Uncertainty quantification using evidence theory in concrete fatigue damage prognosis

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Abstract— **Fatigue failure is the main failure mode of mechanical components in the research of engineering structures. As fatigue life may be a basis for the fatigue reliability design, it is very important to predict it for the normal usage of the structure. Uncertainties rooted in physical variability, data uncertainty and modeling errors of the fatigue life prediction analysis. Furthermore, the predicted life of concrete structures in civil engineering field will be more obviously uncertain than other engineering structures. Due to lack of knowledge or incomplete, inaccurate, unclear information in the modeling, there are limitations in using only one framework (probability theory) to quantify the uncertainty in the concrete fatigue life prediction problem because of the impreciseness of data or knowledge. Therefore the study of uncertainty theory in the prediction of fatigue life is very necessary.**

This study explores the use of evidence theory for concrete fatigue life prediction analysis in the presence of epistemic uncertainty. The empirical formula S-N curve and the Paris law based on the fracture mechanics are selected as the fatigue life prediction models. The evidence theory is used to quantify the uncertainty present in the models' parameters. The parameters in fatigue damage prognosis model are obtained by fitting the available sparse experimental data and then the uncertainty in these parameters is taken into account. In order to alleviate the computational difficulties in the evidence theory based uncertainty quantification (UQ) analysis, a differential evolution (DE) based interval optimization method is used for finding the propagated belief structure. The object of the current study is to investigate uncertainty of concrete fatigue damage prognosis using sparse experimental data in order to explore the feasibility of the approach. The proposed approach is demonstrated using the experimental results of the plain concrete beams and the steel fibred reinforced concrete beams.

Keywords- fatigue damage prognosis, evidence theory, uncertainty analysis, differential evolution algorithm, concrete

I. Introduction

It is acknowledged that reinforcement concrete structure has been widely used in the engineering facilities and manifested their importance in many ways by providing their services to people in their useful life. Meanwhile, these structures experienced a continuous, progressive, permanent deterioration process as a consequence of their exposure to fatigue condition: static or dynamic loads in their service cycle. Similar to most brittle failure, fatigue damage of concrete may not be emerged as obviously plastic deformation or other potential and led to catastrophic accidents. Inspired by this arduous and difficult problem, considerable attention has been concentrated to investigate the security and capability of these concrete facilities under the condition of fatigue load and many constructive achievements were presented during last 10 or so years.

In the initial state of fatigue research, the main efforts were applied to conduct the experiments and summary the obtained empirical results [1]. Oh [2] used the experimental and theoretical study to investigate the fatigue strength of plain concrete and discussed the fatigue life distribution from the S-N curve of concrete components for various stress levels [3]. Singh et.al. [4, 5] conducted a series of studies on the fatigue life of steel fiber reinforced concrete (SFRC) under various stress levels and stress ratios. Theirs studies demonstrated that the two parameters Weibull distribution is suitable to model the distribution of fatigue life. With same valid hypothesis, Mohammadi [6] examined and calibrated the parameters Weibull distribution with a set of experiment included 280 components. From the above studies, the characters of empirical method for fatigue life prediction of concrete materials can be concluded as follows: at first, an object is set to get the fatigue life of concrete materials; then experimental observation is employed to investigate the relationship of external load condition and internal physical characters; at last, probability distribution is introduced to present a formula for fatigue life with a certain level of guaranteed rate.

Besides to above classical method, some researchers make an endeavor on fracture mechanics model to depict the fatigue life of concrete materials. As a classical formulation in fracture mechanics, Paris law [7] and its variant versions were naturally introduced to the domain of fatigue of concrete materials. Baluch [8] verified the validity of crack propagation of plain concrete using Paris law. Bazant [9] investigated the size effect of the fatigue crack development of concrete and attempted to explain the reason of wide variation range of experimental results. Matsumoto [10] revealed that fracture mechanical model links the material structure and S-N diagram in an explicit way. Cheng and Shen [11] discussed the problem to use Paris and Forman law in high percent fibered composite. Considered crack size effect, Spagnoli [12] employed both similarity methods and fractal concepts to derive a crack-size dependent Paris law. Diab et.al. [13] used an improved Paris

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law which incorporated with fracture energy ratio to predict the fatigue life of FRP concrete. Sonalisa and Kishen [14] developed a modified fatigue crack propagation law for plain concrete using the concept of dimensional analysis by incorporating the effect of reinforcement through a pair of closing force. As has been listed in above studies, the fatigue life of concrete materials which derived from S-N curve and fracture mechanics is seemed as a random variable and depicted with probability theory.

It should be noted that the classical probability may need a large number of experimental data to model aleatory uncertainty [15, 16] (random) which rooted in physical variability of materials and environment. However, in practical fatigue analysis of concrete materials, the process of collecting experimental data are high economic and time cost, the research of fatigue mechanics are scanty and the proposed quantification model are too idealistic to reflect the realistic physical mechanism. And these shortcomings led the prediction result of fatigue life is not a aleatory but an epistemic uncertainty [15, 16]. In the past decades, several alternative approaches have been developed to deal with epistemic uncertainty that stem from a lack of knowledge, ignorance, or modeling (epistemic uncertainty). Some of the potential uncertainty theories are the theory of fuzzy set [17], possibility theory [18], the theory of interval analysis [19], imprecise probability theory [20] and evidence theory [21, 22]. Among these promising uncertainty representation models, evidence theory with the ability of handle mixed aleatory-epistemic uncertainty are used to uncertainty quantification [23], risk assessment [24] and reliability analysis [25].

With two complementary measures of uncertainty: belief and plausibility, evidence theory can handle epistemic uncertainty effectively. However, the computationally intensive problem involves the evaluation of the bound values over all possible discontinuous sets is a main shackles of wide application for evidence theory. In order to alleviate the computational costs in the evidence theory based uncertainty quantification analysis, the principle and method of using differential evolution [26] based interval optimization to enhance the computational efficiency as described previously by the authors [23, 27] are introduced.

The main theme of current paper is to investigate uncertainty of concrete fatigue damage prognosis using sparse experimental data and explore the feasibility of the proposed approach. The traditional experimental and theoretical method to obtain fatigue life of concrete materials are introduced in section 2. Then, the basic of evidence theory and differential evolution based uncertainty propagation are presented in section 3. For studying the effectiveness of proposed methodology and the influence of uncertainties rooted in fatigue life prediction, some discussions and remarks for the experimental results of the plain concrete beams and the steel fibred reinforced concrete beams are presented in section 4.

II. FATIGUE LIFE PREDICTION MODEL

A. S-N curve based prediction model

In high-cycle fatigue situations, materials performance is generally characterized with the relationship of the applied-fatigue stress and the fatigue life of concrete which also known as S-N curve or Wöhler curve. As a basic method for predicting the fatigue life, S-N curve is stem from regression analysis of experimental data to represent the median or mean numbers of cycles of failure (N) and a given constant stress ratio (S). The general formulation of S-N curve is given as:

$$\lg N = \lg C - m \lg S \tag{1}$$

where m and $\lg C$ are numerical coefficients, whose value can be obtained by fitting the experimental data N and S. Thus, the fatigue lives of concrete under different stress levels is achieved through S-N curve. With the uncertainty raising from load condition, inherent material variability, sparse statistical data and imperfect knowledge of fatigue mechanics, the fatigue life show considerable scatter and should be modeled by an comprehensive uncertainty quantification measure to handle the mixed aleatory-epistemic uncertainty. Additionally, it should be noted that the prediction results of S-N curve are limited to identical situation and may not be valid because of changing the loading mechanism or boundary condition.

B. Prediction model with Paris law

Apart from the above classical empirical method, fracture mechanics based fatigue life prediction model is also used to depict the relationship of crack growth rate and crack tip stress intensity factor amplitude. The famous Paris law [7] can be formulated as:

$$da/dN = C\left(\Delta K\right)^{n} \tag{2}$$

where a is the crack length; N is the numbers of load cycles; C and n are Paris constants; ΔK is the variation of stress intensity, $\Delta K = K_{max} - K_{min}$, K_{max} and K_{min} are maximum and minimum stress intensities in a fatigue cycle, respectively. Given an initial crack length a_0, the numbers of fatigue failure load cycles N can be calculated from equation (3):

$$N = \int_{a_0}^{a_f} 1\Big/C\left(\Delta K\right)^{n}\, da \tag{3}$$

where a_f is the failure crack length with a threshold and a_0 is the initial crack length. To solve above integration equation, the values of C and n should be previously calibrated by fitting experimental data. The value of K is computed as:

$$K = Pf\left(a/D\right)\Big/ b\sqrt{D} \tag{4}$$

where P is the applied load; b and D are thickness and width of specimen; and $f\,(a/D)$ is the regression formulation of crack length and width of specimen via finite element analysis (FEA). Given an experimental measurement, the value a_f can be obtained.

According to above derivation, the result of fatigue life of concrete is influenced by the mixed aleatory-epistemic uncertainty rooted in load condition, size effect of specimen, initial crack length, regression result of FEA model, criteria of failure and limited number of specimen. Due to these mixed aleatory-epistemic uncertainty, evidence theory is a wise choice to quantify the uncertainty response of fatigue life of concrete.

III. Uncertainty propagation based on evidence theory

A. Basic concept of evidence theory

Evidence theory is proposed by Dempster [21], and developed to be a complete uncertainty model theory by Shafer [22]. Different from the single measurement used in classical probability theory, evidence theory employs two function (*Bel*), and the plausibility function (*Pl*) to represent the uncertainty. Compare to classical probability theory, these measures led evidence theory become less restrictive and more effective to represent the epistemic uncertainty. Frame of discernment Ω is the sample collection consist with exclusive and exhaustive elements and the mathematical representation can be described as: $\Omega=\{\Omega_1, \Omega_2, \ldots, \Omega_n\}$, n is denote to a finite number. Correspondingly, the power set $P(\Omega)$ is denote to the all possible subsets of Ω including the empty set. As a critical concept of evidence theory, mass function m is a mapping from $2^\Omega \rightarrow [0, 1]$. Assume A is arbitrary subset of Ω, mass function is defined as:

$$\begin{cases} m(A) \geq 0, \quad \forall A \in P(\Omega) \\ \sum_{A \in P(\Omega)} m(A) = 1 \end{cases} \tag{5}$$

The basic belief assignment (BBA) $m(A)$ is interpreted as the degree of evidence supporting the claim that a specific element of Ω belong to the set A. Every set A for which $m(A)>0$ is called focal element. In evidence theory, BBA provides a method to express a certain belief to a proposition, $Bel(A)$ is interpreted to be the minimum likelihood that is associated with the set A and plausibility $Pl(A)$ is the maximum amount of likelihood that could be associated with set A. The belief and plausibility functions are given as:

$$\begin{aligned} Bel(A) &= \sum_{B \subseteq A} m(B) \\ Pl(A) &= \sum_{B \cap A \neq \varnothing} m(B) \end{aligned} \tag{6}$$

Apparently, the belief and plausibility functions define an interval as shown in Fig. 1.

B. Uncertainty representation using evidence theory

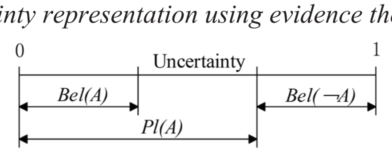

Figure 1. Belief and Plausibility of proposition *A*.

As an effective measurement to handle statistical data, histogram is general used to represent the distribution of collection data. The current paper presented an effective approach which introduced in [28] to model the uncertainty reflected with histogram using evidence theory. To construct BBA for subsets of a universal set, the relationship of two adjacent bins are categorized into three different relationship states: ignorance, agreement and conflict. Assume each variable has a collection, which is constituted by C data points. Suppose I_1, I_2 are two adjacent bins in histogram, the number of data points in each intervals are represented by A and B ($A<B$), respectively. For the case of $A/B<0.5$, the relationship between the above bins is in the form of ignorance, and corresponding BBA can be expressed as:

$$\begin{aligned} m(\{I_2\}) &= B / C \\ m(\{I_1, I_2\}) &= A / C \end{aligned} \tag{7}$$

If the ratio $A/B>0.8$, the relationship of two adjacent bins is defined as agreement. In this situation, the new bin include the all information of two bins, and the new BBA is given as following:

$$m(\{I\}) = (A+B) / C \tag{8}$$

Apart from above two situations, the relationship between the above bins is in the form of conflict and corresponding BBA presented in the form:

$$\begin{aligned} m(\{I_1\}) &= A / C \\ m(\{I_2\}) &= B / C \end{aligned} \tag{9}$$

Find the highest bin in histogram and gradually identify the relationship of adjacent bins until obtain the reciprocal relationship in histogram, the construction of BBA structure for each variable is completed. After represent the uncertainty by evidence theory, the joint BBA structure is constructed by Cartesian product of all variables.

C. Differential evolution based uncertainty propagation

As illustrated in Fig. 2, *f*(), *x, d*, and *y* are denote to the system model, uncertain variable, certain variable and uncertain system response. In evidence theory, the uncertainty is represented by intervals, this made uncertainty propagation becomes a process involves to obtain the boundary response of each joint intervals. Some strategies, like Monte-Carlo sampling, interval analysis and intelligent optimization algorithm are applied to solve this problem. It is well know that the accuracy of Monte-Carlo sample is based on the numbers of population and the interval analysis unable to handle complex problems, especially to calculate a black-box. Differential evolution algorithm (DE) [29] is adopted herein not only to obtain accurate system response but to overcome the barricade of computation cost.

As a stochastic direct search method based on advanced strategies, general selection and fast convergence, DE is always used to solve non-linear, high-dimensional and complex computational optimization problems i.e. interval optimization. Herein, differential evolution global optimization method is used for propagation of the represented uncertainty through fatigue life prediction model and to estimate uncertainty measures, i.e. cumulative belief function (CBF) and cumulative plausibility function (CPF).

IV. NUMERICAL CASE

A. *Unceraint quantifcation of Paris law for plain concrete beams*

In this paper, we choose the experiment in [30] as the research object, use the Paris law fatigue life prediction model to access the fatigue life of concrete. In this experiment, the concrete beam specimens were tested under three-point flexural loading. The fatigue life uncertainty variate in the range which consist of 12 sets of specimens. The information about initial crack length a_0, the numbers of fatigue failure load cycles N and corresponding Paris constants C and n are shown in Table I.

TABLE I. THE VALUE OF EXPERIMENTAL DATA

NO.	**1**	**2**	**3**	**4**	**5**	**6**
a_0**(mm)**	40	40	40	40	40	40
N	25000	2100	14350	11000	18500	3500
lg*C*	-1.745	-1.236	-0.923	-1.796	-2.918	-2.402
n	12.236	8.257	17.154	7.714	3.252	7.778
NO.	**7**	**8**	**9**	**10**	**11**	**12**
a_0**(mm)**	60	80	80	80	100	100
N	6000	5700	20000	1100	1800	3000
lg*C*	-1.581	-1.406	-2.010	-1.332	-1.362	-1.706
n	8.030	15.114	7.749	7.014	2.761	2.420

Using the experimental data in Table I, the histogram and BBA structure of lg*C* and n are listed in Fig. 3.

Based on the BBAs for lg*C* and n given in Fig. 3, the differential evolution algorithm is used to propagate the uncertainty, The cumulative distribution curves of fatigue life under different stress levels are obtained as shown in Fig. 4, in which CBF and CPF denote the cumulative distribution function for belief, plausibility, respectively.

As shown in Fig. 4, the CDF curve, the result of probabilistic method, denotes the cumulative distribution

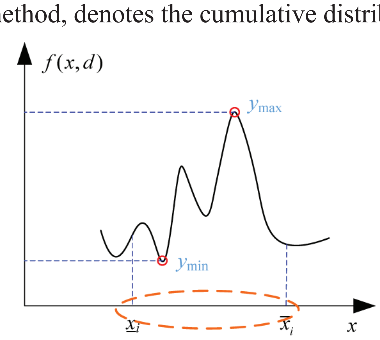

Figure 2. The BBA structure of lg*C* and *n*

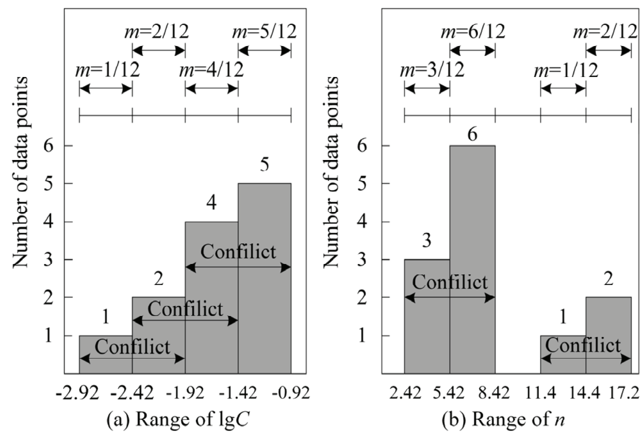

Figure 3. The BBA structure of lg*C* and *n*

function for probability. It can be obtained from Fig. 4 that there is a good compatibility among the CPF curve, the CBF curve and the CDF curve. The CDF curve located in the gap of the CBF curve and the CPF curve, thus belief and plausibility provide bounds as to where the true, unknown cumulative distribution function may fall. Therefore, evidence theory is an alternative method that can deal with the uncertainty effectively when classical probability method is inappropriate because of the insufficient data or lacking of knowledge or information. To compare the detailed quantification results, Table II summaries the predticted fatigue life with 5% confidence failure and complete range using differente uncertainty quantification measurement and experiment.

Table II illustrates a comparison of the experimental data and some information extracted from Fig. 4. Some useful informations can be derived from this table that the experimental data can be included in the intervals of the fatigue life of concrete derived by the two presented approaches. And it is easy to find that the prediction intervals of probabilistic method is bigger than the ones of the evidence theory due to the bigger sampling range of probabilistic method which contains the sample points with a very small probability. The evidence theory method expresses the uncertainty based on the parameters obtained from the experimental data. The range of the predicted results are more close to experimental data, and

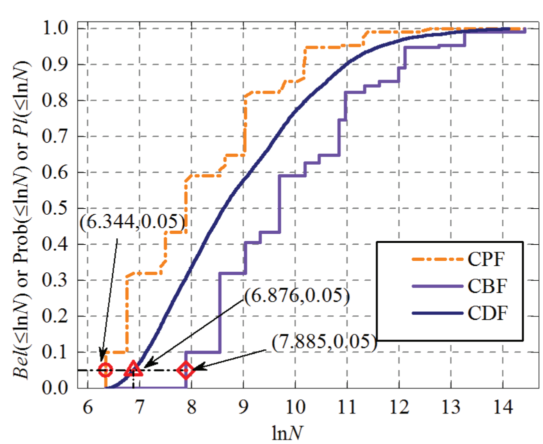

Figure 4. The result of uncertainty propagation for plain concrete beam

TABLE II. THE COMPARISON OF FATIGUE LIFE PREDICTION RESULTS

ln$N$	**evidence theory**	**probability theory**	**experimental data**
5%confidence failure	[6.344,7.885]	6.876	---
fatigue life range	[6.344,14.42]	[6.361,14.11]	[7.497,10.127]

the experimental results are included. This shows that the application of evidence theory to predict the fatigue life of concrete is reliable and more validity than the probabilistic method.

B. Unceraінty quantifcation of S-N for steel fibered concrete beams

For fatigue life prediction of steel fibered concrete, Shi et.al. [31] proposed the concept of equivalent fatigue life and its definition is formulated as:

$$EN = \left(N\right)^{1-R} \tag{10}$$

where R is the stress ratio and the mean of N is the fatigue life or current cycles of failure. Introducing above asummption, the fatigue life prediction can be expressed as:

$$\lg\left(EN\right) = \lg C_1 - C_2 \lg S \tag{11}$$

where lgC_1 and C_2 are constants which is derived from regression analysis for experimental data. In the current papaer, the experimental data conducted by Singh [4] was employed to constrcut the statistical information of evidence theory representation. The detailed of these data is listed in the Table III:

TABLE III. THE EQUIVALENT FATIGUE LIFE (EN) OF STEEL FIBERED CONCRETE BEAM

No.	**Stress level *S***			
	0.90	***0.85***	***0.775***	***0.675***
1	17	151	2416	16111
2	34	170	2727	17458
3	41	170	3021	21745
4	43	195	3165	26424
5	50	212	3366	40837
6	63	276	3679	56300
7	63	299	4332	60834
8	70	445	5078	80114
9	109	591	5534	104038
10	150	608	6404	155218

By means of the least square method to handle the set of experimental data, the value of lgC_1 and C_2 is obtained as Table IV:

TABLE IV. THE VALUE OF CONSTANTS IN S-N

NO.	**1**	**2**	**3**	**4**	**5**	**8**	**9**	**10**	**11**	**12**
C_2	23.435	21.801	22.13	22.515	23.548	23.736	23.982	24.19	23.588	24.182
lgC_1	0.42	0.692	0.715	0.723	0.699	0.779	0.789	0.87	1.047	1.085

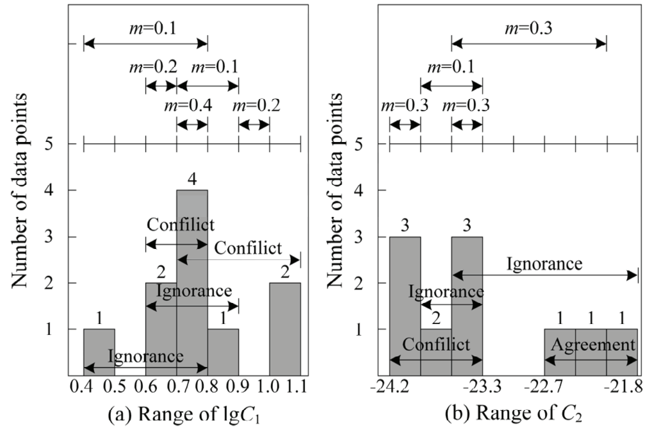

Figure 5. The BBA structure of lgC_1 and C_2

Similar to case study of plain conrete beam, the evidence based uncertainty representation of constants lgC_1 and C_2 are built by means of section III. The BBA structure of lgC_1 and C_2 are listed in Fig. 5. After uncertainty representation, the joint basic belief function is constructed by Cartesian product. The process of uncertainty propagation is successfully implemented to investigate the impact of the variable parameters for fatigue life of steel fibered concrete. The results of uncertainty propagation under different stress level S are shown in Fig. 6.

As show in Fig. 6, the uncertainty quantification results of fatigue life prediction for steel fibered reinforced concrete variate in a wide range. The distance of CBF and CPF denotes to the epistemic uncertainty that derived from the limited experimental data and lack of knowledge for complicated composited materials (e.g. steel fibered reinforced concrete). The points represented by red markers are the fatigue value which have 5% confidence. From the difference of these marked value, it reflected the single uncertainty quantification measurement, like probability theory, may disable to handle the mixed aleatory-epistemic uncertainty. Comparatively, evidence theory have great potential to quantify this mixed uncertainty and the using of differential evolution based interval optimal strategy break down the barricade of computational cost.

V. SUMMARY

Uncertainty quantification of fatigue life prediction is the basis of fatigue reliability design. In this paper, the mixed aleatory-epistemic uncertainty of the parameters of S-N curve and Paris law are considered. The uncertainty quantification measurement that combines evidence theory and differential evolution is proposed to quantify the uncertainty of fatigue life of concrete. In comparison with the quantification results of classical probability theory, the quantification method presented in this paper is more close to experimental data range. And this reflected that evidence theory is a more reliable uncertainty analysis than probability in the condition that sparse statistical data and imperfect knowledge of physical mechanism. Unlike the traditional probabilistic method limited by the sufficient data to depict precise probability distribution, evidence theory with unrestrictive measures to handle the lack of knowledge or incomplete, inaccurate, unclear information in the model is a great potential method.

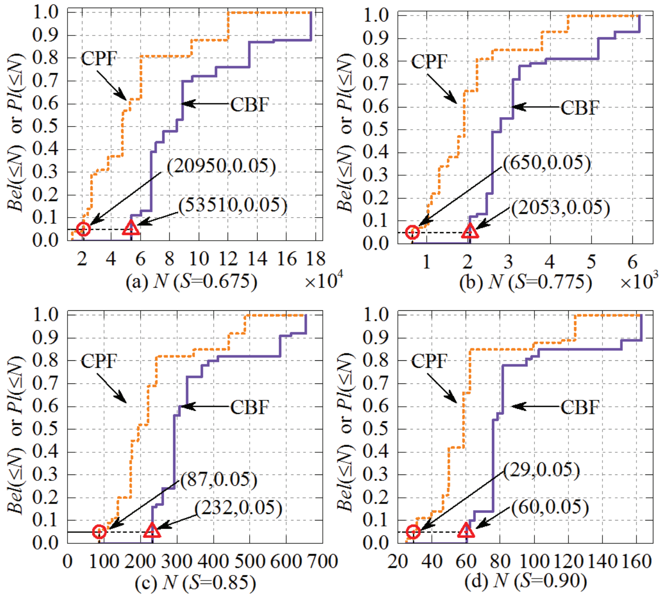

Figure 6. The result of uncertainty propagation for plain concrete beam

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