

Parameter Identification for a Structural System with Particle Tuned Mass Damper Using a Shuffled Complex Evolution Metropolis Algorithm

H. S. Tang^{1,2,*}, X. Y. Guo¹, C. F. Wan³

¹ Research Institute of Structural Engineering and Disaster Reduction, Tongji University, Shanghai, P.R. China

Email: thstj@tongji.edu.cn; guoxueyuan@tongji.edu.cn

² State Key Laboratory for Disaster Reduction in Civil Engineering, Tongji University, Shanghai, P.R. China

³ International Institute for Urban Systems Engineering, School of Civil Engineering, Southeast University, Nanjing, P.R. China

Email: wan_seu@163.com

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ABSTRACT

This paper presents a parameter estimation method using the Shuffled Complex Evolution Metropolis (SCEM-UA) algorithm global optimization for a structural system with the recently proposed particle tuned mass damper (PTMD). The SCEM-UA algorithm is an adaptive evolutionary Markov Chain Monte Carlo (MCMC) sampler, which combines the search capabilities of the Shuffled Complex Evolution (SCE-UA) algorithm, controlled random search, competitive evolution, with the Metropolis algorithm. This algorithm provides an efficient estimate of the most likely parameter set and underlying posterior distribution by continuously updating the proposal distribution and evolving the sampler to the posterior target distribution. The new method overcomes the limitation with traditional structural parameter identification methods in considering the uncertainty problem caused by the available modeling and measurement information. The performance of the proposed method is investigated by means of numerical study of a structure model and the shaking table tests of a structural system consisting of a 5-storey steel frame as the primary structure and a PTMD. The method is employed to identify the parameter set of both the structural system and the PTMD within a single optimization run. The results presented in this paper not only show that SCEM-UA method has great global optimization capability, but also indicate that the method is able to simultaneously identify the uncertainties associated with the posterior distribution of the parameter estimates.

1. Introduction

Parameter identification methods play an important role in the structural analysis as the structural systems usually contain several parameters that are unable to be measured directly. The parameter identification is widely applied for assessing the performance of structures in civil engineering, such as reliability predictions, structural health monitoring and structural control etc. However, traditional identification methods have limitations in practical applications due to the complexity of structure systems and the insufficiency of measurement information. In recent years, heuristic computational intelligence methods, which belong to the global optimization category, have been found to be powerful

* Corresponding author. This is useful to know for communication with the appropriate person in cases with more than one author.

methods to overcome these problems. This kind of methods transforms parameter identification into optimization problems. Among the most important global heuristic optimization methods, such as genetic algorithms (GA) ^[1-3], the simulated annealing ^[4], the Particle Swarm Optimization (PSO) ^[5], the Artificial Bee Colony (ABC) algorithm^[6], the Differential Evolution ^[7], the Big Bang-Big Crunch (BB-BC) optimization algorithm^[8], and the evolution strategy (ES) ^[9].

The classical deterministic global optimization methods, which aim to successfully find a single best set of parameter values, have been well developed. Nevertheless, the obvious presence of uncertainty in the mathematical models and the measurements makes it impossible to find a single point in the parameter space associated with good simulations for the real structure. Recently methods for realistic assessment of parameter uncertainty, such as Markov Chain Monte Carlo (MCMC) algorithms ^[10-13] and bootstrap sampling techniques ^[14-15] have been developed and successfully applied to structural models to update model and estimate parameter uncertainties. MCMC algorithms have become increasingly popular for estimating the posterior probability distribution of parameters in mechanical systems and structures, as they are able to successfully manage nonlinear complex models. However, MCMC methods are based on the evaluation of probability density functions of parameters requiring the priori definition of the sampling distribution. In addition, the design of MCMC samplers is sufficiently challenging to achieve fast convergence with adequate occupation of the lower posterior probability regions of the parameter space.

To improve the search efficiency of MCMC samplers, an evolution of the MCMC method, entitled the Shuffled Complex Evolution Metropolis algorithm (SCEM-UA) was proposed by Vrugt et al. ^[16]. This algorithm is a modification of the original SCE-UA algorithm ^[17], which aims at searching the robust global optimal solution efficiently by running multiple Markov chains as opposed to a single chain in the classical MCMC algorithm. Despite the successful applications of SCE-UA algorithm, it has the tendency to collapse to a single region of attraction, which can be avoided by the stochastic nature of the Metropolis-annealing scheme. Consequently, the SCEM-UA algorithm integrates the advantages of the Metropolis algorithm ^[18], controlled random search ^[19], competitive evolution, and complex shuffling. The SCEM-UA can simultaneously identify the most likely parameter set and its associated posterior probability distribution in every model run.

The SCEM-UA algorithm has been successfully applied to calibrate the conceptual rainfall-runoff model ^[20-22], river basin scale model ^[23] and the groundwater model ^[24-25] for parameter optimization and uncertainty assessment. Previous studies suggested that the adaptive capabilities of the SCEM-UA algorithm can significantly reduce the number of model simulations required to estimate the posterior distribution of the parameters when compared with traditional Metropolis-Hastings samplers ^[16]. Even a few articles have been reported to apply the SCEM-UA method for realistic assessment of parameter and model prediction uncertainties of the hydrologic models, the novelty of the proposed work is that the SCEM-UA algorithm is first investigated for parameter optimization and uncertainty assessment in civil engineering.

In this work, we employ the SCEM-UA algorithm to assess the uncertainty in the parameter estimates of structural systems consisting of the shear frame type structure model and the additional particle tuned mass damper (PTMD). The PTMD is a new device in the vibration control area, which integrates the advantages of the traditional tuned mass damper and the efficient particle damper. Some experimental and numerical studies have been conducted to verify the effective performance of the PTMD and the application in civil engineering ^[26-28]. However, the precise dynamics analyses of the PTMD systems is a great challenge because of the complex computation principle of some parameters. The SCEM-UA global optimization algorithm is used to automatically search through the space of feasible parameter values and to obtain the combination of parameter values that produces the best model performance. Within one optimization run SCEM-UA also yields the underlying posterior parameter distribution, which reflects the uncertainty about the unknown model parameters. The methodology is demonstrated by illustrative examples.

2. Methodology

2.1 SCEM-UA Algorithm

In this paper, the SCEM-UA algorithm is employed to estimate the posterior probability distribution of the structural model parameters. The SCEM-UA algorithm is a modification of the original SCE-UA global optimization algorithm. Fundamentally, the SCEM-UA is an adaptive Monte Carlo Markov Chain sampling technique to a Bayesian scheme where parameters are represented as probabilities, instead of as a single optimal set of values. The SCEM-UA algorithm includes several different Markov Chains, which evolve independently and communicate with each other every a predefined number of generations.

The SCEM-UA algorithm starts with an initial population of s parameter sets randomly distributed throughout the feasible parameter space defined by the joint prior parameter distributions $p(\boldsymbol{\theta})$. A total of s samples from the parameter space $\boldsymbol{\theta}$ are stored in order of decreasing posterior density $p(\boldsymbol{\theta}|\mathbf{y})$. The sample points are partitioned into q complexes with the expectation that $q \ll s$. Partitioning is done by stripping the total population across the complexes. After the samples have been divided into complexes, each complex is evolved independently using the sequence evolution metropolis (SEM) algorithm described below. This algorithm produces new candidate points in each of the parallel sequences S^k by generating draws from an adaptive proposal distribution by using the information induced in the m samples of C^k . After the SEM algorithm completes, all the updated complexes are combined together and stored in order of decreasing posterior density. The evolution and shuffling procedures are repeated until the Gelman–Rubin convergence diagnostic for each of the parameters demonstrates convergence to a stationary posterior target distribution ^[29].

The use of complexes enables the collection of information gained about the search space by each individual sequence during the evolution process. The shuffling of these complexes enhances the survivability of the sequences by a global sharing of the information gained independently by each parallel sequence. This series of operations can give the optimization problem more than one region of attraction and produce a robust collection of MCMC samples capable of facilitating efficient and effective searches of the parameter space

The implementation scheme of the SCEM-UA algorithm is outlined below.

1. Suppose the problem has n dimensions. Initialize the process including the population size s and the number of complexes q , then each complex has m samples, where $m=s/q$.
2. Generate s samples $(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \mathbf{L}, \boldsymbol{\theta}_s)$ randomly from the prior distribution and compute the posterior density $\{(\boldsymbol{\theta}_1|\mathbf{y}), (\boldsymbol{\theta}_2|\mathbf{y}), \mathbf{L}, (\boldsymbol{\theta}_s|\mathbf{y})\}$.
3. Sort s points in order of decreasing posterior density and store them in array $D[1:s, 1:n+1]$, where n is the number of parameters. Samples posterior density is stored in the last column of array D .
4. Initial the starting points of the parallel sequences S^1, S^2, \dots, S^q es, such that S^k is $D[k, 1:n+1]$, where $k=1,2,\dots,q$.
5. Partition the s points of D into q complexes (C^1, C^2, \dots, C^q) , each containing m points.
6. Evolve each of the parallel sequences according to the Sequences Evolution Metropolis (SEM) algorithm.
7. Shuffle complexes. Unpack all the complexes C back into D , rank the points in order of decreasing posterior density.
8. Check the Gelman and Rubin (GR) convergence criteria, If the criteria are satisfied, stop. Otherwise, back to step 5.

Sequences Evolution Metropolis (SEM) algorithm is one of the key components of the SCEM-UA algorithm. It produces new candidate points in each of the parallel sequences S^k by generating draws

from an adaptive proposal distribution by using the information induced in the m samples of C^k . The flow-chart diagram of SEM algorithm is given in Fig. 1.

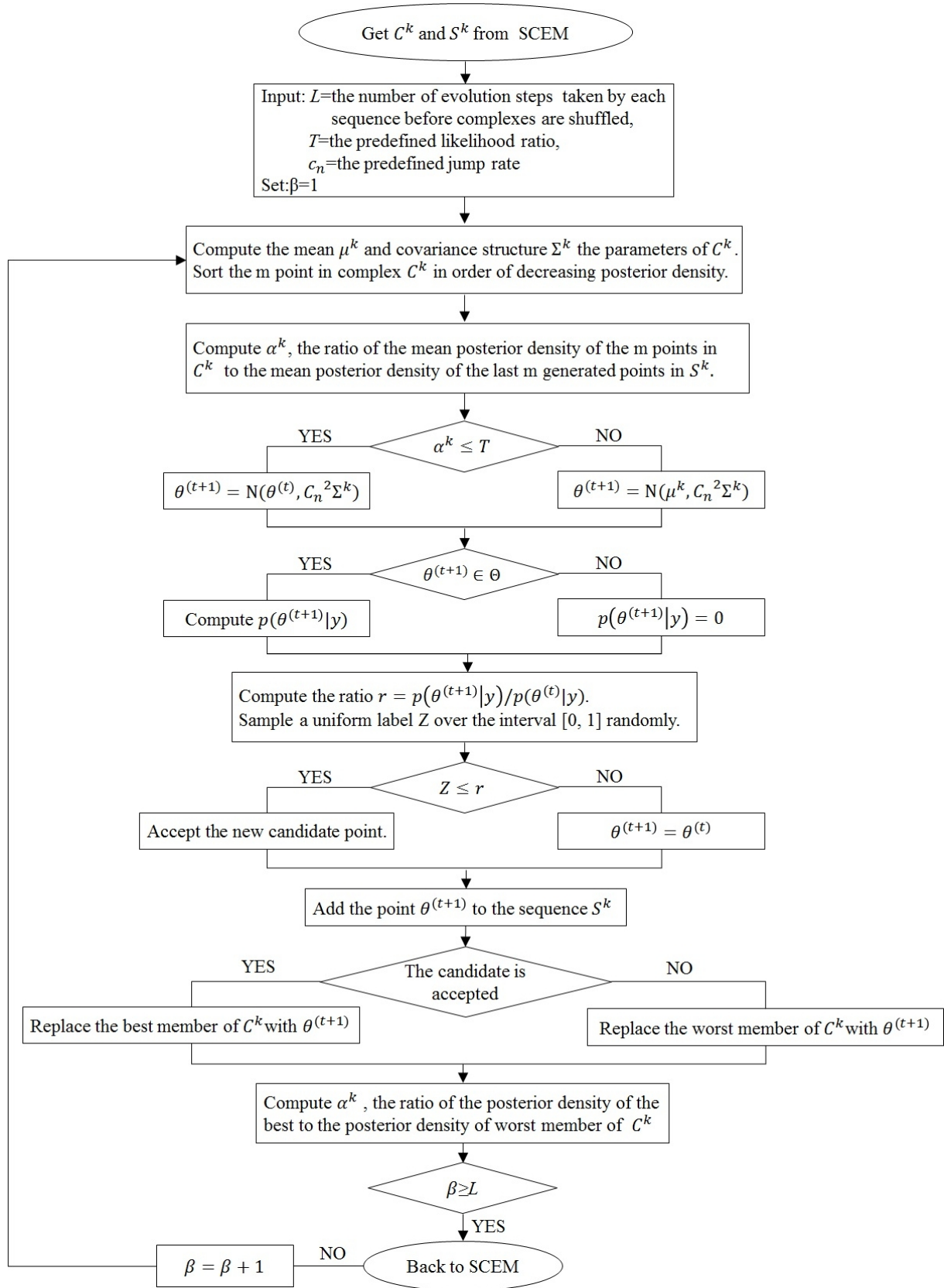


Figure 1. Flowchart of the SEM algorithm

2.2 Modeling

The structural parameter identification is based on the predetermined mathematical model and the input and output information from the actual measurement. The SCEM-UA algorithm is applied to search for an optimal set of parameters to minimize the error between the simulated and the measured structural response. The dynamic equation of the primary multiple degree of freedom (MDOF) structure (Fig. 2(a)) motion can be described as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{u}(t) \quad (1)$$

where \mathbf{M} , \mathbf{C} and \mathbf{K} are the mass, damping and stiffness matrices, \mathbf{x} is the displacement vector and \mathbf{u} is the input force vector. The damping matrix \mathbf{C} is the Rayleigh damping matrix and the modal damping ratio is ζ_r set in the first two modes of vibration ($r=1$ and 2).

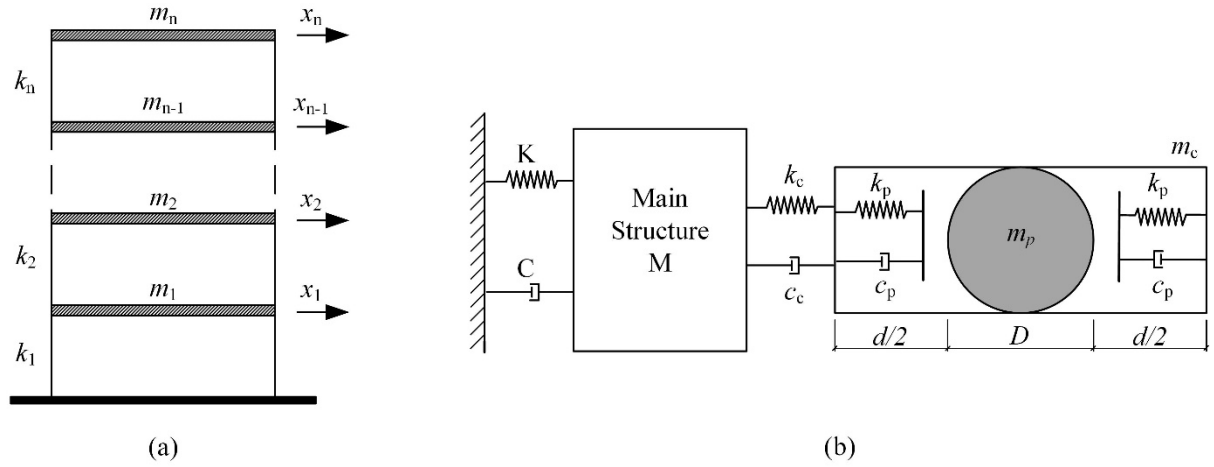


Figure 2. Simplified diagram of the system: (a) primary structure and (b) primary structure with the PTMD

The PTMD, which combines the particle damping technology and the TMDs, possess multiple damping mechanisms, including tuning, impact, collision and friction. The PTMD is suspended on the top of the primary structure considered as a single pendulum. Considering the calculation efficiency, the simplified analytical method proposed by Lu et al. [30] is selected to simulate the vibration reduction effects of the PTMD. The particles in the container are equivalent to a single particle and the schematic diagram of the primary structure with the PTMD illustrated by Fig. 2(b). The suspended container of the PTMD is considered as the $(n+1)$ th DOF with an external force F_p caused by the collisions between the particle and the container. Therefore, the governing equation of the primary structure Eq. (1) can be rewritten as

$$\begin{cases} \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{E}\phi + \mathbf{f} \\ m_c \ddot{y}_1 + c_c \dot{y}_1 + k_c y_1 - c_p H(y_2, \dot{y}_2) - k_p G(y_2) = 0 \\ m_p \ddot{y}_2 + c_p H(y_2, \dot{y}_2) + k_p G(y_2) = 0 \end{cases} \quad (2)$$

$$\mathbf{E} = [-m_1 \quad -m_2 \quad \dots \quad -m_n]^T \quad (3)$$

$$\mathbf{f} = [0 \quad 0 \quad \dots \quad 0 \quad 1]^T \quad (4)$$

where ϕ is the location vector of the control force, $y_1 = x_c - x_s$, $y_2 = x_p - x_c$, and the subscripts c and p represent the container and the simplified particle, respectively. G and H are nonlinear function related to the clearance d of the equivalent single particle damper which can be obtained based on the equivalent principle. More details about the equivalent principle and the simplified analytical method can be referred to Lu et al. [30].

3. Numerical Validation on the Shear Model

The performance of the presented SCEM-UA algorithm is first investigated in a case of a 10-DOF shear frame type structure as shown in Fig. 2(a). In addition, the SCEM-UA algorithm is compared with other global optimization algorithms presented in previous research. The structural parameters are listed in Table 1. The dynamic equation of the structure motion is described as Eq. (1). The fourth-order Runge–Kutta method is employed to carry out the simulation of the structural response to a given excitation. Modal identification is performed and the identified modal parameters of the 10-DOF structure system can be fully described by the parameter set

$$\boldsymbol{\theta} = (m_1, m_2, \frac{1}{4}, m_{10}, k_1, k_2, \frac{1}{4}, k_{10}, z_1, z_2) \quad (5)$$

In this study, similar to other studies^[1], input forces are applied at the 5th level of the structure as random white Gaussian noise with the RMS of the force scaled to 1000 N. Acceleration measurements are obtained at different floor levels: acceleration measurements at floors 1, 2, 4, 6, 8, and 10 are available in the unknown mass case. Parameter estimation of the unknown mass model with 10% noise scenario is investigated for the sake of comparison.

Table 1. Structural parameters of 10-DOF system

Stiffness(kN/m)	Mass (kg)	Damping ratio
$k_{1-4}=5000$		
$k_{5-8}=4000$	$m_{1-5}=6000$	$\zeta_1=5\%$
$k_{9-10}=3000$	$m_{6-10}=4200$	$\zeta_2=5\%$

The SCEM-UA algorithm was implemented with a population size $s=600$ and $q=30$ complexes, 20 points in each complex. The prior information of estimation parameters usually consists of realistic lower and upper bounds on each of the parameters, thereby defining the feasible parameter space and imposing a uniform prior distribution on this rectangle. The evolution of the Gelman-Rubin convergence diagnostic for estimation parameters using the SCEM-UA algorithms is illustrated in Fig. 3. Fig. 4 presents the marginal posterior probability distributions for the structural model estimation parameters constructed using 6000 samples generated after convergence of the SCEM-UA algorithm. Note that for all parameters in Fig. 4 do not correspond to the range specified for the respective prior uniform distributions. Although not presented in this paper, similar results are obtained for the other parameters.

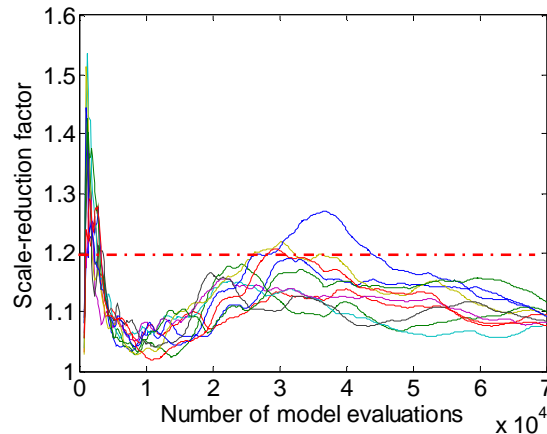


Figure 3. Evolution of the Gelman and Rubin scale-reduction factor

Table 2 presents the final posterior moments derived using those samples that were generated with the SCEM-UA algorithm after convergence to the stationary target distribution has been achieved. Also included are the most likely parameter values of the structural model, identified using the GA, PSO, BB-BC and SCE-UA global optimization algorithms after the same evaluations (50000) for the sake of comparison. When compared to these methods, the SCEM-UA method shows slightly better global optimization capability but also the ability to identify simultaneously the uncertainties associated with

the posterior distribution of the parameter estimates within a single optimization run. Clearly, this feature is an additional benefit of the SCEM-UA algorithm because it makes superfluous the two-step procedure in which the global optimum in the parameter space is first identified, followed by launching parallel Metropolis-Hastings samplers from this starting point to identify parameter uncertainty.

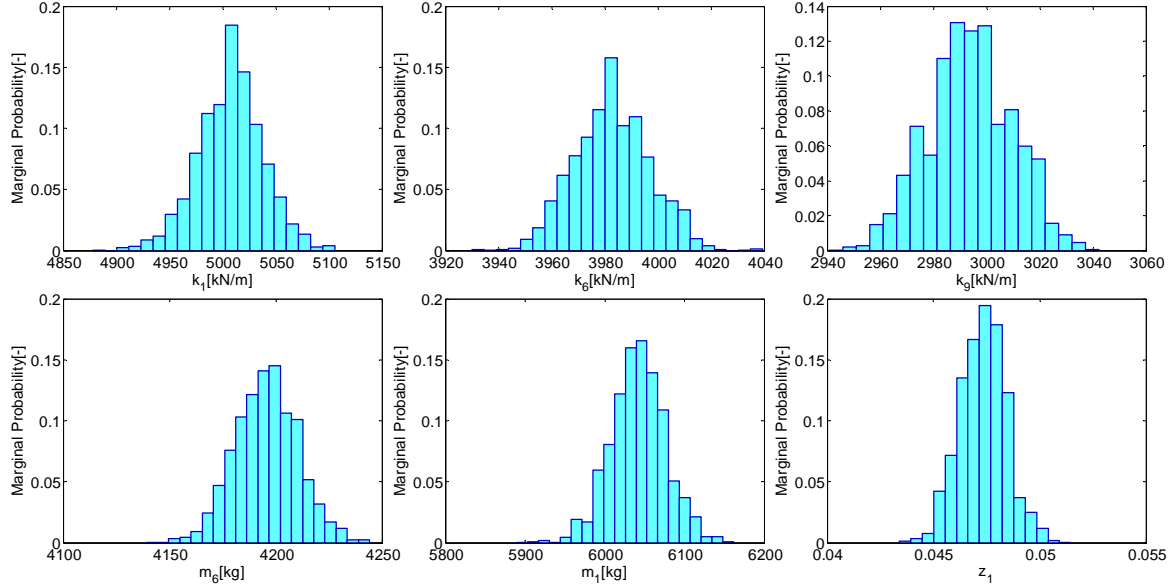


Figure 4. Posterior probability distributions of structural parameters

Table 2. Identification results for comparison

Parameter	GA	PSO	BB-BC	SCE-UA	SCEM-UA
Avg. error- m (%)	2.98	2.74	2.61	1.04	0.68
Max. error- m (%)	6.62	7.11	4.55	3.17	1.49
Avg. error- k (%)	3.00	3.00	2.25	1.12	0.73
Max. error- k (%)	6.81	6.10	3.33	2.62	1.52
Avg. error- c (%)	8.41	7.96	2.79	2.94	2.71
Max. error- c (%)	15.29	10.7	4.72	6.13	5.40

4. Application to Structural System with Particle Tuned Mass Damper

The results of the numerical study indicate that the SCEM-UA algorithm is capable of parameter identification with the incomplete measurements of the structural response and noise existing. In this section, the SCEM-UA algorithm is applied to a realistic five-story steel frame as the primary structure with a PTMD. The shaking table test of a 5-storey steel frame model is carried out by Lu et al. [30]. The $15 \times 180 \times 1060$ mm sized columns are made of Q690 high-strength steel plate, and the $2.0 \times 2.0 \times 0.03$ m sized floors are made of Q345 steel plate. The PTMD with 2.26% total auxiliary mass ratio to the primary system is suspended on the top of the primary steel frame by four steel strands. The dynamic equation of the structure system has discussed above as Eq. (2). The mass matrix of the steel frame $\mathbf{M}=(1032kg, 1032kg, 1032kg, 1032kg, 829.9kg)$ and the total masses of the container and the particles, $m_c=39.345kg$ and $m_p=7.96kg$, are already known. In addition, based on the fundamental theorem of the structural dynamics, the stiffness and the damping coefficient can be determined by the circular frequency and the damping ratio respectively. Therefore, the parameters of the structure system which are unable to be measured can be described by the parameter set

$$\boldsymbol{\theta} = (k_1, k_2, \gamma, k_3, z_1, z_2, w_c, z_c, w_p, z_p) \quad (6)$$

In this study, the structure system is excited by the one-dimensional ground motion El Centro seismic wave (1940, NS) and the peak value of the acceleration is 0.2g (g is the acceleration due to gravity).

Acceleration measurements are obtained at each floor level. The SCEM-UA algorithm was implemented with a population size $s=400$ and $q=20$ complexes, 20 points in each complex. The prior information of estimation parameters consists of realistic lower and upper bounds on each of the parameters with a uniform prior distribution.

The evolution of the Gelman-Rubin convergence diagnostic for estimation parameters using the SCEM-UA algorithm is illustrated in Fig. 5. The line plots indicate that for all parameters the parallel sequences converged to the target distribution after approximately 400 iterations. The rapid convergence of parameters is confirmed by the evolution of the samples generated in the $q=20$ sequences presented in Fig. 6. Similar results are obtained for the other parameters. Fig. 7 present the marginal posterior probability distributions of the estimation parameters constructed after convergence of the SCEM-UA algorithm. Note that for all parameters in Fig. 7 do not correspond to the range specified for the respective prior uniform distributions. The summarizing statistics of the posterior parameter distributions are presented in Table 3.

In order to demonstrate the reliability of the identified values, the identified modal parameters are input to carry out the simulation of the structural response to the given excitation. The comparison results between the actual response and the inversion response are shown in Figure 8. According to the analysis results in Chapter 3, the results of structural parameter identification contain uncertainties depending on the noise exists in the observed data due to the limitation of measurement accuracy. The predicted motion trends with an acceptable accuracy and the reasonably accurate estimates of the maximum response show the feasibility of the SCEM-UA algorithm for structural parameter identification and the ability to identify simultaneously the parameters of the primary structure and the PTMD.

Table 3. Summarizing statistics of the posterior parameter distribution

Parameter	M.V.	S.D.	R.E.
k_1 (kN/m)	4.418E+05	1.90E+03	0.43%
k_2 (kN/m)	5.125E+05	1.29E+03	0.25%
k_3 (kN/m)	4.778E+05	2.61E+03	0.55%
k_4 (kN/m)	5.217E+05	1.31E+03	0.25%
k_5 (kN/m)	5.718E+05	2.89E+03	0.50%
z_1	7.608E-03	1.05E-04	1.38%
z_2	1.487E-03	4.24E-05	2.85%
ω_c (rad/s)	6.799	2.76E-02	0.40%
ω_p (rad/s)	1.585E+02	2.08E+01	13.12%
z_c	3.434E-01	8.14E-03	2.37%
z_p	3.820E-01	2.45E-02	6.41%

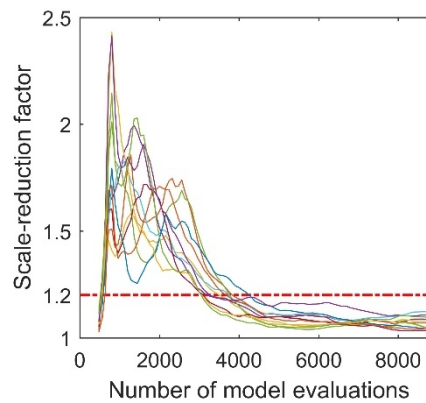


Figure 5. Evolution of the Gelman and Rubin scale-reduction factor for the parameters in the structure system with a PTMD

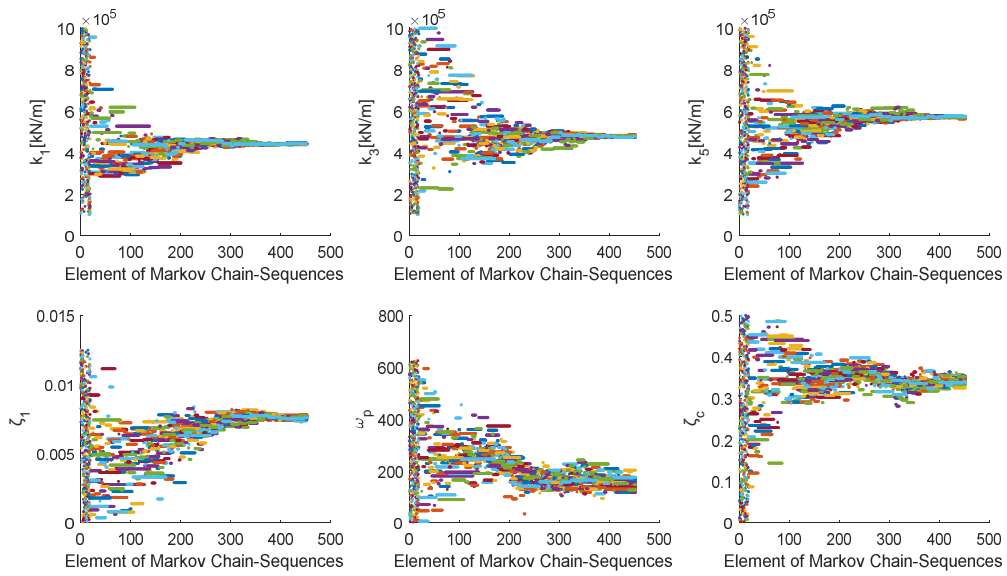


Figure 6. Evolution of the samples for the parameters in the structure system with a PTMD

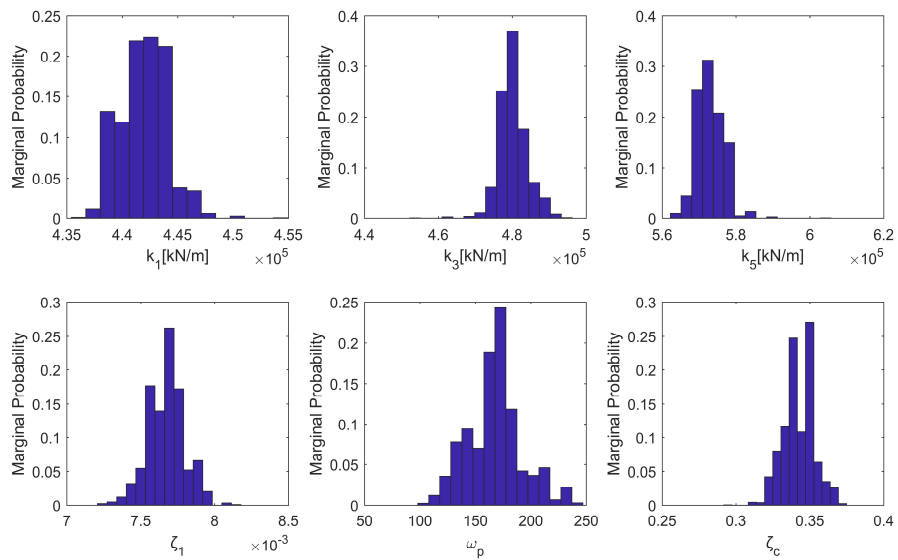


Figure 7. Posterior probability distributions of the parameters in the structure system with a PTMD

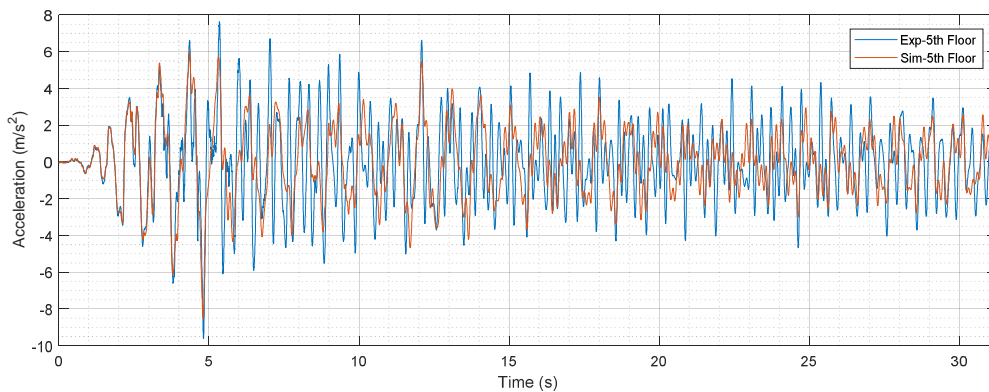


Figure 8. Comparison of experimental and simulated acceleration time series for the 5th floor

5. Conclusions

In this study, the recently developed SCEM-UA algorithm was applied to estimate parameter of structural system models. Results show that the SCEM-UA algorithm is able to efficiently and effectively explore the feasible parameter space and to converge to the target posterior parameter distributions. The efficiency and effectiveness of the newly developed SCEM-UA algorithm for estimating the posterior distribution of the parameters is compared with the other global optimization algorithms. The application of the SCEM-UA algorithm to a controlled structure proves that the parameter identification of both the original structure and the PTMD simultaneously is feasible. It is clear from the results that the SCEM-UA method shows excellent global optimization capability but also the ability to identify simultaneously the uncertainties associated with the posterior distribution of the parameter estimates within a single optimization run. The results also highlights the ability of SCEM-UA method to determine good model parameter vectors without any a priori knowledge of the system.

In this work, only the uncertain source of the insufficient observations with errors is considered. In the future, the idea is to employ the SCEM-UA approach presented here in order to assess the uncertainty in the parameter estimates of real structural models including modeling, observation and excitation uncertainties.

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