7th Asia-Pacific Workshop on Structural Health Monitoring
November 12-15, 2018 Hong Kong SAR, P.R. China

# Vehicle-Track System Identification for High-Speed Railway Using a Cross Entropy Approach

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**KEYWORDS:** High-Speed Railway; Vehicle–Track System; System Identification; Cross Entropy Algorithm

## ABSTRACT

Railway track irregularity is an important indicator of serviceability condition. This paper establishes a track irregularity identification approach using the cross entropy algorithm. The cross entropy algorithm is general algorithm which can be applied to solve global optimization problems. It updates the parameters of distribution based on the generated samples in previous iteration, to realize parameter identification. According to the CRH380AL high speed train and CRTII slab ballastless track system, a quarter vehicle-track coupling model under the load of Chinese random irregularity spectrum of ballastless track is simulated in this paper. And then Newmark explicit numerical integration method is adopted to resolve the large-scale nonlinear dynamic equations and to generate dynamic response of vehicle. The vehicle acceleration are processed by the cross entropy algorithm to identify track irregularity. This paper also discusses the influence of different measurement noise on the identification results and proposes a method for mitigating the effects of high noise environment.

## 1. Introduction

High-speed railway, an important passenger transport, has a vital impact on China's economic development. With the great improvement of vehicle's departure frequency and operating speed, the load of track increases and the damage of track structure is aggravated, which will affect the safety and comfort of the in-service vehicles. Therefore, the real time monitoring of track irregularity is of great significance to ensure the safety of the train operation and to make reasonable maintenance plan.

At present, the key measures for high-speed railway track irregularity are mainly divided into static method and dynamic method. Static methods, such as the use of rod and level equipment or specialised 'dipstick' walking profilometers, are slow and time consuming. Dynamic methods such as track detection vehicles provide an accurate, high-resolution measurement of track irregularity, though they usually have high cost and need special time to monitor track. It is difficult to get the track irregularity in the first time. Therefore, the method using vehicle vibration signal to monitor the track irregularity has gradually entered the scholars' sight.

In 1999, at WCRR, Japanese scholar, Yoichi proposed the method using acceleration signal of train's axle-box to test track irregularity [1]. Based on this, Bocciolone et al. has used the non-stationary wavelet analysis method to process the acceleration signal, and verified the effectiveness of the track irregularity

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test results [2]. In recent years, the acceleration measurement method [3–10] is widely applied to identify the track irregularity. Some scholars use experience mode decomposition, local wave decomposition to study track irregularity [11, 12]. However, these methods didn't consider the coupling between vehicle and track, and the accuracy of the algorithm need to improve.

The cross entropy (CE) algorithm is a relatively novel and rapidly developed mathematical algorithm. The CE method was motivated by Rubinstein (1997), where an adaptive variance minimization algorithm for estimating probabilities of rare events for stochastic networks was presented. It was modified to solve combinatorial optimization problems in Rubinstein's study [13, 14]. In recent years, the CE method has been successfully applied to continuous optimized control, multi-extremal optimization and strategic search optimization, etc. [15-17]. E.J.Obrien's team from University College of Dublin has done a lot of work on the CE algorithm in automobile engineering and road engineering, and they successfully adopted the CE algorithm to identify the vehicle structure parameters. According to the advantages of the CE algorithm and the shortcomings of the current methods, this paper will use the CE algorithm to study the vertical track irregularity based on the vehicle-track coupling dynamic model.

# 2. Vehicle–Track Coupling Model

## *2.1 Vehicle–Track Coupling Model*

The vehicle-track coupling model is established, as shown in figure 1. The vehicle model is simplified as a multi-rigid-body system that consists of carriage, bogie, wheelsets, primary suspensions and secondary suspensions. The carriage and bogies are connected through the secondary suspensions, and bogie is linked with wheelsets through the primary suspensions. The primary and secondary suspensions are represented by parallel spring-damping elements. The track consists of the rail, track plate, base plate. The rail that is discretely supported by the fasteners has been described by the Bernoulli–Euler beam theory, and the rail fasteners are represented by parallel spring damping elements. The vehicle subsystem and the track subsystem are coupled by the vertical force between the wheels and rail. It is presented by Hertz contact model [18].

$$P(t) = [\frac{1}{G}\delta Z(t)]^{3/2} \quad \textbf{(1)}$$

where $G$ is the wheel-rail contact constant (m/ $N^{2/3}$) and $\delta Z(t)$ is the elastic compression between wheels and rails. The values of each structural parameter in the model are shown in Table 1.

**Table 1.** Properties of the vehicle-track coupling model

| Property | Unit | Symbol | Value |
|---|---|---|---|
| Quarter mass of carriage | kg | $M_c$ | 9721 |
| Half mass of bogie | kg | $M_t$ | 1530 |
| Mass of wheel | kg | $M_w$ | 1517 |
| Stiffness of secondary suspension | N/m | $K_{s2}$ | $4.5\times10^5$ |
| Damping of secondary suspension | Ns/m | $C_{s2}$ | $2\times10^4$ |
| Stiffness of primary suspension | N/m | $K_{s1}$ | $4.5\times10^5$ |
| Damping of primary suspension | Ns/m | $C_{s1}$ | $4\times10^4$ |
| Stiffness of fastener | N/m | $K_b$ | $2.5\times10^7$ |
| Damping of fastener | Ns/m | $C_b$ | $7.5\times10^4$ |

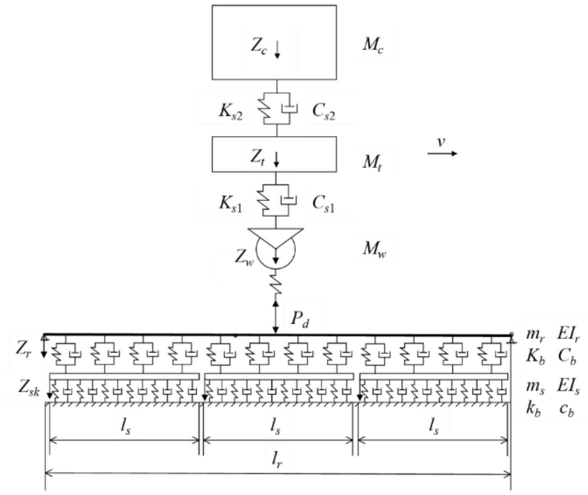


**Figure 1.** The vehicle-track coupling model

### *2.2 The Dynamic Equation*

The vehicle-track coupling dynamic equation can be expressed in a unified form:

$$[M]\{\ddot{Z}\}+[C]\{\dot{Z}\}+[K]\{Z\}=\{P\} \tag{2}$$

This paper uses Newmark explicit integration method to solve the large-scale nonlinear dynamics differential equations [19]. The explicit scheme is proposed as following:

$$\begin{cases} \{Z\}_{n+1}=\{Z\}_n+\{\dot{Z}\}_n\Delta t+(\frac{1}{2}+\psi)\{\ddot{Z}\}_n\Delta t^2-\psi\{\ddot{Z}\}_{n-1}\Delta t^2 \\ \{V\}_{n+1}=\{\dot{Z}\}_n+(1+\varphi)\{\ddot{Z}\}_n\Delta t-\varphi\{\ddot{Z}\}_{n-1}\Delta t \\ \{\ddot{Z}\}_{n+1}=[M]^{-1}\{P\}_{n+1} \end{cases} \tag{3}$$

where

$$\begin{aligned} \{\mathrm{P}\}_{n+1}=&\{\mathrm{P}\}_{n+1}-[\mathrm{K}]\{\mathrm{Z}\}_n-(\mathrm{C}+\mathrm{K}\Delta\mathrm{t})\{\dot{\mathrm{Z}}\}_n-[(1+\varphi)\mathrm{C}+(\frac{1}{2}+\psi)\mathrm{K}\Delta\mathrm{t}]\{\ddot{\mathrm{Z}}\}_n\Delta\mathrm{t} \\ &+(\varphi\mathrm{C}+\psi\mathrm{K}\Delta\mathrm{t})\{\ddot{\mathrm{Z}}\}_{n-1}\Delta\mathrm{t} \end{aligned} \tag{4}$$

To start the integration produce, one can easily let $\varphi=\psi=0$ at the first time step and use the initial condition as well as:

$$\{\ddot{\mathrm{Z}}\}_0=[\mathrm{M}]^{-1}(\{\mathrm{P}\}_0-\mathrm{C}\{\dot{\mathrm{Z}}\}_0-\mathrm{K}\{\mathrm{Z}\}_0) \tag{5}$$

# 3. The Cross entropy Algorithm

The cross entropy algorithm is an adaptive global random optimization algorithm based on Kullback-Leibler distance developed for rare-event simulation by Rubinstein (1997) [13, 14]. It is an iterative approach that employs Monte Carlo simulation to achieve combinatorial optimisation. The CE algorithm involves two distinct phases: (1) generation of a sample according to a specified random mechanism; (2) updating the parameters of the random mechanism, typically parameters of probability density functions, in order to produce improved sample in the next generation [20].

More formally, the process is described as follows: supposing to minimize performance function $S(x)$ over all states $x$ in some set $\chi$ . This paper denotes the minimum by $\gamma^*$ , thus

$$\gamma^* = S(x^*) = \min_{x\in\chi} S(x) \tag{6}$$

To proceed with the CE method, the deterministic problem is first randomized by defining a family of probability density functions $\left\{ f\left(x;v\right), v \in V \right\}$ on the set $\chi$ . Thus Eq. (6) is associated with the estimation of

$$l(\gamma) = P_{v_0}\left(S(x) \le \gamma\right) = E_{v_0}\left[I_{S(x)\le\gamma}\right] = \sum_{x\in\chi} I_{\{\phi(x)\le\gamma\}} f(x;v_0) \tag{7}$$

Where $I_{\{\phi(x)\le\gamma\}}$ is the indicator function? Therefore the optimization problem has been turned into estimation the probability of the event, $S(x) \le \gamma$ . When $\gamma$ is close to $\gamma^*$ , $\{S(x) \le \gamma\}$ is a rare event, and the estimation of probability $l$ is a non-trivial problem. The CE method solve the problem through adaptive changes to the probability density function according to the Kullback-Leibler distance, thus generating a sequence of parameter of probability density function $v_t$ and a sequence of levels $\gamma_t$ which converges quickly to a small neighbourhood of the optimal. In fact each iteration consists of two main phases. In the first phase, $\gamma_t$ is updated, in the second phase, $v_t$ is updated. Gaussian distribution is adopted in the paper, thus $v$ contains two parameters, mean and standard variance. At the beginning of iterative algorithms, we initialize by setting $v_0$ ($\mu_0$ and $\sigma^2{}_0$) and determine the quantile coefficient $\rho$. Then $\gamma_t$ and $v_t$ are updated as follows:

1. Adaptive updating of $\gamma_t$: for fixed $v_{t-1}$, first generating $M$ random samples $X_i$,$i$=1,2,⋯,$M$ according to the probability density function $f(.,v_{t-1})$, then calculating the associated performance function $\tilde{S}(X_i)$ ,$i$=1,2,⋯,$M$. The $M$ values are then sorted in an increasing order, $\tilde{S}_1 \le \tilde{S}_2 \le ... \le \tilde{S}_M$ . We call $X_1, \ldots, X_{\lceil(1-\rho)M\rceil}$ the elite samples. Let $\gamma_t$ be a $(1-\rho)$-quantile of $S(X_i)$ under $v_{t-1}$. A simple estimator of $\gamma_t$ , denoted $\hat{\gamma}_t$ , can be obtained as:

$$\hat{\gamma}_t = \tilde{\boldsymbol{S}}_{(\lceil(1-\rho)M\rceil)} \tag{8}$$

2. Adaptive updating of $v_t$: For fixed $\gamma_t$ and $v_{t-1}$, the elite samples are determined. According to the maximum likelihood method, it is easy to get improved mean and standard variance from the elite samples. An updated estimate, $v_t$, of the probability distribution parameters are then obtained as following:

$$\mu_{n(t)} = \frac{\sum_{m=1}^{M} I_{\{S(\boldsymbol{X}_m)\le\gamma_t\}} x_{mn}}{\sum_{m=1}^{M} I_{\{S(\boldsymbol{X}_m)\le\gamma_t\}}} \tag{9}$$

$$\sigma^2_{n(t)} = \frac{\sum_{m=1}^{M} I_{\{S(\boldsymbol{X}_m)\le\gamma_t\}} (x_{mn} - \mu_{n(t)})^2}{\sum_{m=1}^{M} I_{\{S(\boldsymbol{X}_m)\le\gamma_t\}}} \tag{10}$$

The two process are repeated until a stopping criterion is reached. Since the standard deviation will gradually decrease during the iteration, it is necessary to expand the standard deviation to avoid convergence to local solution and to speed up the iteration. Referring to the results of EJ Obrien's study [21,22], when the iteration is arrived at the iteration number limit, after half of iterations are completed, expand the standard deviation of the T/2th iteration result by 3 times as the initial standard deviation of the T/2+1 iteration

## 4. Track Irregularity Identification

### *4.1 Performance Function*

By the CE method process given in section 3, a performance function is defined. This is taken to be a relatively simple least squares minimisation of the difference. It is represented by the function $S(\chi)$ in Eq(11).

$$S(\chi) = (\hat{\eta}_1\hat{a}_1 - \eta_1 a_1)^2 + (\hat{\eta}_2\hat{a}_2 - \eta_2 a_2)^2 \tag{11}$$

Where $\hat{a}_1$ and $a_1$ are measured and simulated acceleration of the carriage, and $\hat{a}_2$ and $a_2$ are measured and simulated acceleration of the bogie. $\hat{\eta}$ and $\eta$ are weighting parameters which denote the relative importance each acceleration source has in the performance function. To assign equal importance to each acceleration source in the function:

$$\hat{\eta}_i = \mathrm{RMS}(\hat{a}_1) \times \ldots \times \mathrm{RMS}(\hat{a}_{i-1}) \times \mathrm{RMS}(\hat{a}_{i+1}) \times \ldots \times \mathrm{RMS}(\hat{a}_n) \tag{12}$$

$$\eta_i = \mathrm{RMS}(a_1) \times \ldots \times \mathrm{RMS}(a_{i-1}) \times \mathrm{RMS}(a_{i+1}) \times \ldots \times \mathrm{RMS}(a_n) \tag{13}$$

Where RMS is the root mean square of the specified acceleration response.

### *4.2 Optimisation Process*

Firstly, the track irregularity is idealised as an optimisation problem with *K* unknowns. Depending on the sampling interval and length of the track, *K* may be very large. This means that a very large sample size would be required and there is a risk that the algorithm may converge prematurely to a false solution. To overcome this problem, the optimisation is split into *Kp* phases, in which a smaller 'window' of the unknowns is determined before proceeding to the next phase. *k* unknowns are considered at any one time, where *k*=*K*/*Kp*. At the end of each phase, the *k* best estimates are saved as the estimated track irregularity. And the *k*th unknowns in the window are used as the starting elevations of *k* unknowns for the next phase. Standard deviation is also reset to account for the relative uncertainty in track irregularity further along the phase window being analysed. This is achieved by increasing the standard deviation in increments of 1/*k*. The process is repeated until all *K* unknowns have been characterised. Figure 2 provides a schematic of the phasing procedure.

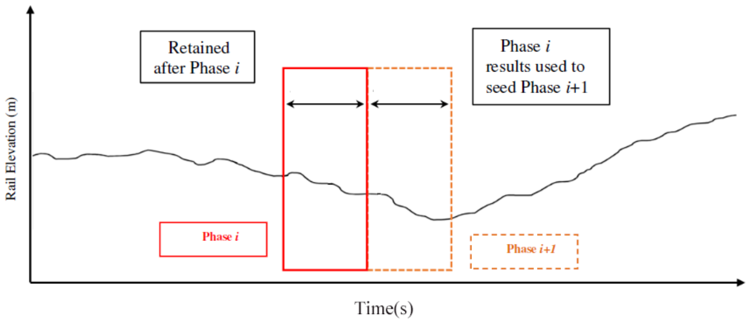

**Figure 2.** 'Windowing' of track irregularity in phases

### *4.3 Numerical Validation of Algorithm*

The algorithm is numerically validated using track irregularity generated by the power spectral density (PSD) function proposed in Chinese PSD of ballastless track irregularities of high-speed railway [23]. The PSD function is shown as following:

$$S(f) = \frac{A}{f^n} \tag{13}$$

Where $f$ is the spatial frequency. In the literature, the sampling interval is 0.25m and the spatial frequency is $4m^{-1}$. Therefore, the length of track is selected to be 200m, and the spatial step length is 0.25m. To increase computational efficiency, this iterative process is limited to 20 iterations during which it is observed that the desired convergence generally occurs. Considering the relationship between the unknown quantity and the computational efficiency, the sample number $M$ is selected to be 250 and the number of unknowns in each phase $k$ is selected to be 5. According to the characteristics of China's high-speed railway, the 5 unknowns of the first window are assumed to have a mean of 0 mm and a standard deviation of 1mm.

In order to investigate the effect of measurement noise on the accuracy of the proposed method, various scales of white noise are added to the measured acceleration. It is generally assumed that the noise is Gaussian random process. In the paper, the signal-to-noise ratio (SNR) is used to investigate measurement noise levels. SNR is the ratio of signal power to noise power, indicating the higher SNR, the more accurate the signal measured. There are three cases tested:
Case 1: SNR=80dB Gaussian white noise are added to the measured carriage and bogie accelerations.
Case 2: SNR=50dB Gaussian white noise are added to the measured carriage and bogie accelerations.
Case 3: SNR=30dB Gaussian white noise are added to the measured carriage and bogie accelerations.

The CE algorithm is used to identify the track irregularity using vehicle acceleration collected in different noise environments. The results are compared with the actual track input to verify the effectiveness of the algorithm.

### *4.4 Result*

The results are compared with the actual track irregularity input as shown in figures 3, 4 and 5. The left ordinate in the figure corresponds to the track irregularity, and the right ordinate corresponds to the absolute error. The estimated track irregularity in the three noise environment are then FFT-transformed to obtain PSD as shown in figure 6.

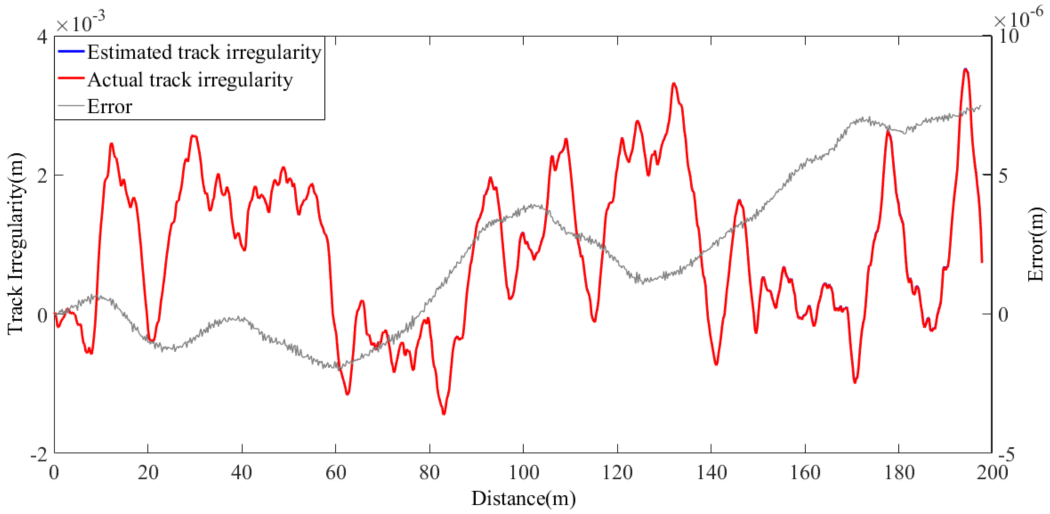


**Figure 3.** Estimated track irregularity in the noise environment with SNR=80dB

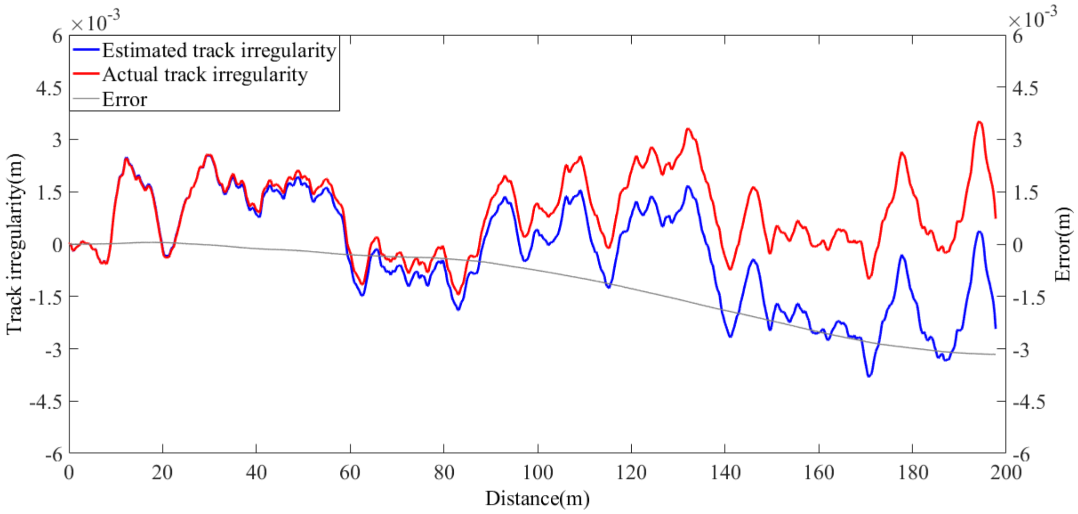


**Figure 4.** Estimated track irregularity in the noise environment with SNR=50dB

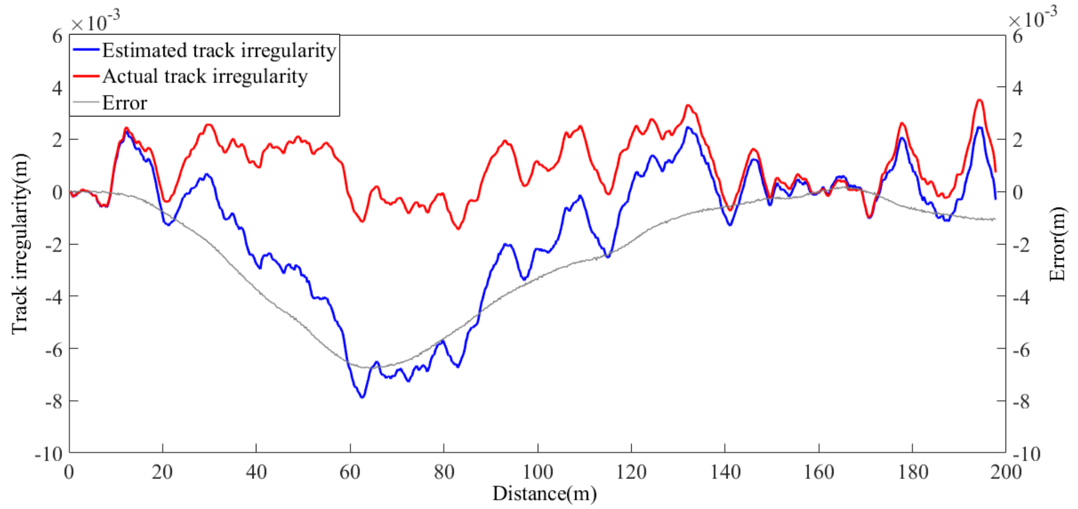


**Figure 5.** Estimated track irregularity in the noise environment with SNR=30dB

It can be seen from figure 3~5 that the CE algorithm can identify the track irregularity effectively and efficiently. The result in the noise environment with SNR=80dB is highly consistent with the actual track irregularity input. The absolute error is maintained on the order of $10^{-9}$m, indicating the effectiveness of the CE algorithm in low noise environments. The results in the noise environment with SNR=50dB and SNR=30dB are relatively inferior to the actual track irregularity, and the absolute error is on the order of $10^{-3}$m.

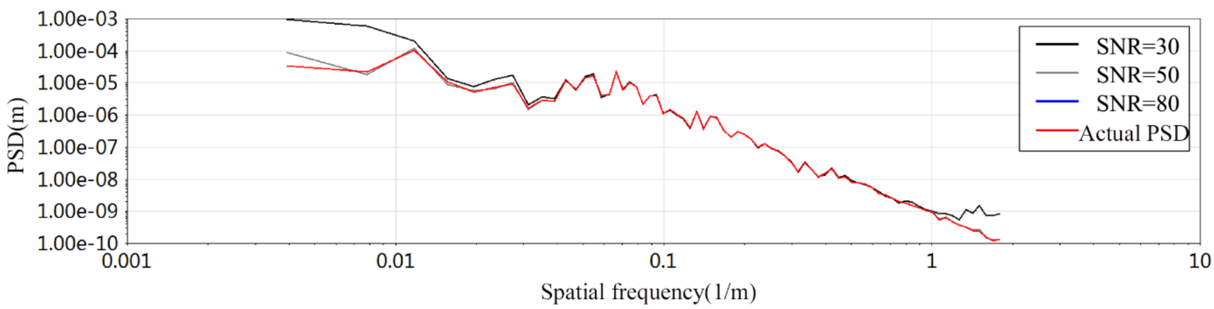


**Figure 6.** Estimated PSD in different noise environment

Figure 6 compares estimated PSDs in different noise environments. In the noise environment with SNR=80dB, the estimated PSD (blue line) of the track irregularity is extremely consistent with the actual PSD. In the noise environment with SNR=50dB, there is error in frequency segment below 0.008 $m^{-1}$ that corresponds to the wavelength greater than 125m. The management wavelength of track irregularity in the paper is 0.25m~120m, so there will be low-frequency trend item above 120m. In the noise environment with SNR=30dB, the error occurs in frequency segments above 1$m^{-1}$ and below 0.04$m^{-1}$. The error in the frequency segment above 1$m^{-1}$ is mainly due to the low SNR noise causing more high-frequency noise components in measured acceleration. At the same time, the high-frequency track irregularity components will lead to noise-like acceleration response in vehicle. Therefore, there will be many error high-frequency components in the estimated PSD. High noise input lead to measured acceleration containing more error signal, which is the most important reason for the error in the frequency segment below 0.04$m^{-1}$.

### *4.5 Result after Filtering out Trend Items*

In the paper, the maximum management wavelength is limited to 120m, so the trend items larger than the maximum management wavelength should be filtered out.

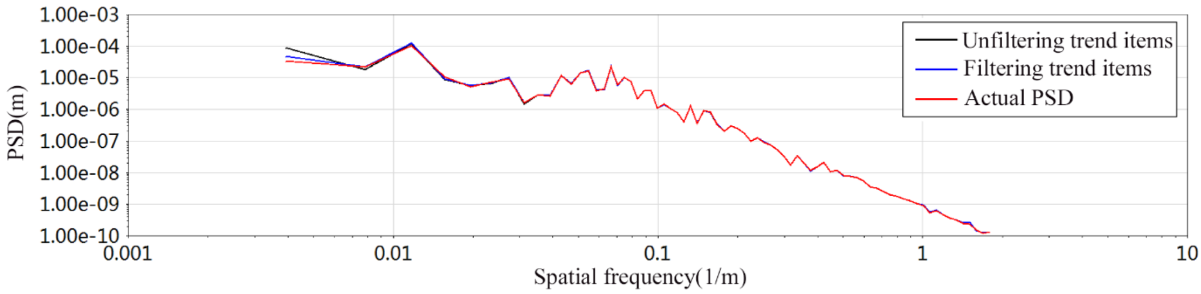


**Figure 7.** Estimated PSD in the noise environment with SNR=50dB

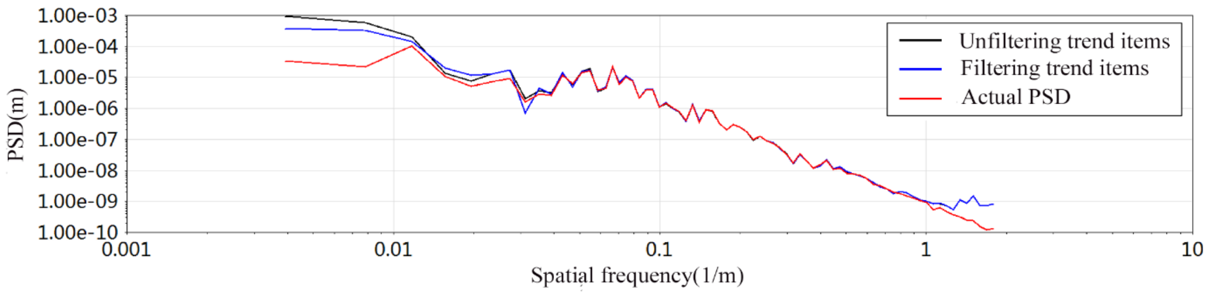


**Figure 8.** Estimated PSD in the noise environment with SNR=30dB

Figure 7 is the estimated PSD after filtering out the trend term above120 m in the noise environment with SNR=50dB. It can be seen that the optimal result after filtering out the trend term is highly consistent with the actual track. The recognition accuracy has been significantly improved indicating that the CE algorithm can identify the track irregularity more accurately after filtering out trend items in the noise environment with SNR=50dB. Figure 8 shows that the accuracy is improved partly in the noise environment with SNR=30dB, however, there are still some differences at low frequency segment. It is not ideal for the excessive noise environment to filter out the trend term larger than the management wavelength. In this case, the acceleration signal should be prioritized for noise reduction and then processed.

## 5. Conclusion

A novel method, the CE algorithm, for the track irregularity identification using vehicle acceleration has been described. The method proposes the use of a combinatorial optimisation technique to determine the track irregularity which causes a set of observed responses in a known vehicle-track model. Initially the vehicle-track coupling model, which consists of the vehicle subsystem and the track subsystem, is established. The CE algorithm is numerically validated using Chinese PSD function proposed by China Academy of Railway Sciences and Southwest Jiaotong University. Using the CE algorithm combined with the moving windows, the vertical track irregularity can be effectively identified. In the noise environment with SNR=80dB, a low noise environment, the absolute error in the 200m track example can be controlled on the order of $10^{-9}$m. In the noise environment with SNR=50dB, a medium noise environment, the method that filters out the trend term above the maximum management wavelength can effectively improve the recognition accuracy. But excessive noise environment will cause too much error in estimated track irregularity. Therefore, when the track irregularity is identified in a high-noise environment, the acceleration signal should be prioritized for noise reduction and then the track irregularity is identified using the CE algorithm.

## Acknowledgement

This study was supported by the National Natural Science Foundation of China (Grant No. 51578140).

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