Additional Damping Force Identification of Structures Equipped with Eddy Current Inerter Dampers based on Kalman Filter

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ABSTRACT

The paper presents an innovative eddy current inerter damper (ECID), which provides the additional damping force through the inerter and eddy current system. In order to obtain the additional damping force generated by ECID, an inverse methodology based on Kalman filter is developed to solve the force identification problem. The additional damping force generated by ECID can be considered as part of the input force of primary structure. On condition that the excitation force and parameters of the structure are known, the inverse solution method can be used to estimate the additional damping force. The inverse method is based on the Kalman Filter and recursive least-squares algorithm. A state space model of damper controlled structure is first built. Kalman filter is then used to generate the residual innovation sequence and the recursive least-square algorithm with forgetting factor is used for computing the magnitude of additional damping force. The proposed method is examined with a series of free vibration experiments of a single-degree-of-freedom structure equipped with ECID. The testing results show that the ECID provides a significant damping effect and the additional damping force generated by ECID can be estimated in high accuracy using the inverse method based on Kalman filter.

1. Introduction

Structural control in civil engineering has been a crucial part of designing new structures and retrofitting existing structures. Over the past several decades, various methods of structural control have been developed and utilized to attenuate vibrations. Among numerous methods, passive energy dissipation systems have been most widely used in the civil engineering, such as metallic yield dampers, friction dampers, viscoelastic dampers, viscous fluid dampers, tuned mass dampers and tuned liquid dampers[1].

As typical passive dampers, tuned mass dampers (TMD) are developed as innovative devices for passive vibration control of structures in the 1970's. TMD systems can be used to tune only a particular frequency of vibration, and they are not effective for structures under seismic excitations[2]. In addition, the performance of TMD systems is restricted by the mass of TMD that provided for the primary structure. In this condition, inerter systems, whose effective masses are amplified by ball screw or rotated flywheel, are developed. Recently, a tuned viscous mass damper (TVMD), which mainly consists of viscous damping system and rotational inertial mass, is proposed by Saito et al.[3]. In the TVMD system, the rotated inerter system can be driven by the ball screw and enhances the energy-dissipation capability. However, the viscous damping system has poor durability[4], some innovative damping elements are introduced, such as eddy

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current damping system. In 1994, eddy current tuned mass damper (ECTMD) is developed in aerospace engineering by Kienholz et al[5]. In 2007, an innovative eddy current damper for rotor systems is developed by Zhu, which can attenuate the vibration of the system.

An innovative eddy current inerter damper (ECID) is presented in this paper, which consists of a stator system and a rotor system. The stator system includes steel plates fixed on the base and permanent magnets absorbed on steel plates. The rotor system includes a roller and circular conducting plates fixed on the roller. The ECID is fixed on the ground and connected to the primary structure with steel cables. When inter-story drift occurs in the structure, the roller can be driven by the steel cables to rotate. The rotated circular conducting plates can be treated as inerters. And at the same time, eddy currents can be induced by relative motion between the rotated circular conducting plates and magnets, which can also dissipate energy as heat in the material. Therefore, the additional damping force provided by ECID is mainly generated by the inerter and eddy current system.

In order to clarify the structural damping, various methods of damping identification are developed. There exist methods which can identify the damping values, like damping ratios. And these methods can be divided into frequency and time domain[6]. On the other hand, additional damping force generated by dampers can also represent structural damping properties. However, the damping force is not easily measurable directly in the actual projects. On the contrary, structural response is much more accessible. Therefore, indirect estimation of the additional damping force generated by damper has been attempted, which is an inverse problem involving force identification from measurement of structural response[7]. To be specific, the additional damping force can be considered as a part of the input force of the structure, then the problem of additional damping force identification can be turned to the problem of input force identification.

For the input force identification problems, Stevens has summarized the force identification process for linear structural system[8]. Some scholars have presented various force estimation methods to identify the input force of structures, such as Hillary[9], Ory[10], Doyle[11], Michaels and Pao[12]. Recently, inverse methodology based on Kalman filter is developed to solve the force identification problem. The Kalman filer (KF) is proposed in 1960[13], it has a recursive structure and can be used to process sequential noisy measurement data. The KF models the dynamic system into a set of state equations. When the mass, stiffness and damping of primary structure are known, the excitation force is known, the unknown additional damping force provided by dampers can be considered as the input parameter to be estimated. Chan et al. developed a solution to solve the problem of tracking a maneuvering target using the generalized least-squares approach[14, 15]. However, this input estimation technique, with the batch form, requires matrix inversions that leads to computational inefficiency. An input estimation algorithm developed by Tuan et al., which consists of KF and a recursive least-squares algorithm, has been proved to have a superior performance in tracking targets and a greater computational efficiency[16]. And weighting in the recursive least-squares algorithm is crucial, especially when the unknown force is time-varying. The most widely used weighting adopts the forgetting factor, which can be used to preserve the updating ability of the algorithm continuously[17]. And the input estimation algorithm with forgetting factor is proved to be more efficient and robust[18]. This algorithm is applied in the structural systems, the results show that the estimated input forces are in good agreement with the actual input forces[19, 20].

In this paper, the mechanical model and operating principle of a single-degree-of-freedom structure with ECID is described. And to estimate the additional damping force generated by ECID, the KF and recursive least-squares algorithm combined with forgetting factor is presented to compute the additional damping force. And to verify the effectiveness of the Kalman filter-based algorithm, a series of free vibration experiments are conducted.

2. Single-Degree-Of-Freedom Structure Equipped with ECID

2.1 ECID mechanical model

The inerter system with eddy current damping devices can be termed as eddy current inerter damper (ECID). There are mainly four basic mechanical elements: an eddy current damping element, an inerter, a friction element and a spring **(Fig. 1)**. Therefore, the additional damping force generated by ECID includes three mainly parts and can be calculated using the following equation:

$$G = k_b x_b = c_d \dot{x}_d + m_d \ddot{x}_d + Q_f \, . \tag{1}$$

Where G donates the additional damping force generated by ECID, Q_f donates the friction force generated by ECID. x_b and x_d donate the deformation of the spring and the eddy current damping element, respectively. k_b, c_d and m_d donate the stiffness, the damping and the mass of ECID, respectively.

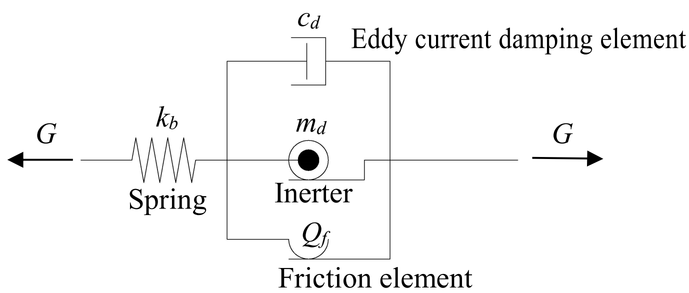


Figure 1. Mechanical model of eddy current inerter damper (ECID)

2.2 Equation of motion for SDOF structure with ECID

Fig.2 shows the analysis model of structure with ECID. The additional damping force generated by the damper can be considered as a part of the input force of the structure. Thus, the equation of motion for the damper controlled structure can be represented as:

$$\mathbf{M}\ddot{\mathbf{Y}}(t) + \mathbf{C}\dot{\mathbf{Y}}(t) + \mathbf{K}\mathbf{Y}(t) = \mathbf{F}(t) + \mathbf{G}(t) \, . \tag{2}$$

Where **M**, **C** and **K** denote the mass matrix, damping matrix and stiffness matrix of the primary structure without the damper, respectively. $\mathbf{F}(t)$ is the excitation force vector, for free vibration $\mathbf{F}(t)$ equals zero vector. $\mathbf{G}(t)$ is the additional damping force vector generated by ECID, and $\ddot{\mathbf{Y}}(t)$, $\dot{\mathbf{Y}}(t)$ and $\mathbf{Y}(t)$ denote the acceleration, velocity and displacement vector, respectively.

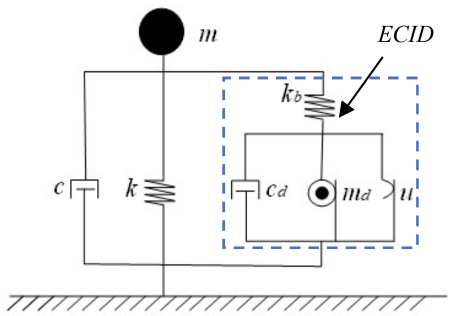


Figure 2. Analysis model

3. Additional Damping Force Identification method based on Kalman filter

3.1 Kalman Filter

In converting to the state-space equation, the state variables of the system with n degrees of freedom are represented by a state vector $\mathbf{X}(t)=\left[\mathbf{Y}(t)\dot{\mathbf{Y}}(t)\right]^{\mathrm{T}}$, the continuous-time state equation can be written as:

$$\dot{\mathbf{X}}(t)=\mathbf{AX}(t)+\mathbf{BF}(t)+\mathbf{BG}(t)\,. \tag{3}$$

Where

$$\mathbf{A}=\begin{bmatrix}\mathbf{0}_{n\times n} & \mathbf{I}_{n\times n}\\ \mathbf{-M^{-1}K} & \mathbf{-M^{-1}C}\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}\mathbf{0}_{n\times n}\\ \mathbf{M^{-1}}\end{bmatrix}$$

Equation (3) can be discretized over time intervals of length Δt, and associated with noise inputs, then equation (3) becomes:

$$\mathbf{X}(k+1)=\mathbf{\Phi X}(k)+\mathbf{\Gamma F}(k)+\mathbf{\Gamma G}(k)+\boldsymbol{\omega}(k)\,. \tag{4}$$

Where

$$\mathbf{\Phi}=\exp(\mathbf{A}\Delta t),\ \mathbf{\Gamma}=\int_{k\Delta t}^{(k+1)\Delta t}\exp\{\mathbf{A}[(k+1)\Delta t-\tau]\}d\tau\mathbf{B}=[\mathbf{I}-\exp(-\mathbf{A}\Delta t)]\mathbf{A}^{-1}\mathbf{B}$$

$\mathbf{\Phi}$ is the state transition matrix, $\mathbf{\Gamma}$ is the input matrix, $\mathbf{F}(k)$ is the sequence of excitation force and $\mathbf{G}(k)$ is the sequence of additional damping force. $\boldsymbol{\omega}(k)$ is the process noise vector, which is assumed to be zero mean and white noise. And the corresponding covariance matrix is $\mathbf{Q}$.

The Kalman filter can be used in the condition that the observation measurement is structural displacement responses. Therefore, the measurement equation is firstly introduced and discretized over time intervals of length Δt, and associated with noise inputs, the measurement equation can be written as:

$$\mathbf{Z}(k)=\mathbf{HX}(k)+\mathbf{v}(k) \tag{5}$$

$\mathbf{H}$ is the measurement matrix, $\mathbf{Z}(t)$ is the observation vector and $\mathbf{v}(k)$ is the measurement noise vector, which is assumed to be zero mean and white noise. And the corresponding covariance matrix is $\mathbf{R}$.

The Kalman filter is then used to generate a residual innovation sequence, and it takes place in a recursive manner in two stages. Firstly, based on the model and the observations until $t(k\text{-}1)$, the prediction of the state at $t(k)$ can be obtained.

$$\overline{\mathbf{X}}(k/k-1)=\mathbf{\Phi}\overline{\mathbf{X}}(k-1/k-1)+\mathbf{\Gamma F}(k-1)\,. \tag{6}$$

$$\overline{\mathbf{P}}(k/k-1)=\mathbf{\Phi}\overline{\mathbf{P}}(k-1/k-1)\mathbf{\Phi}^{\mathbf{T}}+\mathbf{Q} \tag{7}$$

Where $\overline{\mathbf{X}}(k/k-1)$ and $\overline{\mathbf{P}}(k/k-1)$ donate predictions at $t(k)$, $\overline{\mathbf{X}}(k-1/k-1)$ and $\overline{\mathbf{P}}(k-1/k-1)$ donate estimations at $t(k\text{-}1)$. Then using the new information provided by the observation at $t(k\text{-}1)$, the updated prediction at $t(k)$ can be obtained,

$$\overline{\mathbf{X}}(k/k)=\overline{\mathbf{X}}(k/k-1)+\mathbf{K}_a(k)\overline{\mathbf{Z}}(k)\,. \tag{8}$$

$$\mathbf{K}_a(k)=\overline{\mathbf{P}}(k/k-1)\mathbf{H}^{\mathrm{T}}\mathbf{S}^{-1}(k) \tag{9}$$

$$\mathbf{S}(k)=\mathbf{H}\overline{\mathbf{P}}(k/k-1)\mathbf{H}^{\mathrm{T}}+\mathbf{R} \tag{10}$$

$$\overline{\mathbf{Z}}(k)=\mathbf{Z}(k)-\mathbf{H}\overline{\mathbf{X}}(k/k-1) \tag{11}$$

$$\overline{\mathbf{P}}(k/k)=[\mathbf{I}-\mathbf{K}_a(k)\mathbf{H}]\overline{\mathbf{P}}(k/k-1) \tag{12}$$

Where $\mathbf{K}_a(k)$ is the Kalman gain, $\overline{\mathbf{Z}}(k)$ is the innovation and $\mathbf{S}(k)$ is the innovation covariance.

3.2 Recursive Least-squares Algorithm with forgetting factor

With the residual innovation sequence generated by the Kalman filter, the recursion relation between the innovation $\overline{\mathbf{Z}}(k)$ and unknown additional damping force $\mathbf{G}(k)$ can be obtained. The recursive least-squares algorithm is then used to compute the onset time histories of the additional damping force. The detailed formulas are presented in the paper of Tuan et al. The equations are:

$$\mathbf{B}_s(k) = \mathbf{H}(\Phi\mathbf{M}_s(k-1)+\mathbf{I})\Gamma \tag{13}$$

$$\mathbf{M}_s(k) = [\mathbf{I}-\mathbf{K}_a(k)\mathbf{H}][\Phi\mathbf{M}_s(k-1)+\mathbf{I}] \tag{14}$$

$$\mathbf{K}_b(k) = \frac{\mathbf{P}_b(k-1)\mathbf{B}_s^T(k)}{\gamma}[\frac{\mathbf{B}_s(k)\mathbf{P}_b(k-1)\mathbf{B}_s^T(k)}{\gamma}+\mathbf{S}(k)]^{-1} \tag{15}$$

$$\mathbf{P}_b(k) = \frac{1}{\gamma}[\mathbf{I}-\mathbf{K}_b(k)\mathbf{B}_s(k)]\mathbf{P}_b(k-1) \tag{16}$$

$$\hat{\mathbf{G}}(k) = \hat{\mathbf{G}}(k\text{-}1)+\mathbf{K}_b(k)[\overline{\mathbf{Z}}(k)-\mathbf{B}_s(k)\hat{\mathbf{G}}(k-1)] \tag{17}$$

Where $\mathbf{S}(k)$ is the innovation covariance, γ is the forgetting factor, which ranges from 0 to 1. When γ equals 1, the algorithm reduces to the usual sequential least squares, which is suitable only for a constant-parameter system. For $0<\gamma<1$, the algorithm can preserve its updating ability continuously by adjusting $\mathbf{K}_b(k)$, which achieves a balance of high adaptive capability and the loss of accuracy.

In conclusion, the procedure to assess the unknown additional damping force using the method can be summarized as follows:

(1) The state-space equations (4) and (5) are established and the state vector is clarified, which represents the system responses.
(2) Based on the Kalman filter equations (6)-(12), the Kalman gain $\mathbf{K}_a(k)$ and innovation $\overline{\mathbf{Z}}(k)$ can be obtained.
(3) With the residual innovation sequence generated by the Kalman filter, the recursion relation between the innovation $\overline{\mathbf{Z}}(k)$ and unknown additional damping force $\mathbf{G}(k)$ can be established. Then the recursive least-squares algorithm, equation (13)-(14), are used to calculate the unknown additional damping force $\hat{\mathbf{G}}(k)$.

4. Experimental verification and results

In order to calculate the additional damping force provided by dampers and verify the effectiveness of the additional damping force identification method based on Kalman filter, a series of experiments are conducted.

4.1 Experimental equipment

The experimental model consists of a single-degree-of-freedom steel frame as the primary structure and an ECID **(Fig. 3)**. The total mass of the primary structure is 23kg, and the total height is 1.0m. The slab consists of steel plates (Q235) with plane dimensions of 834×390mm and a thickness of 10mm. The columns consist of steel plates (Q235) with height×width×thickness dimensions of 1000×60×3mm. The first natural frequency of the primary structure is adjusted as almost 1 Hz.

The experiment makes use of free vibration method, with an initial displacement of 80mm. To measure structural responses, three types of sensors are installed. The acceleration of the floor is measured by accelerometers. The displacement of the top floor is measured by a displacement meter. In addition, two force sensors are stalled in the steel cables to measure the tension of steel cables **(Fig. 3)**.

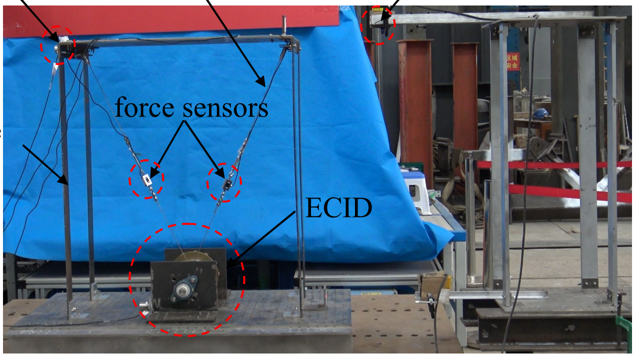


Figure 3. Configuration of the test specimen

4.2 Parameter Identification of Primary Structure without ECID

To calculate the additional damping force generated by the ECID, the unknown parameters of primary structure are required to be identified firstly. Therefore, the free vibration test of primary structure without ECID is conducted firstly. Accordingly, the excitation force vector **F**(t) and additional damping force vector **G**(t) in the equation of motion both equal zero vector. Then the extend Kalman filter (EKF) is adopted to identify system parameters: stiffness and damping.

In the extended Kalman filter algorithm, the extended state vector can be represented as:

$$\dot{\mathbf{X}} = \begin{Bmatrix} \dot{\mathbf{X}}_1 \\ \dot{\mathbf{X}}_2 \\ \dot{\mathbf{X}}_3 \\ \dot{\mathbf{X}}_4 \end{Bmatrix} = \begin{Bmatrix} \mathbf{X}_2 \\ \mathbf{M}^{-1}[-\mathbf{C}\mathbf{X}_2 - \mathbf{K}\mathbf{X}_1] \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \tag{18}$$

Where $\mathbf{X}_1 = \mathbf{Y}, \mathbf{X}_2 = \dot{\mathbf{Y}}, \mathbf{X}_3 = \begin{bmatrix} k_1 & \dots & k_n \end{bmatrix}, \mathbf{X}_4 = \begin{bmatrix} c_1 & \dots & c_n \end{bmatrix}$, the k_i and c_i denote the shear stiffness of the ith story and damping, respectively. The extended Kalman filter consists a time update step and a measurement update step. In the time update step, the predicted state can be represented as:

$$\overline{\mathbf{X}}(k/k-1) = \overline{\mathbf{X}}(k-1/k-1) + \int_{t_k}^{t_{k+1}} f(\overline{\mathbf{X}}(k-1/k-1), t)dt \, . \tag{19}$$

To apply estimation theory in EFK, the non-linear term in the **Eq.(19)** can be linearized by Taylor's expansion. And in this paper, the fourth-order Runge-Kutta method is used to resolve differential equation. In the measurement update step, the basic principles of EKF is similar to KF. Limited by space, the detailed calculating process of EFK will not be described in this paper. As described above, the measurement is the displacement response of the top floor, which is shown in **Fig.4**. The initial values used in the EFK are given as follows: sampling interval $\Delta t = 10^{-3}$ s and covariance matrix of measurement noise **R** = 10^{-12}.

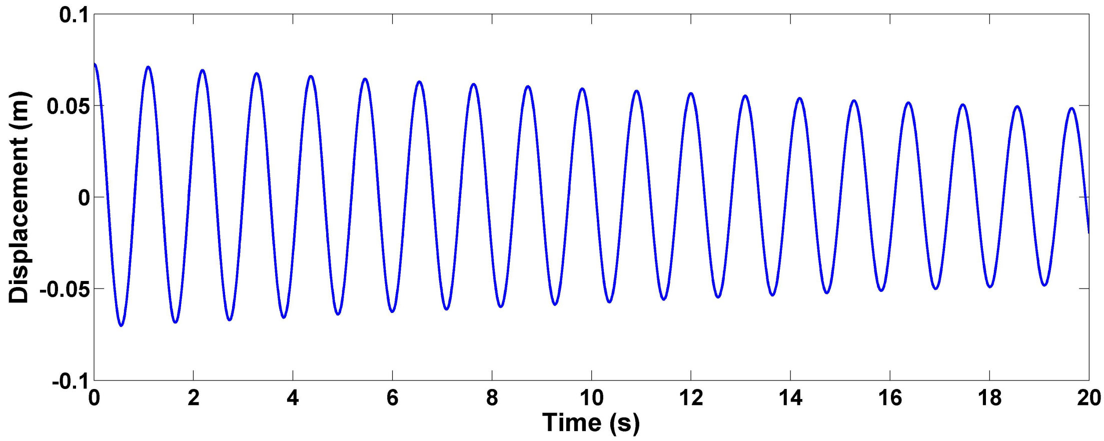
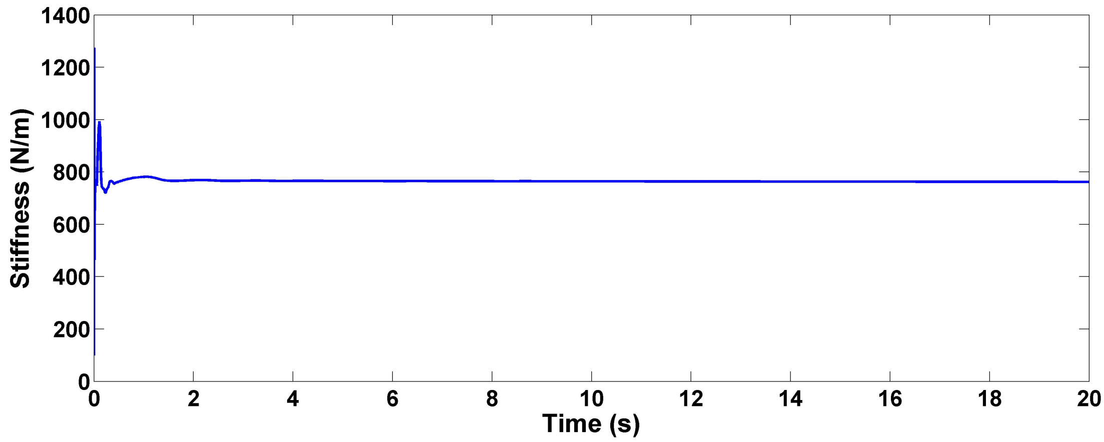
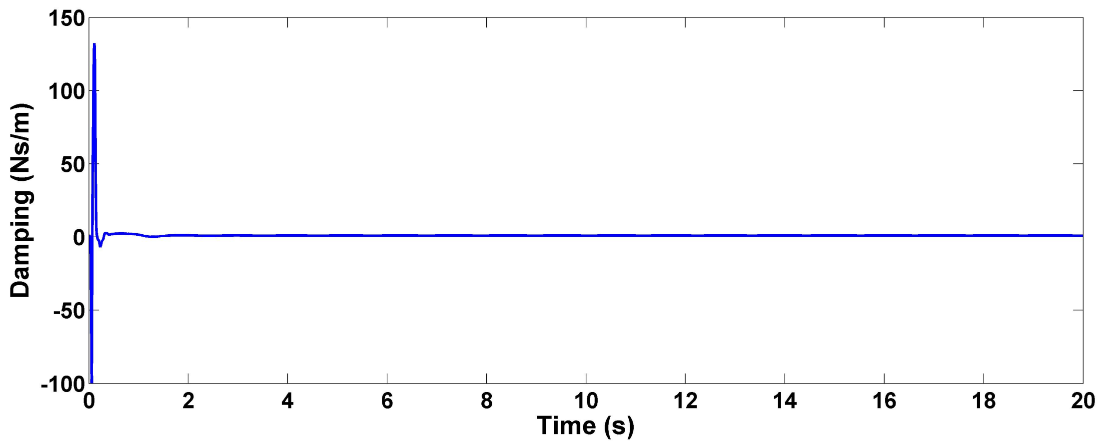


Figure 4. Structural displacement response of primary structure without ECID

Through the analysis, the estimated stiffness and damping of primary structure can be seen in **Fig.5**, which shows the extended Kalman filter algorithm has a good capability of tracking. The estimated k and c equal 762.2316N/m and 0.9574N.s/m, respectively. And to verify the accuracy of estimated parameters, an inversion analysis to calculate the structure response is conducted. As it can be shown in the **Fig.6**, the estimation results are very close to the experimental measurements.

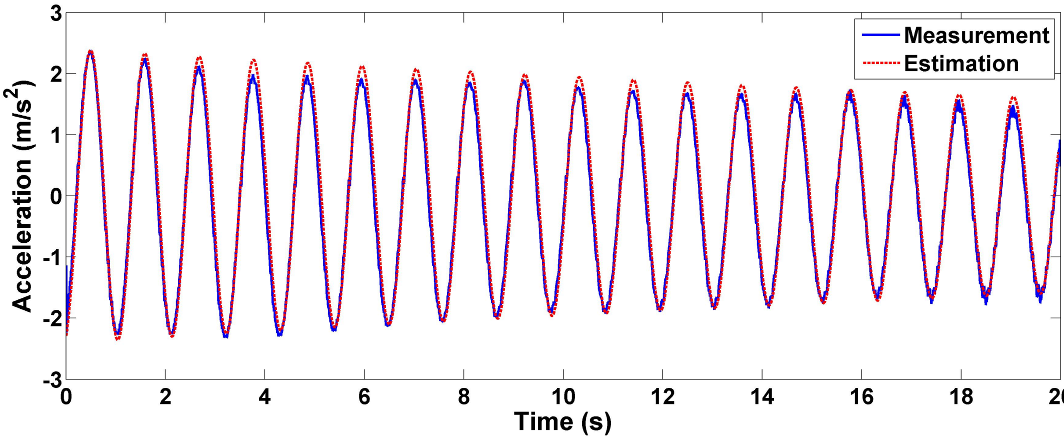


(a) stiffness

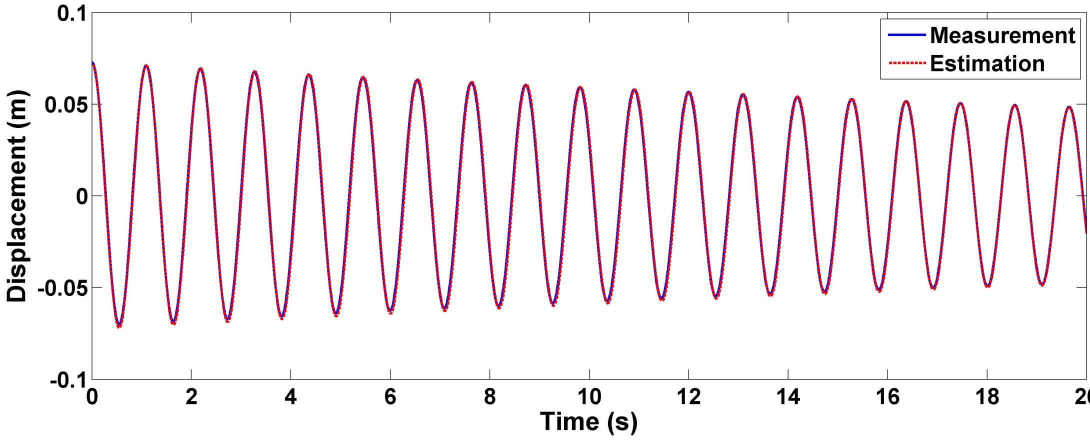


(b) damping

Figure 5. Parameter estimation of primary structure



(a) acceleration response



(b) displacement response

Figure 6. Comparison of response estimation of primary structure

4.3 Additional Damping Force Identification of whole Structure with ECID

For the free vibration, which can be seen in the equation of motion for the damper controlled structure (**Eq. (2)**), the excitation force vector $\mathbf{F}(t)$ equals zero vector. And through the above analysis, the structural parameters **M**, **K** and **C** are determined. As a single-degree-of-freedom structure, the m, k and c equal 23kg, 762.2316N/m and 0.9574N.s/m, respectively.

Then the additional damping force identification method based on Kalman filter, mentioned in the section 3, is adopted to estimate the additional damping force generated by ECID. The measurement is the displacement response of the top floor, which is shown in **Fig.7**. And the initial values used in the Kalman filter-based algorithm are given as follows: sampling interval $\Delta t = 10^{-3}\text{s}$, forgetting factor $\gamma = 0.9$, covariance matrix of process noise $\mathbf{Q} = 10^{-2}$ and covariance matrix of measurement noise $\mathbf{R} = 10^{-11}$.

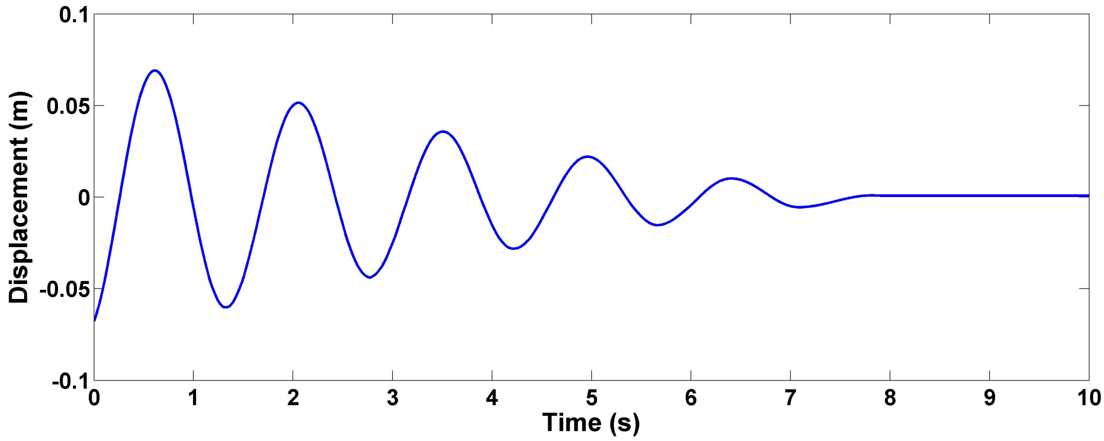


Figure 7. Structural displacement response of whole structure with ECID

The experimental additional damping force $\hat{G}$ is calculated by **Eq. (20)**, which is based on measurements of the steel cables tension T_1 and T_2. The calculation principle can be seen in **Fig.8**. The estimation and measurement comparison of additional damping force time history is plotted in **Fig.9**. As it can be seen from the above plots, the Kalman filter-based algorithm is proved to have a good approximation capability of tracking, and the estimations are very close to experimental results.

$$\hat{G} = (T_1 - T_2)\cos\theta \,. \tag{20}$$

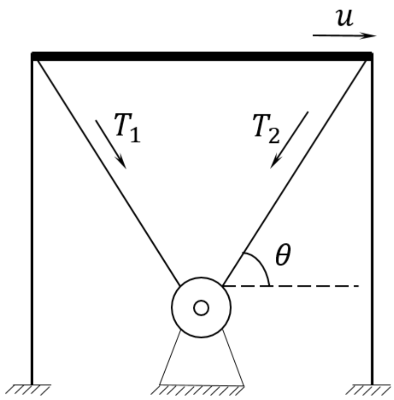
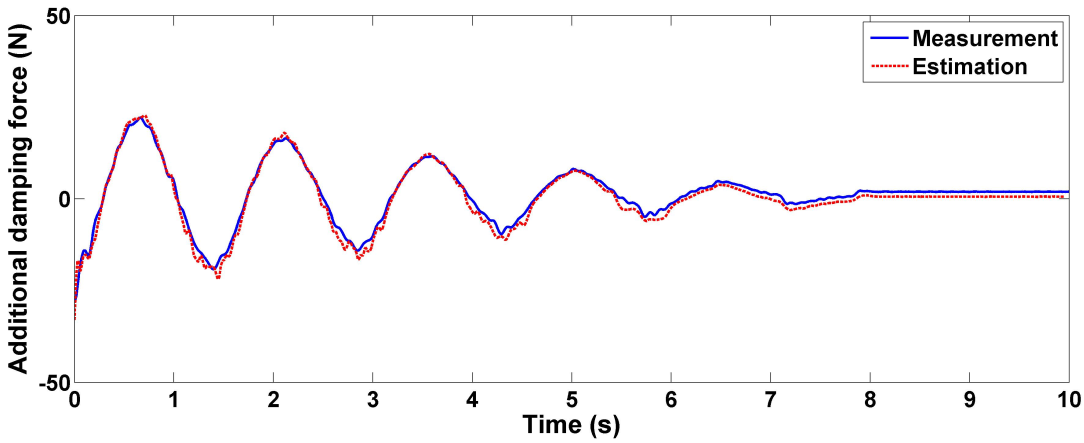
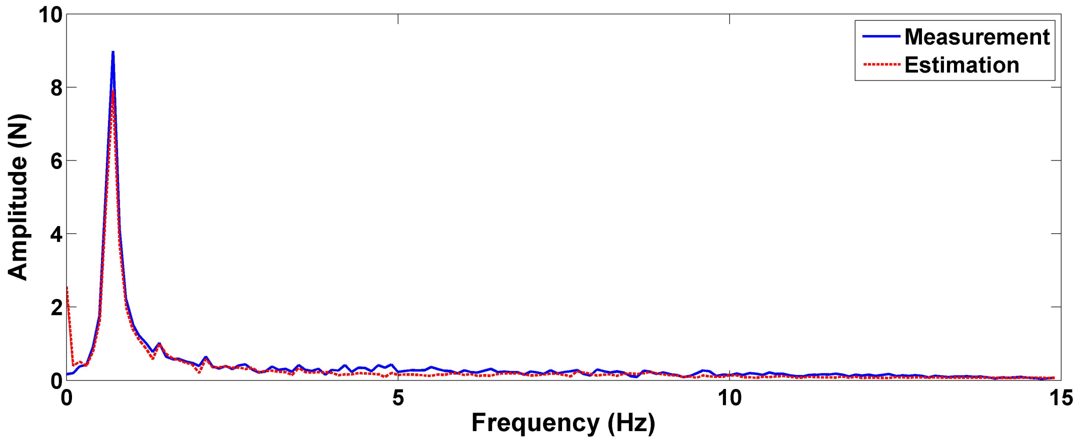


Figure 8. Calculation principle of additional damping force



(a) time domain



(b) frequency domain

Figure 9. Comparison of additional damping force time history (γ=0.9)

In the Kalman filter-based algorithm, the forgetting factor γ is adopted to improve the adaptive ability. To evaluate the accuracy of estimation, the error aimed to quantify the differences between the estimated and experimental results can be defined as below:

$$\text{Error}(\%) = \frac{\sqrt{\sum\left(G-\hat{G}\right)^2}}{\sqrt{\sum G^2}} \times 100 . \tag{21}$$

Where G and $\hat{G}$ represent the experimental and estimated additional damping force, respectively. And the relationship between the error and the forgetting factor γ can be shown in **Fig.10**.

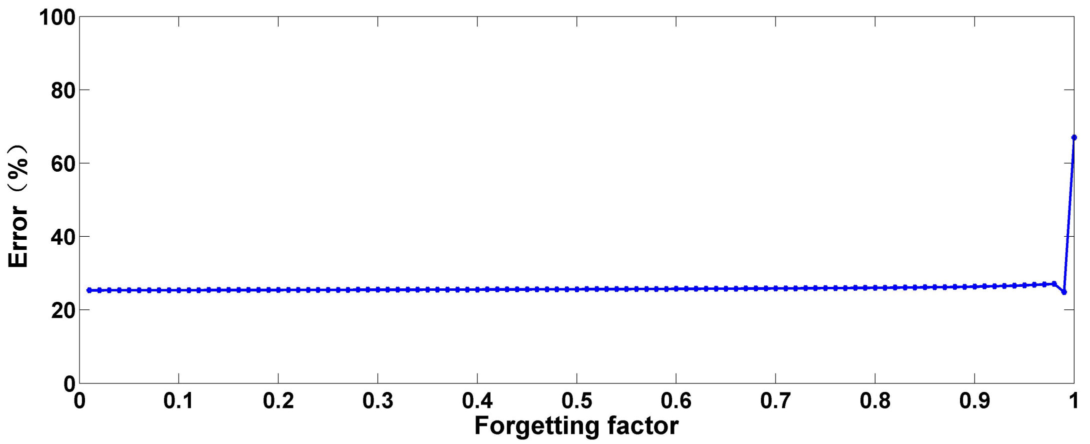


Figure 10. Relationship between the forgetting factor and relative error

5. Conclusions

In this paper, an innovative eddy current inerter damper (ECID) is presented and in order to assess the performance of ECID, an inverse methodology based on Kalman filter is developed to estimate the additional damping force generated by ECID. The unknown parameters of primary structure without ECID, like stiffness and damping, are identified firstly by extended Kalman filter. The additional damping force generated by ECID is considered as the input force of primary structure, then the additional damping force is estimated by the inverse method presented. The feasibility of the method is examined by a series of free vibration experiments of a single-degree-of-freedom structure with ECID. The testing results show that the ECID provides a significant damping effect, the additional damping force generated by ECID can be estimated in high accuracy using the inverse method based on Kalman filter, and the forgetting factor can preserve the updating ability of the inverse method continuously.

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