

# Numerical Computing Method Based on Cellular Automata

Fan Dewei<sup>+</sup> and Tang Hesheng

Research Institute of Structural Engineering and Disaster Reduction, Tongji University

Shanghai, china

**Abstract**—Cellular automata (CA) models are decentralized spatially extended systems consisting of large numbers of simple identical components with local connectivity. Such systems have the potential to perform complex computations with a high degree of efficiency and robustness, as well as to model the behavior of complex systems in nature. A novel numerical computing method based on CA is presented in this paper. In this method the integer analysis of structure is changed into a series of part analysis by using the idea of CA. Using the principle of minimum potential energy for the displacement convergence of all nodes, the non-equilibrium force of every cell is obtained. The overall finite element stiffness matrix and balance equation do not require. It may have a good future in the aspect of large structure because it hardly has any special request on the capability of computer. Simultaneity, It can be developed into a high parallel arithmetic to suit for the request of parallel computer in the future. This method is a good numerical method with great potential. The correctness and effectiveness of the proposed method are demonstrated by some numerical examples.

**Keywords-** Cellular Automata; numerical computing; minimum potential energy Introduction (Heading 1)

## 1. Introduction

The concept of cellular automata is proposed by J. Von Neumann and Stan Elam in the last century[1]. Von Neumann believes that cellular automaton is a general model can be used in different areas. In 1970 the famous mathematician John Conway proposed the concept of the life game[2]. The motivation is to find complex behavior can lead to simple rules. He envisioned a similar two-dimensional square grid board, each of which may be a living cell (state 1) or (state 0). Game of life update rules are as follows: Live by the more than 3 dead cell surrounded by a cellular response cell survival; by two or three more the following live cell surrounded by a living cell or congestion due to isolation and death. The results show that the game of life rich with unexpected act. A lot of paper about the game of life found that there are many such structures[3]. According to Von Neumann rules, life is a game of calculating universal cellular automata. In addition to these theoretical results, the cellular automata in the 20th century 50's also used in image processing[4].

In the last century, Stephen Wolfram studied a group of simple one-dimensional cellular automaton[5], showed that despite the simple rules (now known as Wolfram rules) is also able to model complex behavior. He noted that the cellular automaton is a discrete dynamic system, so even in the very simple structure, it also shows that many of the behavior encountered in continuous systems. His research was devoted to the application of CA model researchers and pointed out the direction. In the current application, cellular automata is considered an ideal natural system that has been successfully applied to different natural phenomena such as crystal growth. The applications are the macroscopic response of CA applications through the use of cellular to simulate a complex continuous response. After 90 years of the last century, cellular

---

<sup>+</sup> Corresponding author.

E-mail address: vanderveer@hotmail.com

automata in various fields has been widely used. James M. Skoda is the first to use cellular automata for social science [6]. But the first clearly defined in the CA framework is economics economist Peter S., is the first CA in sociology emphasizes the important role of the dynamic evolution [7, 8]. CA has also been applied in other areas such as urban development, simulation and prediction, thermal diffusion, the field of parallel computing[9].

Cellular Automata is a discrete space and time, physical parameters which take a limited set of values the ideal of physical systems. It is the edge of the multidisciplinary field of complex systems such as mathematics, physics, computer science, biology, systems science. It is an important research method. As Cellular Element Method is a kind of clear steps, the mechanical calculation of programming convenience, and it is easy to master, it will solve the whole structure into a local analysis, by force local delivery to achieve the ultimate balance between the overall balance which did not like the finite element need global stiffness matrix and solving the overall balance equation, the computer capacity requirements is very low, therefore the calculation in the large complex structure has good prospects. At the same time, borrowed the idea of cellular automata, can form a highly parallel algorithm, is a rich potential for development of new numerical methods.

CA used in structural analysis, and the results of CA algorithm was compared with the FEM results; the CA confirmed the feasibility of application of structural analysis.

## 2. Cellular Automata

Cellular Automata contains cellular, cellular state space, neighbors and local rules.

Cellular automaton model allows simple unit connected by the local rules under the action of the system state. Cellular automata as complex systems can evolve through local rule, making it possible to apply truss structure. The following were analyzed cellular automaton model elements and the similarity of space truss.

### 2.1. Cell

May be called a unit, or primitive, is the most basic cellular automaton part, Cellular distribution in the discrete one-dimensional, two-dimensional Euclidean space. Cellular space of the distribution grid is a collection of cellular space; it can be any dimension by the rules of Euclidean space. Space truss structure and the connected nodes can be abstracted as cellular pole.

### 2.2. State

Cellular state is the state variable used to describe the way cellular response. Practical applications often are extended with more variables in the cellular state.

### 2.3. Cell space

Cellular networks are distributed in space is called the set of cellular space. Figure 1 shows an example of two-dimensional space is divided. Cellular automaton model truss will use more complex three-dimensional form used to describe the spatial truss structure.

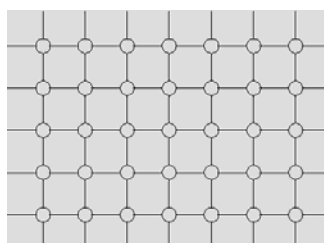


Fig.1. Cellular automata Space

### 2.4. The boundary cell

Cellular automata simulation with real problems, it is clear that cellular space is not infinite. The border cells have different cell neighbor state so the boundary is a special cell. Truss is the application of cellular automata used to describe the different local laws specificity of cellular boundary, the boundary can also be seen as cellular state function outside the boundaries of the rod will be the interface attributes assigned 0 to achieve.

## 2.5. Neighbor

The evolution of cellular automata rules are local, for the specified cell's status updates only need to know the status of its neighbor cell, in fact, the application of local laws of space. Figure 2 shows two commonly used two-dimensional neighbors. This article is used in three-dimensional neighbors.

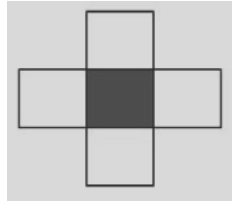


Fig.2. Von Neumann neighbor

## 2.6. The local rules

According to the current state of cellular and the next time the neighbor state to determine the state of cellular function, also known as state transition functions. It reflects the cellular interaction between neighbors.

## 2.7. Time

Cellular Automata is a dynamic system that changes in the time dimension is discrete, namely the time  $t$  is an integer value, and continuous equal spacing. Cellular state in time  $t + 1$  depends only on time  $t$  the state of the cell and its neighbor cell of the state, obviously, in time  $t-1$  of the cell and its neighbor, the state indirectly affect cellular cell in  $t + 1$  times the state.

# 3. The Structure Analysis Method based on CA

## 3.1. A. Three-dimensional cellular truss structure

Cellular truss structure connected by a bar node and its composition, it is clear that all of the space truss structure composed of the same cell. This cellular automaton is similar to the basic form. Cellular automata theory can be applied to the analysis of up truss structure. Figure 3 is the three-dimensional truss and abstract cellular structure.

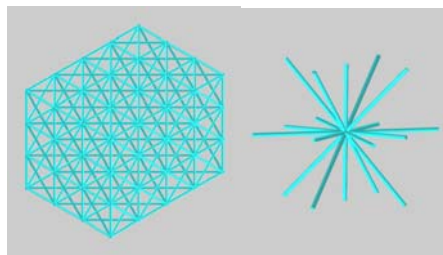


Fig.3. 3-D truss structure and abstract cellular

## 3.2. The state of cellular

The state of cellular automaton evolution of cellular functions that may change. Cellular state, including the displacement of three directions, bar cross-sectional area, elastic modulus, the node force. The state of the cellular expression by the following formula:

$$S_n = \{[u, v, w], [F_x, F_y, F_z], [A_1, A_2, A_3, A_4, A_5, \dots, A_{18}], E\} \quad (1)$$

$[E]$ —The elastic modulus of bar;

$[A_1, A_2, A_3, A_4, A_5, \dots, A_{18}]$ —cellular cross-sectional area of bar;

$[F_x, F_y, F_z]$ —applied to the cell along the x, y, z-axis force;

$[u, v, w]$  —Truss displacement;

## 3.3. Neighbor of the three-dimensional truss structure

Truss structure is a three-dimensional neighborhood structure of a neighbor, the neighbor with the state shown in Figure 4, the center cell has 18 neighbors, apparently with 19 local rules of the state of cell.

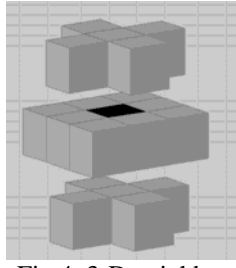


Fig.4. 3-D neighbor

### 3.4. Local law

Truss structure for the minimum potential energy principle of local law, that is, the displacement caused by neighbors cellular response always makes the potential energy minimum.

$$\Pi = U + V = \sum_{k=1}^{18} \frac{1}{2} (EA_k \ell_k \varepsilon_k^2) - F_x \times u - F_y \times v - F_z \times w \quad (2)$$

$$\varepsilon = \frac{\Delta L}{L} \quad (3)$$

$$\Delta L = \frac{L_d - L}{L} \quad (4)$$

$$L_d - L = (u_k - u) \cos \theta \cos \varphi + (v_k - v) \sin \theta \cos \varphi + (w_k - w) \sin \varphi \quad (5)$$

$[\varepsilon_k]$ —The k-bar strain;

$[\ell_k]$ —The k-bar displacement;

$[A_k]$ —The k-bar interface area;

$[u, v, w]$ —Truss displacement;

$[F_x, F_y, F_z]$ —x, y, z-axis force;

$[E]$ —The elastic modulus of bar;

$[\Delta L]$ —The length of the change in bar;

$[L]$ —The length of bar;

$[L_d]$ —Length of bar after deformation;

$[\theta]$ —the angle between the xoy plane and the x-axis;

Obtained from the minimum potential energy principle:

$$\frac{\partial \Pi}{\partial x} = 0 \quad (6)$$

$$\frac{\partial \Pi}{\partial y} = 0 \quad (7)$$

$$\frac{\partial \Pi}{\partial z} = 0 \quad (8)$$

$$\sum_{k=1}^{18} EA_k \frac{1}{\ell_k} \left[ (u_k - u) \cos \theta_k \cos \varphi_k + (v_k - v) \sin \theta_k \cos \varphi_k + (w_k - w) \sin \varphi_k \right] \times \cos \theta_k \cos \varphi_k + F_x = 0 \quad (9)$$

$$\sum_{k=1}^{18} EA_k \frac{1}{\ell_k} \left[ (u_k - u) \cos \theta_k \cos \varphi_k + (v_k - v) \sin \theta_k \cos \varphi_k + (w_k - w) \sin \varphi_k \right] \times \sin \theta_k \cos \varphi_k + F_y = 0 \quad (10)$$

$$\sum_{k=1}^{18} EA_k \frac{1}{\ell_k} \left[ (u_k - u) \cos \theta_k \cos \varphi_k + (v_k - v) \sin \theta_k \cos \varphi_k + (w_k - w) \sin \varphi_k \right] \times \sin \varphi_k + F_z = 0 \quad (11)$$

### 3.5. Cellular automata calculation

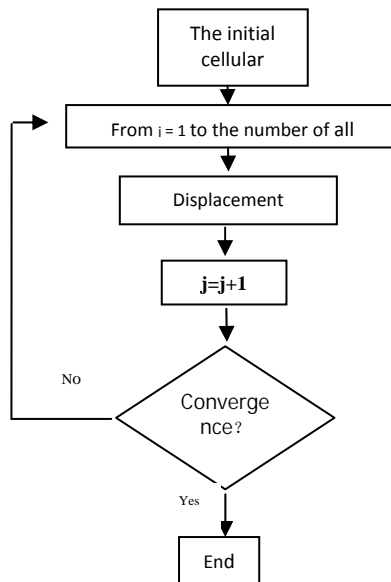


Fig.5. Calculation chart

With the evolution of cellular automata for structural response to flow under the external force as shown in Figure 5, The initial model of the state of all cellular components of the displacement functions are 0, Cellular automata is the basis for the evolution of the local energy minimization of local rules derived (local rule) applied to each cell. The state of all cellular function as a full update time step, the time step is not really the time, but the evolution of structural response to changes in the number of iterations. Cellular automata evolution for the evolution of the displacement is not continuous in time, this is not the continuity of performance at each time step the changes on the structural displacement is not continuous change, The evolution of cellular automata that convergence to the displacement of the structure out of the loop, the end of the evolution of cellular automata.

## 4. Numerical Example

### 4.1. Plane truss

Plane truss structure shown in Figure 6, Section size is  $20 \times 10^{-4} \text{m}^2$ , The length of bar is 1m, Length of diagonals is 1.414m. Concentration is 100kN, One end is hinged bearing, one end of the free.

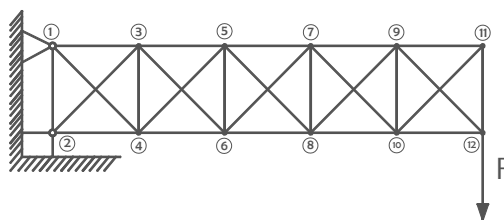


Fig.6. A case of 2-D truss

From the previous derivation, the state function cellular models have 8-under-sectional area, the actual structure may not have a bar, in this example there are 6 in the state function. In cellular ① and ②, obviously, the state of the surrounding cellular changes do not affect the state of cellular changes in displacement, which implements the displacement boundary conditions. For the remaining free boundary conditions, we shot as long as the area corresponding to the assignment of zero can be achieved. In this case, 40 times the number of iterations taken (Iteration 40 the amount of displacement is less than 1%).

The results of Cellular Automata algorithm and FEM algorithm such as Table 1. Table 2 shows the cellular automata algorithm iterations and changes of node displacement table can reflect the concentration effect on the system by the concentrated force of nodes spread throughout the system to eventually reach the final balance of the system.

We can see from Figure 7, the higher the displacement of more convergence of iterations, iterative 40 times the amount of displacement is less than 1%

TABLE I. COMPARISON OF VERTICAL DISPLACEMENT OF 2-D STRUCTURE

number	Cellular algorithm	FEM algorithm
1	0 (m)	0 (m)
2	0 (m)	0 (m)
3	0.0167 (m)	0.01677 (m)
4	0.0167 (m)	0.01667 (m)
5	0.0178 (m)	0.01785 (m)
6	0.0180 (m)	0.01820 (m)
7	0.0183 (m)	0.01832 (m)
8	0.0186 (m)	0.01867 (m)
9	0.0188 (m)	0.01885 (m)
10	0.0193 (m)	0.01928 (m)
11	0.0203 (m)	0.02050 (m)
12	0.0213 (m)	0.02160 (m)

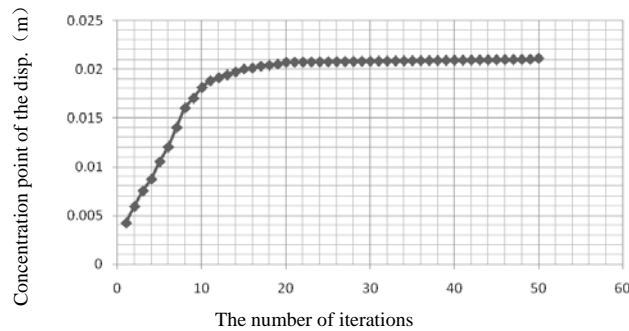


Fig.7. the relationship between the concentration point of the displacement and the number of iterations

TABLE II. RELATIONSHIP BETWEEN ITERATIVE NUMBER AND NODE DISPLACEMENT

number	1 iter- ations(m)	2 iter- ations(m)	3 iter- ations(m)	4 iter- ations(m)	5 iter- ations(m)	6 iter- ations(m)	20 iter- ations(m)
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0.0022	0.0038	0.0160
4	0	0	0	0	0.0041	0.0051	0.0161
5	0	0	0	0.0018	0.0026	0.0041	0.0178
6	0	0	0	0.0036	0.0044	0.0055	0.0179
7	0	0	0.0008	0.0020	0.0030	0.0043	0.0183
8	0	0	0.0025	0.0037	0.0048	0.0059	0.0186
9	0	0.0004	0.0013	0.0026	0.0035	0.0049	0.0188
10	0	0.0017	0.0028	0.0039	0.0051	0.0062	0.0198
11	0	0.0017	0.0025	0.0040	0.0050	0.0063	0.0199
12	0.0042	0.0059	0.0075	0.0086	0.0099	0.0110	0.0207

#### 4.2. 3-D Truss Examples

Space truss structure shown in Figure 8, Section size is  $25 \times 10$  -4m<sup>2</sup>, length of bar is 1m. Diagonals of length 1.414m, Concentration force is 100kN, One end is hinged bearing, one end of the free.

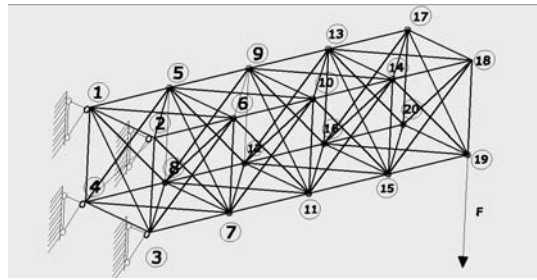


Fig.8. A case of 3-D truss

In three-dimensional lattice of the cellular models, the state function contains 18 cross-sectional areas of bar. In this example there are not all 18 cross-sectional areas in every bar. For example, there are 6 cross-sectional areas in ③ bar; there are 7 cross-sectional areas in ⑦ bar. In this case, we take 80 iterations (80 iterations displacement of less than 1%).

Finite element method algorithm and cellular automata vertical displacement shown in Table 2:

Table III. COMPARISON OF VERTICAL DISPLACEMENT OF 3-D STRUCTURE

number	Cellular algorithm(m)	FEM algorithm(m)
1	0	0
2	0	0
3	0	0
4	0	0
5	0.00136	0.001379
6	0.00137	0.001374
7	0.00137	0.001370
8	0.00137	0.001368
9	0.00170	0.001738
10	0.00172	0.001730
11	0.00182	0.001839
12	0.00183	0.001835
13	0.00242	0.002398
14	0.00265	0.002688
15	0.0027	0.002689
16	0.00257	0.002585
17	0.00349	0.003517
18	0.00398	0.004022
19	0.00425	0.004136
20	0.00368	0.003677

## 5. Conclusion

In these paper cellular automata method is applied to structural mechanics, structural balance is to meet the local minimum potential energy principle of self-organizing process. Cellular automaton rule applies only to a single local cellular, so avoid the establishment of global stiffness matrix, only for solving linear equations, so computation. The numerical examples demonstrate that the method is applied to the feasibility of space truss, and the better the convergence of the displacement. Since the methods to solve them one by one only of each cell does not require the formation of the structure of the overall balance equation, can overcome the finite element calculations inlarge-scale problems in data storage, computing and large-scale structure is expected in large-scale numerical simulation of nana-mechanical role. As a new numerical method, this article is limited to static analysis. Cellular automata applied to structural analysis of research is still in its infancy, there are many reasons for problems such as convergence, computational speed, power analysis should be further explored and deepened.

## 6. References

- [1] J. V. Neumann, "The Theory of Self-Reproducing Automata," A. W. Burks (ed), Univ. of Illinois Press, Urbana and London, 1966.

- [2] M.Gardner, "The fantastic combinations of John Conway's new solitaire game life,"*Scientific American*,220(4):120,1970
- [3] E.R.Berlekamp,J.O.Conway,and R.K.Guy, "Winning Winning Ways for your Mathematical Plays," volume 2.Academic Press,1982.Chapter 25.
- [4] K.Preton and M.Duff.Modern Cellular Automata, "Theory and Applications. Plenum Press,"1984.
- [5] S. Wolfram, "Theory and Applications of Cellular Automata. World Scientific,"Singapore,1986. ISBN 9971-50-124-4 pbk.
- [6] J. M. Sakoda, "The Checkerboard Model of Social Interaction. *J Math. Socio*,"1:119-132., 1971.
- [7] P. S. Albin, "The Analysis of Complex Socioeconomic Systems. Lexington,"MA: D. C. Heath and Company/Lexington Books, 1975.
- [8] P. S. Albin and D. K. Foley(ed.) , "Barriers and Bounds to Rationality: Essays on Economic and Dynamics in Interactive Systems,"Princeton University Press, Princeton, NJ, 1998.
- [9] jixin yang. , "Parallel Cellular Element Method,"*Journal of Computational Mechanics*2006, 23(1), 35-37.