonlinear systems. This method treats the response as the input, and the excitation force as the output. The nonlinear term is considered as a feedback term. It tracks unknown parameters in frequency domain. Rice & Fitzpatrick [4] extended this method to identify two DOFs nonlinear systems treating each response location as a SDOF mechanical oscillator. However, this approach requires excitations to be applied at every nonlinear location.

Richards & Singh [5] developed a conditioned reverse path method (CRP). The improved technique separates the nonlinear part of the system response from the linear part and constructs a hierarchy structure of uncorrelated response components in frequency domain. CRP removes the restriction that the excitation must be applied at the location of the nonlinearity in order to identify its unknown parameters. Adams & Allemang [6] proposed a frequency domain method for estimating parameters of nonlinear parametric models by using the spatial information and treating the nonlinear forces as internal feedback forces in the underlying linear system. This method is called Nonlinear Identification through Feedback of the Outputs (NIFO), and it requires measurements

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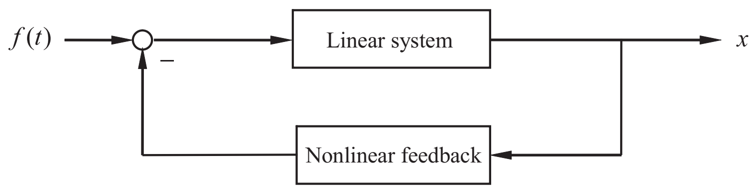

Figure 1: Nonlinear model with feedback

of both inputs and outputs and identifies the Frequency Response Function (FRF) of the underlying linear system as well as the parameters related to nonlinearities with light computational effort. Unlike CRP methods, NIFO methods estimate the linear and nonlinear coefficients in a single step.

Recently, Marchesiello & Garibaldi [7] developed an efficient time domain method called Nonlinear Subspace Identification (NSI) for identifying nonlinear systems by exploiting subspace identification methods. Later, Marchesiello & Garibaldi[8] discussed the identification problem for clearance-type nonlinearity by NSI methods. NSI is able to treat many nonlinearities at the same time, several ad hoc functions are defined and adopted in order to identify the clearance-type characteristic, showing advantages with respect to the traditional polynomial approach.

The advantage of reverse path formulation is that it can separate the linearity and nonlinearity of the structure, extract the nonlinearity and identify the underlying linear structure. Therefore, the modal and physical parameters of the underlying linear structure become meaningful again.

Here, we use NIFO methods to identify the underlying linear model of hysteretic systems subjected to external force. The hysteretic restoring force is modeled by polynomial approximation of displacement and velocity. Numerical simulation is carried out to investigate the identification performance.

2 Reverse Path

The reverse path method exploits traditional spectral analysis techniques to identify nonlinear systems, treating the input-output relationship of excitation and response in a reversal way. The advantage of this method is that it can utilize the well-established linear analysis techniques for nonlinear systems. This idea is illustrated via a simple example. Let's consider the symmetric Duffing equation

$$m\ddot{x} + c\dot{x} + kx + k_3x^3 = f(t) \tag{1}$$

where m, c and k are the mass, damping coefficient and stiffness of the underlying linear system, respectively. k_3 is the coefficient of the cubic nonlinear term. $f(t)$ is the external excitation.

Generally, a nonlinear system can be described by Figure 1, as a linear system with nonlinear feedback. The underlying linear system is defined by the linear frequency response function

$$H(\omega) = \frac{1}{-\omega^2 m + i\omega c + k} \tag{2}$$

where $i = \sqrt{-1}$. This transfer function relates an "effective" excitation $f_e(t)$ with displacement response x, where the effective excitation force is defined as

$$f_e(t) = f(t) - k_3x^3 \tag{3}$$

with k_3x^3 as nonlinear feedback.

Now, we consider the above nonlinear system in an alternative viewpoint. Taking the Fourier transform $\mathcal{F}[\bullet]$ of Eq. (1) gives

$$B(\omega)X(\omega) + A(\omega)Y(\omega) = F(\omega) \tag{4}$$

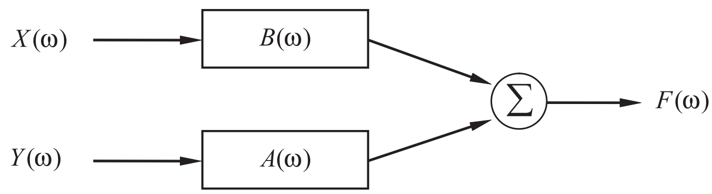

Figure 2: Reverse path model

where $B(\omega) = 1/H(\omega)$, $X(\omega) = \mathcal{F}[x(t)]$ and $F(\omega) = \mathcal{F}[f(t)]$. For the nonlinear term, $A(\omega) = k_3$ and $Y(\omega) = \mathcal{F}[x^3(t)]$.

By exchanging the roles of the input and output, the displacement response x is considered as the input and the excitation force $f(t)$ as the output. Thus, the SDOF nonlinear system is viewed as a two-input/single-output system as illustrated in Figure 2.

Multiplying Eq. (4) by $X(\omega)$ and taking the expectation, we obtain

$$B(\omega)S_{xx}(\omega) + A(\omega)S_{xy}(\omega) = S_{xf}(\omega) \tag{5}$$

Similarly, Multiplying Eq. (4) by $Y(\omega)$ gives

$$B(\omega)S_{yx}(\omega) + A(\omega)S_{yy}(\omega) = S_{yf}(\omega) \tag{6}$$

These two equations form a set of simultaneous equations. By solving them, we can obtain unknown $B(\omega)$ and $A(\omega)$ at every frequency.

$$\begin{bmatrix} S_{xx}(\omega) & S_{xy}(\omega) \\ S_{yx}(\omega) & S_{yy}(\omega) \end{bmatrix} \begin{pmatrix} B(\omega) \\ A(\omega) \end{pmatrix} = \begin{pmatrix} S_{xf}(\omega) \\ S_{yf}(\omega) \end{pmatrix} \tag{7}$$

It is worth noting that the coefficient $A(\omega)$ is estimated as a complex function with respect to frequency. The real part of the result represents the coefficient k_3, and the imaginary part is meaningless with magnitude much smaller than the real part. The limitation of this method for MDOF systems is that the excitation must be applied at the location of nonlinearities.

3 Nonlinear Identification through Feedback of Output Method

The NIFO method is a spectral approach for identifying MDOF nonlinear systems, developed by Adams & Allemang [6]. It interprets the nonlinearities as the unmeasured internal feedback forces, and exploits the spatial information of type of nonlinearities.

Let's consider the Duffing equation again. We can rewrite Eq. (4) in the frequency domain as

$$B(\omega)X(\omega) = F(\omega) - A(\omega)Y(\omega) \tag{8}$$

The nonlinear forces, viewed as internal feedback forces, are the functions of the measured output (cubic function of the displacement x^3). Eq. (8) shows that the underlying linear system is excited by two forces: one is the external force $F(\omega)$, the other is the internal nonlinear feedback due to the nonlinearity $Y(\omega)$. Premultiplying Eq. (8) by the transfer function of the underlying linear system $H(\omega)$ gives

$$X(\omega) = H(\omega)F(\omega) - H(\omega)A(\omega)Y(\omega) \tag{9}$$

in matrix form, we have

$$X(\omega) = \begin{bmatrix} H(\omega) & H(\omega)A(\omega) \end{bmatrix} \begin{pmatrix} F(\omega) \\ -Y(\omega) \end{pmatrix} \tag{10}$$

where $Y(\omega) = \mathcal{F}[x^3(t)]$ is a known nonlinear function. This suggests that the type of nonlinearities should be known as a function of measured outputs before we identify nonlinear systems. The configuration of nonlinearity types is the spatial information of nonlinear systems. Solving Eq. (10) gives the estimates of the FRFs of the underlying linear system $H(\omega)$ and the coefficient $A(\omega)$ of nonlinearities in a single step. A stable least-square solution with spectral averaging is suggested to identify the parameters related to structural nonlinearities.

If the NIFO method is applied to MDOF nonlinear systems, Eq. (10) can be given in matrix form

$$\mathbf{X}(\omega) = \begin{bmatrix} \mathbf{H}(\omega) & \mathbf{H}(\omega)\mathbf{A}_1(\omega) & \cdots & \mathbf{H}(\omega)\mathbf{A}_n(\omega) \end{bmatrix} \begin{pmatrix} \mathbf{F}(\omega) \\ -Y_1(\omega) \\ \vdots \\ -Y_n(\omega) \end{pmatrix} \tag{11}$$

where $\mathbf{X}(\omega)$ is a p-dimensional output vector, and $\mathbf{F}(\omega)$ is a q-dimensional input vector. $Y_n(\omega)$ represents different types of nonlinearities. $\mathbf{H}(\omega)$ is the frequency response function of the underlying linear structure, a $p \times q$-dimensional matrix. $\mathbf{A}_n(\omega)$ indicates the location of nonlinear types, a $q \times 1$-dimensional matrix.

Rewrite Eq.(11) in a compact form

$$\mathbf{X}(\omega) = \mathbf{H}_o(\omega)\mathbf{F}_o(\omega) \tag{12}$$

where $\mathbf{H_o}(\omega)$ is a $p \times (q+n)$-dimensional matrix, and $\mathbf{F_o}(\omega)$ is a $(q+n)$-dimensional vector.

3.1 Least Square Solution

Assuming that data length of measurements is N, we divide it in n_s segments possible overlapped. Then, an overdetermined set of linear equations is obtained.

$$\{\mathbf{Z}(\omega)\}_{n_s \times p} = \{\mathbf{P}(\omega)\}_{n_s \times (q+n)} \{\mathbf{H}_o(\omega)^T\}_{(q+n) \times p} \tag{13}$$

and

$$\{\mathbf{Z}(\omega)\}_{n_s \times p} = \begin{bmatrix} \mathbf{X}(\omega)_1 & \mathbf{X}(\omega)_2 & \cdots & \mathbf{X}(\omega)_{n_s} \end{bmatrix}^T \tag{14}$$

$$\{\mathbf{P}(\omega)\}_{n_s \times (q+n)} = \begin{bmatrix} \mathbf{F}_o(\omega)_1 & \mathbf{F}_o(\omega)_2 & \cdots & \mathbf{F}_o(\omega)_{n_s} \end{bmatrix}^T \tag{15}$$

where the superscript T means transpose of a matrix.

Eq. (13) is the well-known least-square problem with the solution at fixed frequency

$$\mathbf{P}(\omega)^T\mathbf{P}(\omega)\mathbf{H}_o(\omega)^T = \mathbf{P}(\omega)^T\mathbf{Z}(\omega) \tag{16}$$

where $\mathbf{P}(\omega)^T\mathbf{P}(\omega)$ is called the information matrix. In order to avoid possible ill-conditioning from the formation of $\mathbf{P}(\omega)^T\mathbf{P}(\omega)$, the orthogonal decomposition is suggested via Gram-Schmidt orthogonalization.

For convenience, ω is omitted in matrix representation. For example, $\mathbf{P}(\omega)$ is abbreviated as $\mathbf{P}$. The classical Gram-Schmidt orthogonalization factorizes $\mathbf{P}$ by Cholesky decomposition as

$$\mathbf{P} = \mathbf{W}\mathbf{S} \tag{17}$$

where

$$\mathbf{S} = \begin{bmatrix} 1 & s_{12} & s_{13} & \cdots & s_{1(q+n)} \\ & 1 & s_{23} & \cdots & s_{1(q+n)} \\ & & \ddots & \ddots & \vdots \\ & & & 1 & s_{(q+n-1)(q+n)} \\ & & & & 1 \end{bmatrix} \tag{18}$$

is a $(q+n) \times (q+n)$ unit upper triangular matrix, and

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}_1 & \cdots & \mathbf{w}_{q+n} \end{bmatrix} \tag{19}$$

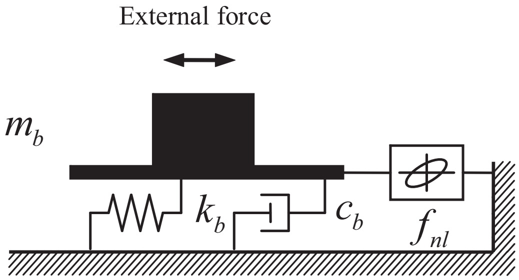

Figure 3: SDOF model

is a $n_s \times (q+n)$ matrix with orthogonal columns that satisfy

$$\mathbf{W}^T\mathbf{W} = \mathbf{D} \tag{20}$$

and $\mathbf{D}$ is a positive diagonal matrix.

The Gram-Schmidt procedure calculates $\mathbf{S}$ one column at one time and orthogonalizes $\mathbf{P}$ as follows.

$$\begin{array}{l} \mathbf{w}_1 = \mathbf{p}_1 \\ \left.\begin{array}{l} s_{ik} = \frac{<\mathbf{w}_i, \mathbf{p}_k>}{<\mathbf{w}_i, \mathbf{w}_i>}, 1 \le i < k \\ \mathbf{w}_k = \mathbf{p}_k - \sum\limits_{i=1}^{k-1} s_{ik}\mathbf{w}_i \end{array}\right\} k = 2, \cdots, q+n \end{array} \tag{21}$$

where $<,>$ denotes the inner product.

Define

$$\mathbf{g} = \mathbf{D}^{-1}\mathbf{W}^T\mathbf{Z} \tag{22}$$

then the unknown matrix $\mathbf{H}_o$ can be obtained using backward substitution.

$$\mathbf{S}\mathbf{H}_o{}^T = \mathbf{g} \tag{23}$$

4 Numerical Simulation

4.1 Simulation Model

According to the experimental observation, Yoshida et al. [9] concluded that the elastomeric bearings and friction-type isolators exhibit little rate dependence and quite stable hysteresis loops, a smooth and differential hysteresis model is adopted for the hysteretic dampers in the dynamic simulation. A widely accepted one is the Bouc-Wen hysteresis model proposed by Bouc [10] and generalized by Wen [11]. Baber & Noori [12] and other researchers incorporate the deterioration of hysteretic characteristics.

The differential equation of the Bouc-Wen model is

$$D_y\dot{z} = A\dot{u} - (\gamma \mathrm{sgn}(z\dot{u}) + \beta)\left|z\right|^n \dot{u} \tag{24}$$

and the hysteretic force of the isolation layer is given by

$$f_{nl} = F_y z$$

The terms D_y and F_y are respectively the yield displacement and force of the hysteretic damper; z is a dimensionless parameter; A,β and γ are parameters that describe the shape of the hysteresis loop; and u and $\dot{u}$ are respectively the displacement and velocity of the isolation layer. The smoothness of transition from elasticity to plasticity is determined by n, and when $n \to \infty$ the hysteresis model is reduced to a bilinear case.

Table 1: Parameter values

Argument	Value
m_b	8000 ton
k_b	475000 kN/m
c_b	6000 kN·s/m
β	0.6
γ	0.4
n	1
D_y	1.5 mm
F_y	655.4 kN

A simplest case is considered to study the performance of proposed methods. A single DOF base-isolated system, as shown in Figure 3, is subjected to the external force. The equation of motion of this system is

$$m_b\ddot{x} + c_b\dot{x} + k_b x + f_{nl} = F \tag{25}$$

where m_b, c_b and k_b are the mass, damping coefficient and stiffness of the SDOF system, respectively. The parameters of the system are defined as in Table 1. This system is excited by Gaussian noise with various variance. Obviously, the response depends on the intensity of excitation.

4.2 Nonlinearity Function

The nonlinearity function is required to be known before application. This problem can be overcome by assuming a function space as a search space, when the nonlinearity type is not clear. Masri & Caughey [13] and Masri et al. [14] proposed a nonparametric method named restoring force surface (RFS) methods by using polynomial approximation to fit the experimentally determined restoring force. The nonlinear restoring force is considered as a function of displacement and velocity by an ordinary or Chebyshev polynomial series expansion.

When hysteretic nonlinearity is involved, the nonlinear restoring force appears as a multivalued function of displacement and velocity due to its path-dependent nature. The choice of nonlinearity type poses a cumbersome matter. The polynomial approximation of displacement to fit the nonlinear restoring force will result in a memoryless nonlinear stiffness model, losing hysteretic characteristics. The polynomial approximation of displacement and velocity representing the hysteretic force could relax the memoryless property in a certain level, but could not eliminate its multivalued-function property. Hammond et al. [15] proposed to plot the surface of nonlinear force in a selected space (subset of the state vector, SSV). They defined the restoring force derivative with respect to time as a function of velocity and restoring force. Benedettini et al. [16] studied the performance of polynomial approximation of hysteretic systems in detail. It was shown that the SSV formula could fit the experimental response time histories much better than the traditional formula. However, a recursive procedure is required to solve the problem that the hysteretic force is a evaluated through the integration of its derivative. Here, the traditional formula are used to describe the nonlinear force, further investigation will explore the way of integrating SSV models with NIFO methods.

The hysteretic force is approximated by the following series

$$F_h = \sum a_{ij} x^i v^j \tag{26}$$

where v is velocity and a_{ij} are coefficients of polynomial terms. The linear terms of displacement and velocity are excluded because they are remained to be identified as a part of the underlying linear system. Each polynomial term could be viewed as a nonlinearity type.

4.3 Results by NIFO

The SDOF system is excited by a zero-mean Gaussian random input, whose standard deviation is 798.8 kN so that the maximum displacement response is comparable with the yield displacement of the hysteretic damper,

Table 2: Simulation and NIFO parameters

Argument	Value
$\Delta t(s)$	0.005
Time span (s)	500
Block size	10000
Overlap (%)	90
Averages n_s	89

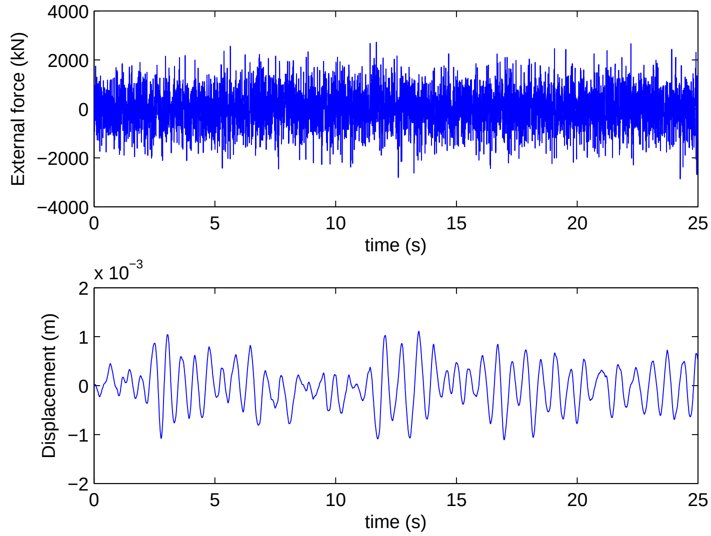

Figure 4: Excitation and response (standard deviation of excitation is 798.8 kN)

as shown in Figures 4 and 5. Numerical integration is conducted by using Simulink MATLAB® , and a total number of 10^5 samples has been generated for the NIFO application (Table 2).

Polynomial approximation of hysteretic force is up to third order.

$$F_h = a_{20}x^2 + a_{11}xv + a_{02}v^2 + a_{30}x^3 + a_{21}x^2v + a_{12}xv^2 + a_{03}v^3 \tag{27}$$

As shown in Figure 6, the NIFO and linear estimates are compared with the underlying linear frequency response function. Although the NIFO estimate achieved better performance than the linear estimate, it still can not retrieve the accurate information of the underlying linear system. The reason is that the multivalued-property remains so that the polynomial approximation can not represent the hysteretic force accurately. Figure 7 shows the NIFO estimate by using polynomial approximation of displacement only. The memoryless property of the nonlinear approximation function makes the performance worse near the vicinity of resonant frequency.

The system is subject to a more intensive Gaussian random input, whose standard deviation is 2403 kN. The maximum displacement is more than two times of the yield displacement, as shown in Figures 8 and 9. The NIFO estimate in Figure 10 deviates further away from the linear value, when the nonlinearity gets stronger.

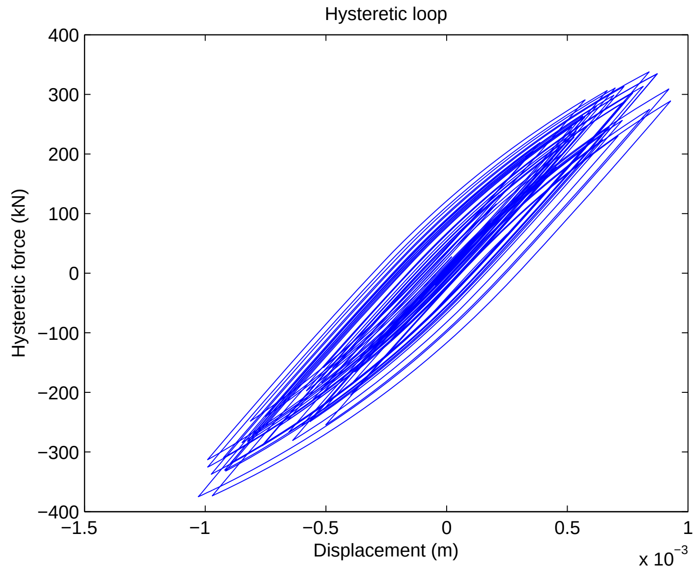

Figure 5: Hysteretic loop (standard deviation of excitation is 798.8 kN)

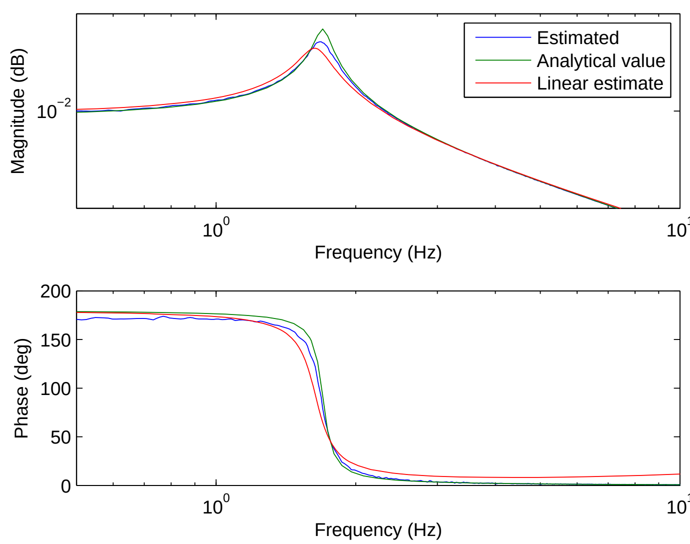

Figure 6: Frequency response function by NIFO using polynomial approximation of displacement and velocity

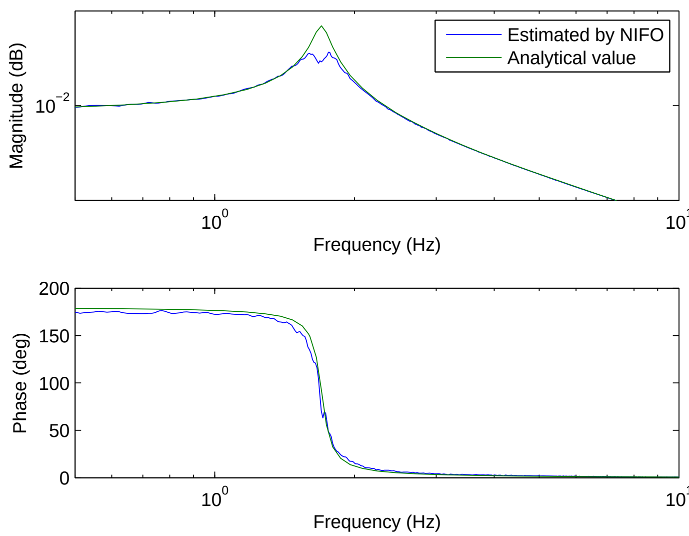

Figure 7: Frequency response function by NIFO using polynomial approximation of displacement

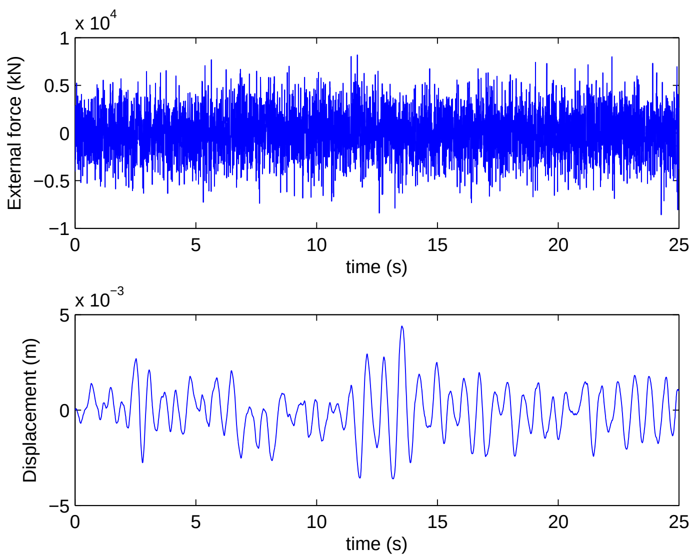

Figure 8: Excitation and response (standard deviation of excitation is 2403 kN)

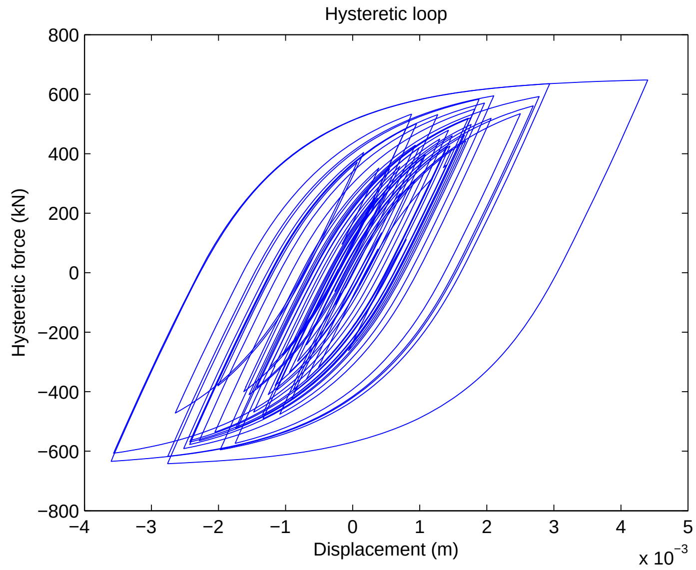

Figure 9: Hysteretic loop (standard deviation of excitation is 2403 kN)

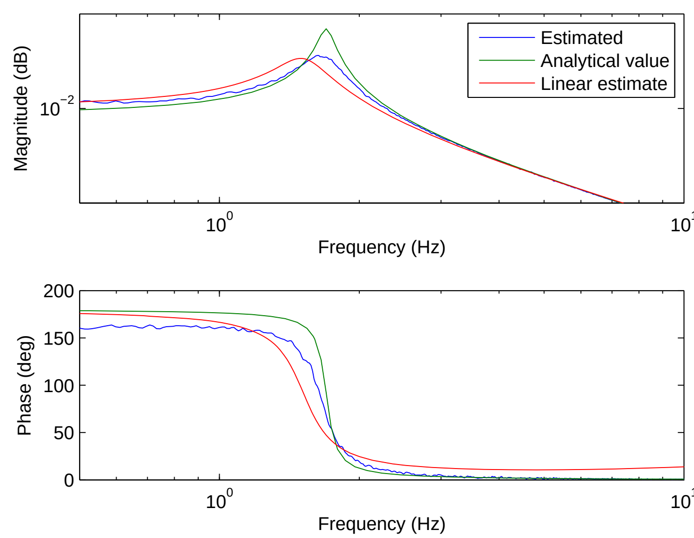

Figure 10: Frequency response function by NIFO using polynomial approximation of displacement and velocity

5 Conclusions

A nonlinear identification technique named NIFO has been presented to identify hysteretic systems. The hysteretic restoring force is modeled by polynomial approximation of displacement and velocity. The idea of reverse path is attractive for its simplicity. It is able to extract the underlying linear model out of nonlinear systems, and it makes the traditional dynamic characteristics (modal frequencies, mode shapes, etc.) meaningful for indicating the state of nonlinear systems. However, polynomial approximation can not yield the exact hysteretic force due to the inherent multivalued-property of hysteretic systems, so that NIFO can not identify the exact underlying linear model. To overcome the difficulties, SSV model is suggested to model the nonlinear hysteretic force. Further study should be made to explore possible algorithms of nonlinear identification methods.

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