Detection of Damage to Frame Structures from Changes in Eigenfrequencies

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Abstract

Based on its effect on the normal modes of a structure, a method for damage detection is proposed. The method uses frequency measurements before and after damage to locate the damage and estimate its severity in shear buildings. Numerical simulation is performed of a 10-story shear building, with different cases including complete eigenfrequency measurements, incomplete frequency measurements, and differing measurement noise levels. The method is further evaluated by vibration tests of two frame models. Numerical simulation and experimental verification clearly show that, for shear buildings, damage severity and locations can be accurately inferred using the present method.

Keywords: damage detection; sensitivity analysis; frame structure; modal sensitivity; vibration test

1. Introduction

Structural damage assessment, or structural health monitoring, is the process of gaining knowledge of the current condition of a structure. In the last thirty years many researches have sought to establish effective local and global methods for damage assessment in civil, mechanical, and aerospace structures; for example, Chen and Garba (1988), Doebling *et al*. (1996), Housner *et al*. (1997), Xu and Zhu (2005). Detection of civil structural damage, as determined by changes in the dynamic properties or responses of structures, has received considerable attention. The basic idea is that modal parameters, notably frequencies, mode shapes, and modal damping, depend on the physical properties of the structures: mass, stiffness, damping, shape. Changes in the physical properties therefore induce changes in the modal properties. Many methods, including system identification approaches (Lam *et al*. (1998), Zhao and Dewolf (1999)), model updating techniques (Mottershead (1993), neural network methods (Masri *et al*. (1996)), have been developed to infer the location and severity of damage from those changes.

A damage detection technique should be tested not only with simulated data, but also with real measurement data from dynamic tests of structures. A method that has been verified in simulation might sometimes fail on a real structure, because noise and measurement errors are always involved in practice.

Accordingly, there have been many studies of dynamic tests on models. Casas and Aparicio (1994) tested four pairs of simply supported concrete beams. In each pair, one beam was undamaged, whereas the other was damaged. The damage was simulated by means of cracks having different length and spacing. Rytter *et al*. (2000) used two finite element methods to analyze the results of tests on hollow section cantilevers containing fatigue cracks. Vestroni and Capecchi (2000) treated simply supported beams with one or two damaged sections by both numerical simulation and dynamic experiment. Ren and Roeck (2002a, 2002b) proposed a damage identification method through a concrete beam test. They used a series of step loaded static tests to generate progressive damage to the beams. For building structures, Morassi and Rovere (1997) simulated the damage in a five story steel frame by cutting a notch at the bottom of the second story column. Lam *et al*. (1998) tested a two-story steel plane frame model, to detect damage. Damage was simulated as the removal of both the top and seat angles of the beam-column connection. Morita *et al*. (2001) presented damage tests of a five-story steel frame. They simulated damage by removing studs from a single story, loosening bolts, cutting part of the beams and extracting braces from a single story.

Although many structural models have been created for dynamic tests with a wide variety of damage detection methods, the methods have rarely been used for a real structure because not many modal parameters can be measured in practice. For a real building, the basic resonance frequency is the most easily measured modal parameter; it can also be measured to high precision and so is widely used. We propose below to use frequency measurements before and after damage in order to specify damage to building structures, both in location and extent. A sensitivity-based damage detection approach is first put forward. Using the sensitivity coefficient of the modal frequency to

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stiffness and supposing that damage in the structures reduces stiffness without mass change, an expression for the ratio of frequencies before and after damage is derived. Numerical simulation is performed using a 10-story shear building with different cases including complete frequency measurements, incomplete frequency measurements and different measurement noises. The method is further evaluated by vibration tests of some frame models.

2. Damage Detection Method

2.1 Sensitivity Coefficient of Modal Frequency to Stiffness

The equation for an undamped n-DOF structural dynamic system can be written as:

$$[M]\{\ddot{x}\}+[K]\{x\}=0 \tag{1}$$

where $\{x\}$=vector of displacement in the physical coordinate system, and $[K]$ and $[M]$ respectively denote stiffness and mass matrices, which are real symmetric matrices with dimension $n\times n$.

The eigenproblem may be stated as:

$$[K]\{\phi\}_r=\lambda_r[M]\{\phi\}_r \tag{2}$$

where $\{\phi\}_r$, λ_r =the rth eigenvector and eigenvalue of the system, respectively, and $\lambda_r=\omega_r^2$ with ω_r as the rth natural frequency of the system.

The eigenvector $\{\phi\}_r$ can be normalized to be of unit-mass mode shape. Using Eq. (2), we can relate the natural resonance frequencies to stiffness; sensitivity coefficients of resonance frequencies to stiffness can be written as:

$$\frac{\partial\omega_r}{\partial k_{ij}}=\begin{cases}\dfrac{1}{\omega_r}\phi_{ir}\cdot\phi_{jr} & (i\neq j)\\ \dfrac{1}{2\omega_r}\cdot\phi_{ir}{}^2 & (i=j)\end{cases} \tag{3}$$

where, $\phi_{i,j}$ is the ith element of the jth mode.

3. Frequency-change-ratio Method for Shear-Type Building Damage Detection

3.1 Complete Frequency Measurement

The modal frequency ω_r and mode shape $\{\phi_r\}$ are functions of mass and stiffness m_{ij}, k_{ij}:

$$(\omega_r,\{\phi_r\})=f(m_{ij},k_{ij}) \tag{4}$$

Let us Taylor-expand Eq. (4) and neglect second- and higher-order terms, assume also that damage in the structures reduces stiffness without mass change. The change in the frequency can then be written as:

$$\Delta\omega_r=\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\frac{\partial\omega_r}{\partial k_{ij}}\right)\Delta k_{ij} \tag{5}$$

In dynamic analysis, inter-story shear type buildings are invariably modeled as a mass-spring system. The mass distribution of an inter-story shear type structure is, from first floor to the top, $m_1,m_2,\cdots m_n$. The inter-story stiffness distribution is $k_1,k_2,\cdots k_n$. If there is damage in the ith story, the stiffness reduction in the ith story decreases only the elements of the stiffness matrix that are related to the ith story, $k_{i,i}$,$k_{i,i-1}$, and $k_{i-1,i-1}$. Suppose the inter-story stiffness of the ith story of the system decreases by Δk_i, elements $k_{i,i}$ and the component $k_{i-1,i-1}$ of the matrix decreases by Δk_i, while components $k_{i,i-1}$ and $k_{i-1,i}$ of the matrix increase by Δk_i. Other components do not change. The rth frequency shift can be written as:

$$\Delta\omega_r=\frac{\Delta k_i}{2\omega_r}\cdot\left(\phi_{ir}^2+\phi_{i-1,r}^2-2\phi_{ir}\phi_{i-1,r}\right) \tag{6}$$

The frequency change ratio can be rewritten as:

$$\frac{\Delta\omega_r}{\omega_r}=\frac{\Delta k_i}{2\lambda_r}\left(\phi_{ir}-\phi_{i-1,r}\right)^2 \tag{7}$$

For the unit-stiffness decrease, the rth frequency change ratio can be expressed as:

$$\frac{\Delta\omega_r}{\omega_r}=\frac{1}{2\lambda_r}\left(\phi_{ir}-\phi_{i-1,r}\right)^2 \tag{8}$$

From this expression, if the ith story has a unit stiffness reduction, each frequency change ratio can be written as a vector $\{\gamma_i\}$:

$$\{\gamma_i\}=\left[\frac{\Delta\omega_{1i}}{\omega_1},\frac{\Delta\omega_{2i}}{\omega_2},\cdots\frac{\Delta\omega_{ni}}{\omega_n}\right]^T \qquad (i=1,2,\cdots n) \tag{9}$$

Here $\Delta\omega_{ji}/\omega_j$ represents the j th frequency change ratio due to the i th story unit stiffness reduction.

After damage, the measured frequency change ratio vector is:

$$\left\{\frac{\Delta\tilde{\omega}}{\omega}\right\}=\left[\frac{\Delta\tilde{\omega}_1}{\omega_1},\frac{\Delta\tilde{\omega}_2}{\omega_2},\cdots,\frac{\Delta\tilde{\omega}_n}{\omega_n}\right]^T \tag{10}$$

where $\Delta\tilde{\omega}_i$ is the shift in the ith modal frequency.

The equation can therefore be written as:

$$\left\{\frac{\Delta\tilde{\omega}}{\omega}\right\}=[\gamma]\cdot\{\alpha\} \tag{11}$$

in which $[\gamma]$ is the theoretical frequency change ratio matrix. It can be shown that:

$$\{a\}=[\gamma]^{-1}\cdot\left\{\frac{\Delta\tilde{\omega}}{\omega}\right\} \tag{12}$$

where $\{\alpha\}=[\alpha_1,\alpha_2,\cdots,\alpha_n]^T$ is a damage extent vector, with subscripts $1,\cdots n$ indicating the damage locations.

3.2 Incomplete Frequency Measurement

In practice, it is difficult to obtain all the frequencies of a structure. Assume that only the first m ($m<n$) frequencies can be measured successfully; then the measured frequency change ratio vector is

$$\left\{\frac{\Delta\tilde{\omega}}{\omega}\right\}=\left[\frac{\Delta\tilde{\omega}_1}{\omega_1}\ \frac{\Delta\tilde{\omega}_2}{\omega_2}\ \cdots\ \frac{\Delta\tilde{\omega}_m}{\omega_m}\right]^T \tag{13}$$

The frequency-change-ratio vector (10) can be rewritten as:

$$\{\gamma_i\}=\left[\frac{\Delta\omega_{1i}}{\omega_1},\cdots\frac{\Delta\omega_{ni}}{\omega_n}\right]^T \quad (i=1,\cdots,m) \tag{14}$$

Eq. (11) becomes:

$$\left\{\frac{\Delta\tilde{\omega}}{\omega}\right\}_{m\times 1}=[\gamma]_{m\times n}\cdot\{\alpha\}_{n\times 1} \tag{15}$$

in which, $[\gamma]_{m\times n}$ is the theoretical frequency change ratio matrix, with dimension $m\times n$.

Thus, the damage vector can be expressed as:

$$\{\alpha\}_{n\times 1}=[\gamma]^+_{n\times m}\cdot\left\{\Delta\tilde{\omega}/\omega\right\}_{m\times 1} \tag{16}$$

where, $[\gamma]^+_{n\times m}$ is the matrix inverse to $[\gamma]_{m\times n}$. Since $[\gamma]_{m\times n}$ is a row-full rank matrix, the general inverse can be written as:

$$[\gamma]^+_{n\times m}=[\gamma]^T_{n\times m}\cdot\left([\gamma]_{m\times n}\cdot[\gamma]^T_{n\times m}\right)^{-1} \tag{17}$$

The damage vector can now be rewritten as:

$$\{\alpha\}_{n\times 1}=[\gamma]^T_{n\times m}\cdot\left([\gamma]_{m\times n}\cdot[\gamma]^T_{n\times m}\right)^{-1}\cdot\{\Delta\tilde{\omega}/\omega\}_{m\times 1} \tag{18}$$

4. Numerical Verification

4.1 Cases

A ten-story shear type structure is used to demonstrate the frequency change ratio method. From the first story to the top, the mass and inter-story stiffness distributions are identical, with values 7×10^5 kg and 6×10^{10}N/m. Damage was treated as an inter-story reduction in stiffness without mass variation. In the numerical simulation, the damage was distributed on the 1, 4, 7, 10 storys of the building at the same location or different. Damage severity was expressed as $\Delta \boldsymbol{K}_i / \boldsymbol{K}_i$, the ratio of the change in stiffness to its original value.

4.2 Results of Complete Frequency Measurement

Suppose first that all the ten modal frequencies can be obtained. Both single-location damage and multi-location damage were simulated. Using this method, the results of identification are shown in Fig.1. In Fig.1., "simulation" means the simulated damage in the model, while "calculation" means the identified results in the model by the method.

From the identification results for different cases, it is clear that this method provides good damage identification both for the locations and also the extent of the damage. The single damage case has a greater precision of identification than multiple-damage cases. Identification of damage severity in multiple-damage cases has larger errors in some cases, but the locations can easily be obtained without doubt.

4.3 Results of Incomplete Frequency Measurement

In practice, observers can seldom obtain all the resonant frequencies of a structure at an on-site survey. The first few eigenfrequencies can be easily measured, and to greater precision, while the higher order frequencies of the structures have lower measurement precision and sometimes cannot be obtained. Suppose only the first seven frequencies can be measured successfully. Based on the general inverse matrix analysis, the identification results are shown in Fig.2. Based on the incomplete frequency measurements, the identification results have errors in the determination of damage severity, but the location of the damage points can be accurately determined. Those errors arise mainly from the indeterminate equations. In such cases, only approximate results can be obtained, and inference of damage severity may contain significant errors.

4.4 Consideration of Measurement Noise

To simulate noise in the measurement, the following model of noise is used:

$$\tilde{\omega}_i=\omega_i\left(1+\boldsymbol{r}_i^{\omega}\cdot\boldsymbol{p}^{\omega}/100\right) \tag{19}$$

where $\tilde{\omega}_i$ is the frequency with the noise, and ω_i is the calculated frequency. Here, $\boldsymbol{r}_i^{\omega}$ is a random component with mean 0 and variance 1, and $\boldsymbol{p}^{\omega}$ is the noise level added to the calculation result.

In this case, assume that all the frequencies can be measured, with a certain level of noise. We considered two noise levels, 1% and 2%. Based on the frequency change ratio method, results of damage identification are shown in Fig.3. The accuracy of damage identification varies, not surprisingly, with the noise level. For low-level noise, both the location and severity of the damage can be obtained. As the noise increases the results become less precise. In this case, gross damage can be identified easily, but lesser damage may impair identification results. If the damage is not serious and the measurement noise is high, the inference can be wholly inaccurate, since the small change in the frequency is masked by the noise.

5. Experimental Study of Frame Model

5.1 Test results

Our approach is evaluated using modal test data from a 5-story plexiglass-made plane frame model with a span of 330mm and a floor height of 350mm.

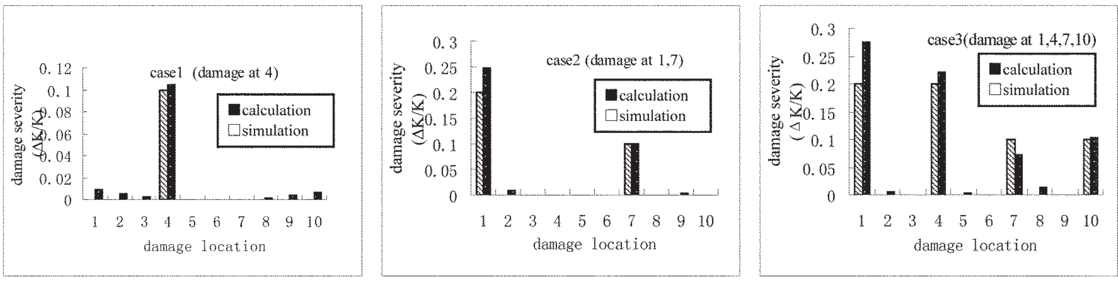


Fig.1. Identification Results by Complete Frequency Measurements

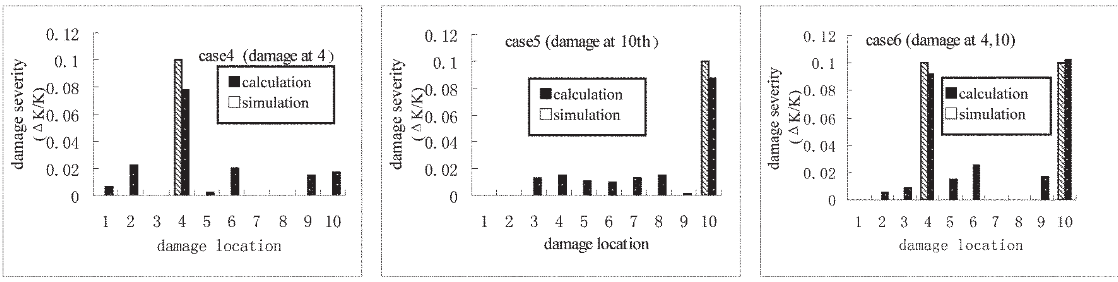


Fig.2. Identification Results by Incomplete Frequency Measurements (the First 7 Frequencies)

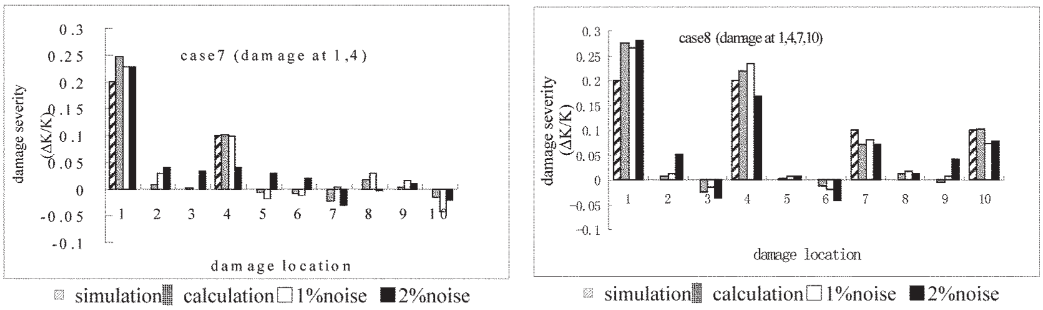


Fig.3. Identification Results for Different Measurement Noise Levels

The beam is T-shape, consisting of two rectangular section members each having cross-section 30×10mm^2. The column is rectangular, with a cross-section of 70 ×10mm^2. Table 1 shows parameters of the 5-story model.

To minimize out-of-plane vibration in the model, the out-of-plane stiffness was designed to be much larger than the in-plane stiffness. A chemical solvent was used to glue the connections between the columns and beams. Fig.4.(a) shows the connection between basement and column. Damage that reduces inter-story stiffness was caused by cutting many thin, small

Table 1. Parameters of the Frame Test Model

Elements	Cross section	Cross-section of area (m^2)	Moment of inertia (m^4)	Modulus of elasticity (N/m^2)	Density (kg/m^3)
Beam	T-shape	6×10^{-4}	8.50×10^{-8}	3×10^{9}	1.2×10^{3}
Column	Rectangle	7×10^{-4}	5.8333×10^{-9}	3×10^{9}	1.2×10^{3}

Table 2. Single Damage Test Case: 5-Story Model

Cases	Moment of inertia (m^4) Undamaged	Damaged	Stiffness reduction (%)	Damage location
Case1	1.16667E-008	1.0833E-008	7.143	5th story damage
Case2	1.16667E-008	0.9167E-008	21.43	5th story damage
Case3	1.16667E-008	0.58333E-008	50.0	5th story damage

Table 3. Test Results of Frequency Change Ratio under Different Severity: 5-Story Model

Cases	ω_1	$\Delta\omega_1/\omega_1$(%)	ω2	$\Delta\omega_2/\omega_2$(%)	ω3	$\Delta\omega_3/\omega_3$(%)	ω4	$\Delta\omega_4/\omega_4$(%)	ω5	$\Delta\omega_5/\omega_5$(%)
Undamaged	6.3477	0	19.409	0	31.9824	0	44.067	0	50.781	0
Case1	6.3477	0	19.287	0	31.6162	0.758	43.701	0.824	50.625	0.478
Case2	6.3477	0	18.9541	0.629	30.7007	1.894	42.578	1.648	50.1448	0.784
Case3	6.3477	0	19.409	2.39	31.9824	4.73	44.067	4.176	50.781	1.726

(a) (b)

Fig.4. Model Test Photos:
(a) Column-to-Basement Connection; (b) Damage Simulation

notches in the columns. Simulation of the reduction in inter-story stiffness is shown in Fig.4.(b).

Table 2 shows the model damage tests run. Table 3 shows the results of frequency measurement. When the 5th story shows damage, the first frequency remains the same as that of the undamaged model. The third frequency of the model changes the most of the five frequencies, followed by the fourth. This matches the calculation results. It can also be observed that, although Case 3 has damage causing 50% reduction in stiffness, the first frequency-change-ratio remains zero because the 1st frequency is the least sensitive of the 5 frequencies to 5th story damage.

5.2 Results of Identification with the five story model

Fig.5. shows the results of damage identification using the frequency-change method with complete frequency measurement. In the cases of single damage, the damage location can be determined using the frequency-change method. The severity of the damage can also be inferred, although with some inaccuracy. The more severe is the damage, the more accurate is the inference.

5.3 Multiple-damage Test: 4-story Model

This model was a 4-story plane frame with the same parameters as those of the former 5-story model (Table 1). Multiple-damage was simulated in both the second and third stories of the model. First, single damage was studied again and simulated at the 2nd story. After the 2nd story was given 50% stiffness reduction, the 3rd story damage was simulated while the 2nd story damage was unchanged. Damage to two stories was thereby simulated and tested. Test cases are set out in Table 4. Table 5 gives the corresponding results of frequency measurement. Using the frequency change method, damage identification in the cases of single and multiple damage to the 4-story model are shown in Figs.6. and 7. respectively.

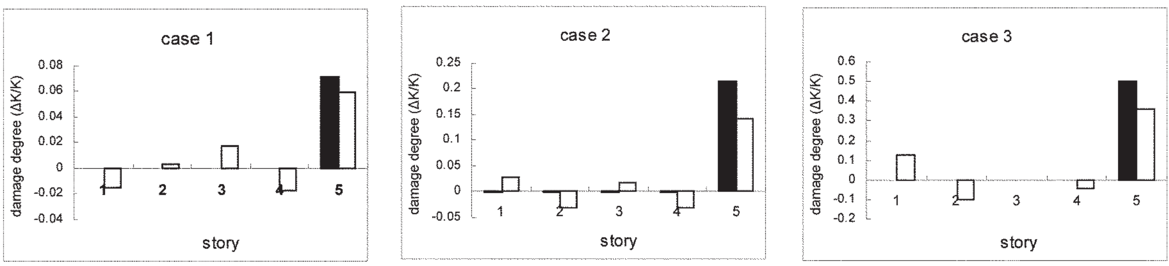
Table 5 shows that when there is a single damage point in the 2nd story of the 4-story model, the 2nd frequency remains unchanged even when the damage to the 2nd story becomes severe. This is because, when the 2nd story has damage, the second frequency change ratio is the lowest among the four frequencies, and is almost zero. The first frequency change ratio is the highest, followed by the fourth.

When the model has damage in two stories, as in Cases 4 through 6 in Table 5, it can be observed that when the 3rd story has damage, the first frequency change ratio remains the same as in single damage cases, whereas the second frequency change ratio increases as the 3rd story damage becomes serious. Because the sensitivity coefficient of the first frequency to the 3rd story stiffness is the lowest of the four frequencies, the first frequency change ratio remains essentially unchanged whatever the severity of damage to the 3rd story. This observation coincides with the analysis results.

Fig.6. reveals that, in cases of a single damage point, the damage location can be detected using the frequency change ratio method. The severity of the damage can be obtained, although with some errors. In the case of multiple damage points, as shown in Fig.7., the location and degree of the damage can be identified through the frequency change ratio method. When the 2nd story damage remains the same, while the 3rd story damage becomes serious, the identification accuracy for the latter damage improves. But for identification of the first damage, the precision becomes lower. Apart from these cases, the locations of any damage can easily be located.

6. Conclusions

This paper has proposed a frequency sensitivity-based method for determining damage parameters. Based on numerical simulation and experimental verification, the following conclusions can be drawn:



■ ***simulation damage*** □***test results***

Fig.5. Damage identification results of the 5-story model

Table 4. Damage Test Cases of 4-Story Model

Cases	Moment of inertia (m^4)		Stiffness reduction (%)	Damage locations
	Undamaged	Damaged		
Case1	1.16667E-008	0.9998E-008	14.3	Damage at 2ed story
Case2	1.16667E-008	0.9167E-008	21.43	Damage at 2ed story
Case3	1.16667E-008	0.5833E-008	50.0	Damage at 2ed story
Case4	1.16667E-008	0.9998E-008	50.0,14.3*	2nd, 3rd damage
Case5	1.16667E-008	0.9167E-008	50.0,21.43*	2nd, 3rd damage
Case6	1.16667E-008	0.833E-008	50.0,28.6*	2nd, 3rd damage
* The former means the damage degree of the second story, and the latter, the third story				

Table 5. Test Results of Frequency Change under Different Damage Degree

Cases	ω_1 (Hz)	$\Delta\omega_1/\omega_1$(%)	ω_2 (Hz)	$\Delta\omega_2/\omega_2$(%)	ω_3 (Hz)	$\Delta\omega_3/\omega_3$(%)	ω_4 (Hz)	$\Delta\omega_4/\omega_4$(%)
Undamaged	8.1787	0	24.689	0	40.1	0	51.086	0
Case1	8.0566	1.49	24.689	0	39.856	0.608	50.476	1.194
Case2	8.0261	1.866	24.689	0	39.703	0.99	50.022	2.083
Case3	7.7515	5.223	24.872	-0.74	39.139	2.397	49.377	3.345
Case4	7.7515	5.223	24.628	0.247	39.047	2.626	48.798	4.479
Case5	7.7515	5.223	24.384	1.235	39.017	2.701	48.462	5.136
Case6	7.7515	5.223	24.094	2.41	38.986	2.778	48.126	5.794

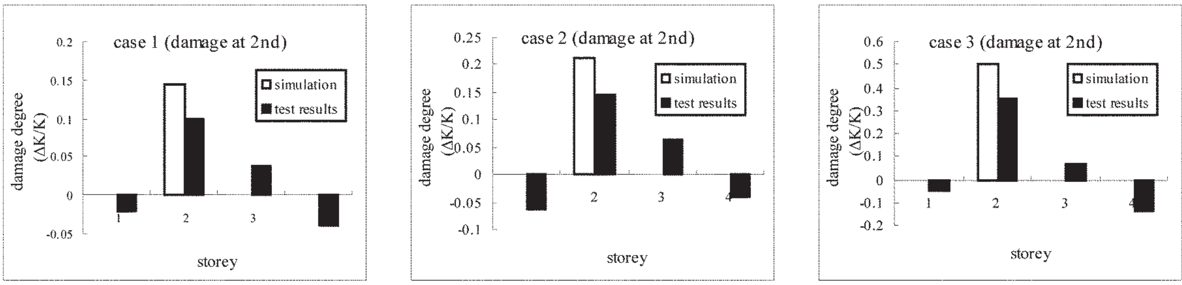


Fig.6. Damage Identification Results of the 4-Story Model Single Damage

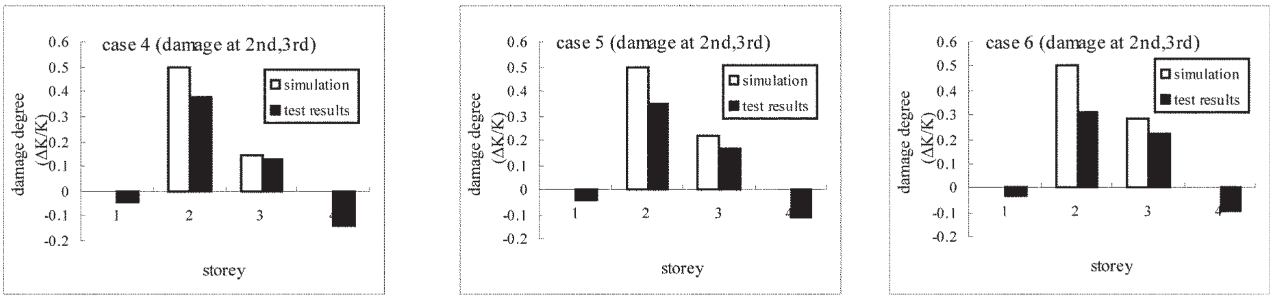


Fig.7. Damage Identification Results of the 4-Story Model: Multiple Damage

1. Sensitivity varies significantly according to the location of the damage. Some types of damage do not change all eigenfrequencies of a structure.

2. A damage detection method based on the frequency-change-ratio can be used in shear-type buildings for identifying not only damage location but also the severity of the damage.

3. The more severe is the damage; the more accurate are the results obtained. When the damage is not serious, it can be located accurately but its extent is less certain.

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