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Differential evolution strategy for structural system identification

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ABSTRACT

Differential evolution (DE) is a heuristic method that has yielded promising results for solving complex optimization problems. The potentialities of DE are its simple structure, easy use, convergence property, quality of solution, and robustness. This paper utilizes a DE strategy to parameters estimation of structural systems, which could be formulated as a multi-modal numerical optimization problem with high dimension. Simulation results for identifying the parameters of structural systems under conditions including limited output data, noise polluted signals, and no prior knowledge of mass, damping, or stiffness are presented to demonstrate the effectiveness of the proposed method.

1. Introduction

System identification plays a key role in health monitoring, non-destructive evaluation, and active control of civil infrastructures. Because of their wide applicability, system identification methods have been studied in civil engineering for various purposes. In structural identification, considerable efforts have been invested in developing methods for identification of system models and their parameters. Currently, a wide range of analytical methods exists for linear or nonlinear systems identification, such as the least square method [1–3], the extended Kalman filter [4], H_∞ filter method [5], particle filter method [6,7] and so on. These methods often have certain traits in common that tend to limit their applicability and success due to the complexity of systems in real world. Most of these methods require an initial guess so that the process can start. The problem can be very sensitive to the choice of these initial estimates, which makes them a poor choice if no prior knowledge is available. Instead, some successes have been achieved with various heuristic optimization algorithms such as genetic algorithms (GAs), evolution strategy (ES) and simulated annealing (SA). These heuristic stochastic search techniques seem to be a promising alternative to traditional approaches. Cunha et al. [8] used GAs to identify the elastic constants of composite materials. Franco et al. [9] used ES to identify multiple degree-of-freedom (DOF) systems. Perry et al. [10] used a modified GA to identify structural systems. Chou and Ghaboussi [11] introduced GAs method to identify damage severity of trusses. Koh et al. [12–14] applied GAs method to solve the global system identification problem in shear-type building structures. Levin and Lieven [15] applied SA method to optimize a finite element model for describing the dynamic behavior of structures.

As a novel evolutionary computation technique, differential evolution (DE) has gained much attention and wide applications for solving complex optimization problems since Storn and Price introduced the algorithm in 1995 [16]. It resembles the structure of an evolutionary algorithm (EA), but differs from traditional EAs in its generation of new candidate solutions and by its use of a 'greedy' selection scheme. Another main characteristic of DE is with its ability to search with floating point representation instead of binary representation that is being used in many basic EAs. DE is one such hybrid, taking the concepts of 'larger populations' from GAs, and 'self-adapting mutation' from ESs. The characteristics together with other factors of DE make it a fast and robust algorithm as an alternative to EA.

Given the characteristics and advantages of DE over the other optimization methods, DE algorithms have becoming more and more popular in solving complex, nonlinear, non-differentiable and non-convex optimization problems. Over the recent years, DE has been successfully applied in different fields mainly for various optimization problems, such as reservoir system optimization [17], optimal design of shell-and-tube heat exchangers [18], beef property model optimization problems [19], generation planning problems [20], distribution network reconfiguration problems [21], capacitor placement problems [22], induction motor identification problems [23], optimal design of gas transmission network [24], chaotic systems control and synchronization [25], to name a few.

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It is worth noting that another recently proposed heuristic algorithm, particle swarm optimization (PSO) [26] has a similar structure in its search mechanism. Owing to its simple concept, easy implementation and quick convergence, nowadays PSO has gained wide applications in different fields including the civil engineering, such as structural reliability assessment [27], optimal design [28–31], structural system identification [32].

Compared with GA and PSO, DE has some attractive characteristics [25]. It uses simple differential operator and one-to-one competition scheme to avoid complicated generic searching operators in GA. It has constructive cooperation between individuals and memory of the good solutions, whereas in GA, previous knowledge of the problem is destroyed once the population changes and in PSO, a secondary archive is needed. In [20], the authors compared the performance of some meta-heuristic techniques including DE on solving the generation expansion planning problem. The results show that DE outperforms other techniques including GA, ES, ant colony optimization (ACO), PSO, SA, tabu search (TS) and hybrid approach (HA). Although many GA versions have been developed, they are still time consuming. SA has proven to be thorough and reliable, but is generally too slow and inefficient to be of practical use with larger modeling problems [33]. Vesterstrøm and Thomsen [34] have investigated the performance of DE, PSO and EA on a selection of 34 numerical benchmark functions. The experimental results show that DE is far more efficient and robust compared to PSO and the EA. Despite the fact that the DE has wide applications in different fields mainly for various optimization problems, the DE has not been used widely in the field of civil engineering.

Numerous traditional approaches in literature tackled the problem of system identification in the field of civil engineering. However, it is difficult for these approaches to extract the physical characteristics of the system like mass, damping, or stiffness in a structural system unless some of these are assumed known a priori. Meanwhile, the measurements of inputs and outputs from a real structural system tend to be complex and expensive. Thus, there is a significant interest in the development of an algorithm that uses as few measurements as possible to obtain the physical characteristics of the system without a priori knowledge of this system. In this study, a parameter estimation technique based on DE is presented to overcome some of the difficulties encountered in the field, which could be formulated as multimodal numerical optimization problems with high dimension. Some numerical examples are presented from which the effectiveness and efficiency of the DE are investigated. The influence of incomplete availability of measurements on the performance of DE for system identification is also discussed.

2. Problem formulation

The basic idea in system identification is to compare the time dependent response of the system and a parameterized model by a norm or some performance criterion giving a measure to how well the model response fits the system response. In order to show this in more detail, let us consider a physical system with input $\boldsymbol{u}$ and output $\boldsymbol{y}$. Let $\boldsymbol{y}(t_i)$ for $i = 1, \ldots, T$ denotes the value of the actual system at ith discrete time step. Suppose that a parameterized model that is able to capture the behavior of the physical system is developed and this model depends on a set of n parameters, i.e., $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^T \in R^n$. Let $\hat{y}(t_i)$ for $i = 1, \ldots, T$ denotes the value of the parameterized model, i.e., the identified system at ith discrete time step. Hence, the objective of system identification is to find a set of parameters that minimize the prediction error between system output $\boldsymbol{y}(t_i)$, i.e., the measured data, and model output $\hat{\boldsymbol{y}}(\boldsymbol{x}, t_i)$ at each time instant t_i (see Fig. 1).

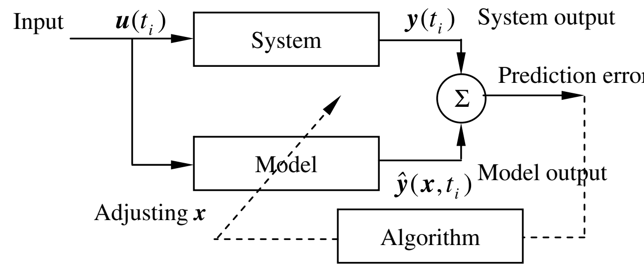

Fig. 1. The principle of system identification.

Therefore, our interest lies in minimizing the predefined error norm of the outputs, e.g., the following mean square error (MSE) function.

$$f(\boldsymbol{x}) = \frac{1}{T}\sum_{i=1}^{T} \|\boldsymbol{y}(t_i) - \hat{\boldsymbol{y}}(\boldsymbol{x}, t_i)\|^2 \tag{1}$$

where $\|\cdot\|$ represents the Euclidean norm of vectors. Formally, the optimization problem requires finding a set of n parameters $\boldsymbol{x}^* \in R^n$, so that a certain quality criterion is satisfied, namely that the error norm $f(\bullet)$ is minimized. The function $f(\bullet)$ is commonly called a fitness function or objective function. Typically, an objective function is used which reflects the goodness of solution in DE. The identification problem thus is treated as a linearly constrained multi-dimensional optimization problem, namely

$$\begin{aligned} &\text{Minimize} \quad f(\boldsymbol{x}), \boldsymbol{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \\ &\text{s.t.} \quad \boldsymbol{x} \in S, \quad S = \{\boldsymbol{x} : x_{\min,i} \leqslant x_i \leqslant x_{\max,i}, \ \forall i = 1, 2, \ldots, n\} \end{aligned} \tag{2}$$

where $f(\boldsymbol{x})$ = objective function which maps decision variable $\boldsymbol{x}$ into the objective space $f = R^n \to R$, S is the n-dimensional feasible search space, $x_{\max}$ and $x_{\min}$ denote the upper bounds and the lower bounds of the n parameters respectively.

Obviously, the fitness landscape of this problem type may have many local optima and a highly complex topology. For Eq. (2), the local and global minima can be easily calculated if the region of realizability of $\boldsymbol{x}$ is convex. However, for structural system identification problem, it may not always be convex, and therefore requires heuristic algorithms such as DE to solve it.

3. Differential evolution algorithm

The DE algorithm [35,36] is a population based algorithm like genetic algorithms using the similar operators: crossover, mutation and selection. In DE, a population of NP (population size) solution vectors is initialized randomly at the start, which is evolved to find optimal solutions through the mutation, crossover, and selecting operation procedures.

An optimization task consisting of n parameters can be represented by an n-dimensional vector. Let $S \in R^n$ be the search space of the problem under consideration. Then, the DE algorithm utilizes NP, n-dimensional vectors

$$\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in})^{\mathrm{T}} \in S, \quad i = 1, 2, \ldots, \mathrm{NP} \tag{3}$$

as a population for each iteration, called a generation of the algorithm.

3.1. Mutation

The objective of mutation is to enable search diversity in the parameter space as well as to direct the existing object vectors with suitable amount of parameter variation in a way which will lead to better results at a suitable time. It keeps the search robust and explores new areas in the search domain.

According to the mutation operator, for each individual, $\boldsymbol{x}_i^{(G)}$, $i = 1, \ldots$, NP, at generation G, a mutation vector $\boldsymbol{v}_i^{(G+1)} = (v_{i1}^{(G+1)}, v_{i2}^{(G+1)}, \ldots, v_{in}^{(G=1)})^{\mathrm{T}}$ is determined using one of the following equations [37]:

$$\boldsymbol{v}_i^{(G+1)} = \boldsymbol{x}_{r1}^{(G)} + F(\boldsymbol{x}_{r2}^{(G)} - \boldsymbol{x}_{r3}^{(G)}) \tag{4}$$

$$\boldsymbol{v}_i^{(G+1)} = \boldsymbol{x}_{\text{best}}^{(G)} + F(\boldsymbol{x}_{r1}^{(G)} - \boldsymbol{x}_{r2}^{(G)}) \tag{5}$$

$$\boldsymbol{v}_i^{(G+1)} = \boldsymbol{x}_i^{(G)} + F_1(\boldsymbol{x}_{\text{best}}^{(G)} - \boldsymbol{x}_i^{(G)}) + F(\boldsymbol{x}_{r1}^{(G)} - \boldsymbol{x}_{r2}^{(G)}) \tag{6}$$

$$\boldsymbol{v}_i^{(G+1)} = \boldsymbol{x}_{\text{best}}^{(G)} + F_1(\boldsymbol{x}_{r1}^{(G)} - \boldsymbol{x}_{r2}^{(G)}) + F(\boldsymbol{x}_{r3}^{(G)} - \boldsymbol{x}_{r4}^{(G)}) \tag{7}$$

$$\boldsymbol{v}_i^{(G+1)} = \boldsymbol{x}_{r1}^{(G)} + F_1(\boldsymbol{x}_{r2}^{(G)} - \boldsymbol{x}_{r3}^{(G)}) + F(\boldsymbol{x}_{r4}^{(G)} - \boldsymbol{x}_{r5}^{(G)}) \tag{8}$$

where $\boldsymbol{x}_{\text{best}}^{(G)} =$ best individual of the population at generation G; F and $F_1 > 0$ = real parameters, called mutation constants, which control the amplification of difference between two individuals so as to avoid search stagnation; and r_1, r_2, r_3, r_4 and r_5, are mutually different integers, randomly selected from the set $\{1, 2, \ldots, i-1, i+1, \ldots, \text{NP}\}$.

Here the choice of Eqs. (4)–(8) lead to different variants of DE, such as DE/rand/1/bin, DE/best/1/bin, DE/current-to-best/1/bin, DE/best/2/bin, and DE/rand/2/bin, respectively. In this study we use the DE/current-to-best/bin scheme (Eq. (6)). Fig. 2 illustrates the vector-generation process defined by Eq. (6).

3.2. Crossover

Following the mutation phase, the *crossover* operator is applied on the population. For each mutant vector, $\boldsymbol{v}_i^{(G+1)}$, a trial vector $\boldsymbol{u}_i^{(G+1)} = (u_{i1}^{(G+1)}, u_{i2}^{(G+1)}, \ldots, u_{in}^{(G+1)})^{\mathrm{T}}$ is generated, with

$$u_{ij}^{(G+1)} = \begin{cases} v_{ij}^{(G+1)} & \text{if } (\text{rand}(j) \leqslant \text{CR}) \text{ or } (j = \text{randn}(i)) \\ x_{ij}^{(G)} & \text{if } (\text{rand}(j) > \text{CR}) \text{ or } (j \neq \text{randn}(i)) \end{cases} \tag{9}$$

where $j = 1, 2, \ldots, n$; rand(j) is the jth independent random number uniformly distributed in the range of [0, 1]. randn(i) is a randomly chosen index from the set $\{1, 2, \ldots, n\}$, and CR is user defined crossover constant $\in [0, 1]$ that controls the diversity of the population [37].

3.3. Selection

After producing the offspring, the performance of the offspring vector and its parent is compared and the better one is selected. If the parent is still better, it is retained in the population. DE employs a greedy selection process that the better one of new offspring and its parent wins the competition providing significant advantage of converging performance over genetic algorithms.

To decide whether the vector $\boldsymbol{u}_i^{(G+1)}$ should be a member of the population of the next generation, it is compared to the corresponding vector $\boldsymbol{x}_i^{(G)}$. Thus, if f denotes the objective function under consideration, then

$$\boldsymbol{x}_i^{(G+1)} = \begin{cases} \boldsymbol{u}_i^{(G+1)} & \text{if } (f(\boldsymbol{u}_i^{(G+1)}) < f(\boldsymbol{x}_i^{(G)})) \\ \boldsymbol{x}_i^{(G)} & \text{otherwise} \end{cases} \tag{10}$$

Thus, each individual of the trial vector is compared with its parent vector and the better one is passed to the next generation, so the best individuals in the population are preserved. These steps are repeated until specified termination criterion is reached.

3.4. Operational parameters

DE has three key parameters: scaling factor of the difference vector – F, crossover control parameter – CR and population size – NP. An additional control variable, F_1, is introduced in DE/current-to-best/bin scheme. The idea behind the additional control variable F_1 is to provide a means to enhance the greediness of the scheme by incorporating the current best vector $\boldsymbol{x}_{\text{best}}^{(G)}$. The operational parameters control the balance between exploitation and exploration. Proper configuration of the above parameters would achieve good tradeoff between the global exploration and the local exploitation so as to increase the convergence velocity and robustness of the search process. Depending on the problem and available computational resources, the population size can be in the range as low as $2n$ (n is the problem dimension) to as high as $100n$ [38]. Generally, with a population size of $20n$, $F_1 = 0.95$ and $F = 0.8$ appear to be reasonably good value to generate satisfactory results. The test results in [37] show that a satisfactory range of CR appears to be within 0.8–1.0.

3.5. Feasible possible parameter space

Theoretically speaking, the search of DE for an optimum in the feasible search space S could be carried out like the other stochastic search optimization algorithms. However, in structural system identification using dynamic analysis, not all sets of parameters in the specified search space might provide physically plausible solutions to the problem. Restricting the search space to the feasible region might be difficult because the constraints are not simple [9]. In this paper, a penalty strategy [39,9] is implemented in the DE algorithm to tackle this problem. If a candidate parameter set is not a physically plausible solution, that is the system is unstable, then an exaggerated cost function value is returned. As this value is uncommonly large in comparison to usual cost function values, these "unstable" offspring are usually eliminated in a single generation.

3.6. Implementation of DE

The procedure of DE methodology can be summarized in the following steps.

- *Step 1*: Input the required DE parameters. Initialize the population of individual for DE, randomly in the limits of specified decision variables.
- *Step 2*: Check all individuals. Eliminate non-physically plausible individuals. Evaluate the objective values of all individuals, and determine $\boldsymbol{x}_{\text{best}}$ which has the best objective value.

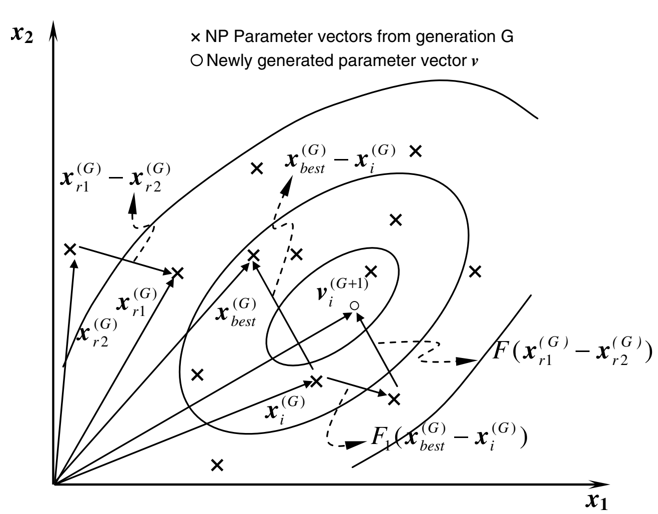

Fig. 2. Two-dimensional example of an objective function showing its contour lines and the process for generating v in scheme DE/current-to-best/bin from vectors of current generation.

Step 3: Perform mutation operation for each individual according to Eq. (6) in order to obtain each individual's mutant counterpart.
Step 4: Perform crossover operation between each individual and its corresponding mutant counterpart according to Eq. (9) in order to obtain each individual's trial individual.
Step 5: Evaluate the objective values of the trial individuals.
Step 6: Perform selection operation between each individual and its corresponding trial counterpart according to Eq. (10) so as to generate the new individual for the next generation.
Step 7: Check all individuals. If a candidate parameter set is not a physically plausible solution, then an exaggerated cost function value is returned. Eliminate "unstable" individuals.
Step 8: Determine the best individual of the current new population with the best objective value. If the objective value is better than the objective value of $\boldsymbol{x}_{\text{best}}$, then update $\boldsymbol{x}_{\text{best}}$ and its objective value with the value and objective value of the current best individual.
Step 9: If a stopping criterion is met, then output $\boldsymbol{x}_{\text{best}}$ and its objective value; otherwise go back to Step 3.

4. Illustrative examples

To illustrate the effectiveness of the parameter estimation technique with the differential evolution strategy presented above, two different structural systems are considered. One is an 8-DOF structural system, the other is a 20-DOF system that has been previously used by other authors to test other structural system identification methods.

For structural identification problems it is common that the mass of the structure is assumed to be known and the identification aims to identify structural stiffness properties. In this paper this common problem is considered and extension is made to the much more difficult case of unknown mass systems, where the mass, stiffness and damping of the structure are to be identified.

In order to compare the performance of the methodology of identification presented here with the recently developed method that has been suggested in literature based on the particle swarm optimization, the system represented in Fig. 3 is analyzed. The structural system considered is two-dimensional shear frame type structures with properties as given in Table 1. The structure consists of rigid beams and flexible columns, effectively reducing the motion to a single translational degree of freedom at each floor level.

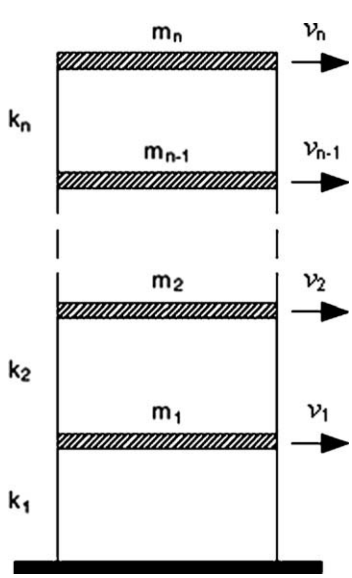

Fig. 3. *n*-DOF structure.

Table 1
Structural properties of 8-DOF system

Stiffness (kN/m)	
Levels 1	5.529e3
Levels 2–8	2.723e3
Mass (kg)	
Levels 1–7	49.48
Levels 8	45.06
Damping (kN s/m)	
Levels 1	100.31
Levels 2–8	52.167

The dynamic equation of motion of the structural system can be written as

$$\boldsymbol{M}\ddot{v}(t) + \boldsymbol{C}\dot{v}(t) + \boldsymbol{K}\boldsymbol{v}(t) = \boldsymbol{u}(t) \tag{11}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices, $\boldsymbol{v}$ is the displacement vector and $\boldsymbol{u}$ is the input force vector.

Therefore, an n-DOF system is fully described by the set of parameters

$$\boldsymbol{x} = (m_1, \ldots, m_n, k_1, \ldots, k_n, c_1, \ldots, c_n) \tag{12}$$

4.1. 8-DOF system

In this example, the mass distribution of the structure is supposed to be known and unknown priori. It is assumed that the structure is excited by known force (Niigata earthquake excitation (Japan, 2000)) and that the response of the structure, in terms of accelerations, is recorded at some given points. The acceleration output measurements error norm is used as the fitness function. The influence of limited availability of measurements on the performance of DE for parameters estimation is discussed in this study. In the "full output" scenario, measurements at all floors are available, whereas in the second "partial output" scenario, only floors 1, 3, and 5 are available. The time records used span a total length of 20 s with a sample time of 0.01 s. The strategy parameters are $F_1 = 0.95$, $F = 0.8$, CR = 0.85, maximum generations = 500 and population sizes = 100. The search space is taken as 0.5–2.0 times the exact values.

The statistical simulation results of 20 independent runs for the known mass system with the usage of the DE strategy are carried out, along with the results obtained with the PSO method for the sake of comparison. The input and output (I/O) data are polluted (in the cases considering noise) with Gaussian, zeromean, white-noise sequences, whose root mean-square (RMS) value is adjusted to be a certain percentage of the unpolluted time histories. The mean results of the parametric identification for full output scenario are summarized in Table 2 with 0% and 10% RMS noises. In addition, a typical DE search performance for the noise-free scenario is provided in Fig. 4. Fig. 5 shows the convergence graphs for this problem.

It can be seen in Table 2 that results of the PSO and DE are comparable and the relative errors obtained in the estimation of the parameters are quite similar in the noise-free scenarios. In the noise-polluted scenarios, the results of the Table 2 show that the errors are slightly higher, ranging from 1.3% to 2.3% for DE in the stiffness parameters, ranging from 2.7% to 6.1% for PSO, and quite higher in the damping parameters, ranging from 1.9% to

Table 2
Results for known mass system with full output scenario-comparison with PSO

Parameters	True value	0% Noise		10%Noise	
		PSO	DE	PSO	DE
k_1	5.529e3	*5.529e3* (*0*)[a]	*5.529e3* (*0*)	*5.380e3* (*2.7*)	*5.458e3* (*1.3*)
k_2	2.723e3	2.723e3 (0)	2.723e3 (0)	2.617e3 (3.9)	2.780e3 (2.1)
k_3	2.723e3	2.723e3 (0)	2.723e3 (0)	2.834e3 (4.1)	2.673e3 (1.9)
k_4	2.723e3	2.723e3 (0)	2.723e3 (0)	2.626e3 (3.8)	*2.784e3* (*2.3*)
k_5	2.723e3	2.723e3 (0)	2.723e3 (0)	2.644e3 (2.9)	2.677e3 (1.7)
k_6	2.723e3	2.723e3 (0)	2.723e3 (0)	*2.889e3* (*6.1*)	2.685e3 (1.4)
k_7	2.723e3	2.723e3 (0)	2.723e3 (0)	2.878e3 (5.7)	2.766e3 (1.6)
k_8	2.723e3	2.723e3 (0)	2.723e3 (0)	2.5923e (4.8)	2.672e3 (1.9)
c_1	100.31	100.31 (0)	100.31 (0)	*106.03* (*5.7*)	102.61 (2.3)
c_2	52.17	*52.22* (*0.1*)	*52.17* (*0*)	55.14 (7.3)	51.13 (2.0)
c_3	52.17	52.17 (0)	52.17 (0)	48.21 (7.6)	*54.15* (*3.8*)
c_4	52.17	52.17 (0)	52.17 (0)	56.39 (8.1)	50.76 (2.7)
c_5	52.17	52.17 (0)	52.17 (0)	55.18 (7.4)	53.83 (3.2)
c_6	52.17	52.23 (0.1)	52.17 (0)	49.11 (5.9)	*53.16* (*1.9*)
c_7	52.17	52.17 (0)	52.17 (0)	48.89 (6.3)	50.71 (2.8)
c_8	52.17	52.12 (0.1)	52.17 (0)	*57.17* (*9.6*)	53.42 (2.4)

[a] Relative errors of identification are in parentheses expressed in %.

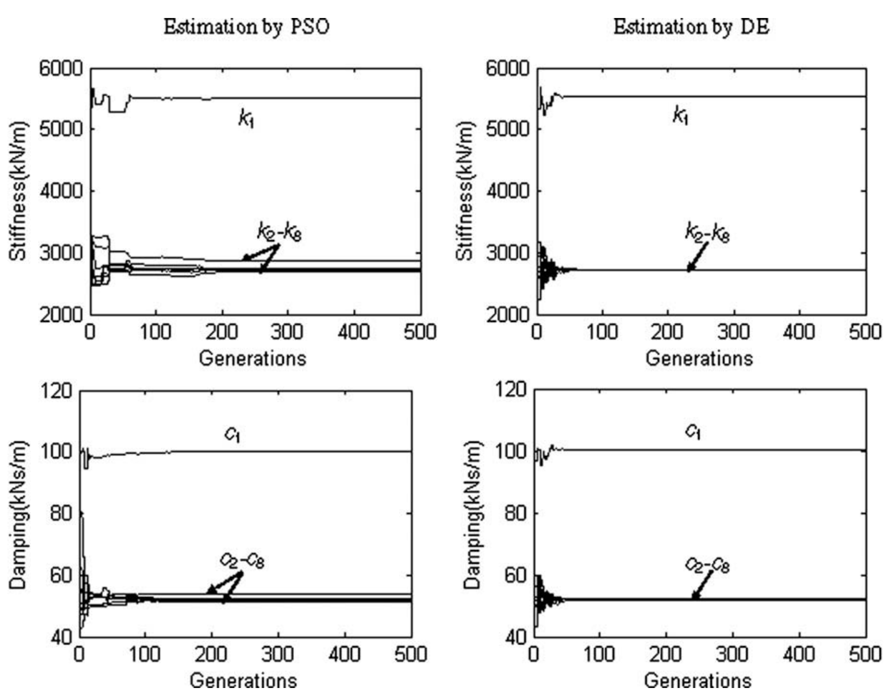

Fig. 4. Typical identification results for 8-DOF known mass system.

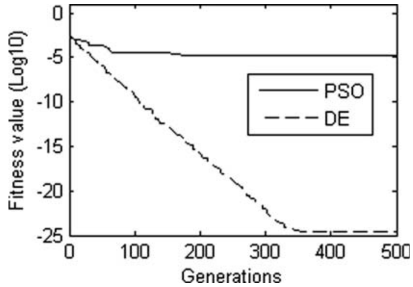

Fig. 5. Typical convergence characteristics of estimation for 8-DOF known mass system.

3.8% for DE, ranging from 5.7% to 9.6% for PSO. The results show that the DE and the PSO seem to perform well to less polluted cases, yielding very accurate results for the noise-free case but accruing more error as the noise level increases. Nevertheless, the results obtained by DE obviously outperform those obtained by the PSO in the noise-polluted case. The largest relative errors are usually observed in the damping coefficients. Due to the fact that the damping parameter has only a small contribution to the overall response, its value is generally poorly estimated. This is a fact that has been reported in other studies [9,10] as well.

Fig. 4 shows that both of two algorithms are able to find near optimum solutions quickly for the noise-free known mass problem. Although the fitness values of the two algorithms (Fig. 5) are very small, the DE converges to the optimum at an exponentially

Table 3
Results for unknown mass system with partial output scenario-comparison with PSO

Parameters	True value	0% Noise		10% Noise	
		PSO	DE	PSO	DE
m_1	49.48	48.49 (2.0)[a]	48.94 (1.1)	46.02 (7.0)	48.30 (2.4)
m_2	49.48	48.74 (1.5)	49.18 (0.6)	*51.65 (4.4)*	50.56 (2.2)
m_3	49.48	*49.02 (1.1)*	50.12 (1.3)	45.42 (8.1)	*48.54 (1.9)*
m_4	49.48	50.42 (1.9)	49.13 (0.7)	46.51 (6.1)	47.90 (3.2)
m_5	49.48	*46.50 (4.1)*	*48.20 (2.6)*	52.89 (6.9)	48.10 (2.8)
m_6	49.48	48.24 (2.5)	50.22 (1.5)	53.08 (7.3)	50.76 (2.6)
m_7	49.48	50.51 (2.1)	48.64 (1.3)	45.57 (7.9)	*51.16 (3.4)*
m_8	45.06	45.87 (1.8)	*44.88 (0.4)*	*53.68 (8.5)*	50.91 (2.9)
k_1	5.529e3	5.412e3 (2.1)	5.470e3 (1.5)	*5.275e3 (4.7)*	5.373e3 (2.8)
k_2	2.723e3	2.774e3 (1.9)	2.688e3 (1.3)	2.573e3 (5.5)	2.807e3 (3.1)
k_3	2.723e3	2.772e3 (1.8)	2.686e3 (1.3)	2.916e3 (7.1)	*2.782e3 (2.2)*
k_4	2.723e3	2.647e3 (2.3)	2.766e3 (1.6)	2.897e3 (6.4)	2.644e3 (2.9)
k_5	2.723e3	*2.772e3 (1.8)*	2.691e3 (1.2)	2.592e3 (4.8)	2.812e3 (3.3)
k_6	2.723e3	2.793e3 (2.6)	2.666e3 (2.1)	2.568e3 (5.7)	2.655e3 (2.5)
k_7	2.723e3	*2.607e3 (4.2)*	2.804e3 (3.0)	*2.935e3 (7.8)*	*2.615e3 (4.0)*
k_8	2.723e3	2.623e3 (3.7)	2.771 (1.8)	2.910e3 (6.9)	2.807e3 (3.1)
c_1	100.31	*95.60 (4.7)*	102.70 (2.4)	109.73 (9.4)	105.72 (5.4)
c_2	52.17	54.83 (5.1)	54.20 (3.9)	56.81 (8.9)	55.61 (6.6)
c_3	52.17	48.52 (7.0)	54.41 (4.3)	46.95 (10.7)	55.97 (7.3)
c_4	52.17	48.62 (6.8)	50.56 (3.1)	*60.56 (16.1)*	49.46 (5.2)
c_5	52.17	56.13 (7.6)	50.50 (3.2)	59.89 (14.8)	*54.79 (5.1)*
c_6	52.17	*47.48 (9.1)*	53.68 (2.9)	46.06 (11.7)	*56.13 (7.6)*
c_7	52.17	54.88 (5.2)	49.25 (5.6)	*56.70 (8.7)*	49.20 (5.7)
c_8	52.17	55.50 (6.4)	50.34 (3.5)	58.01 (11.2)	55.45 (6.3)

[a] Relative errors of identification are in parentheses expressed in %.

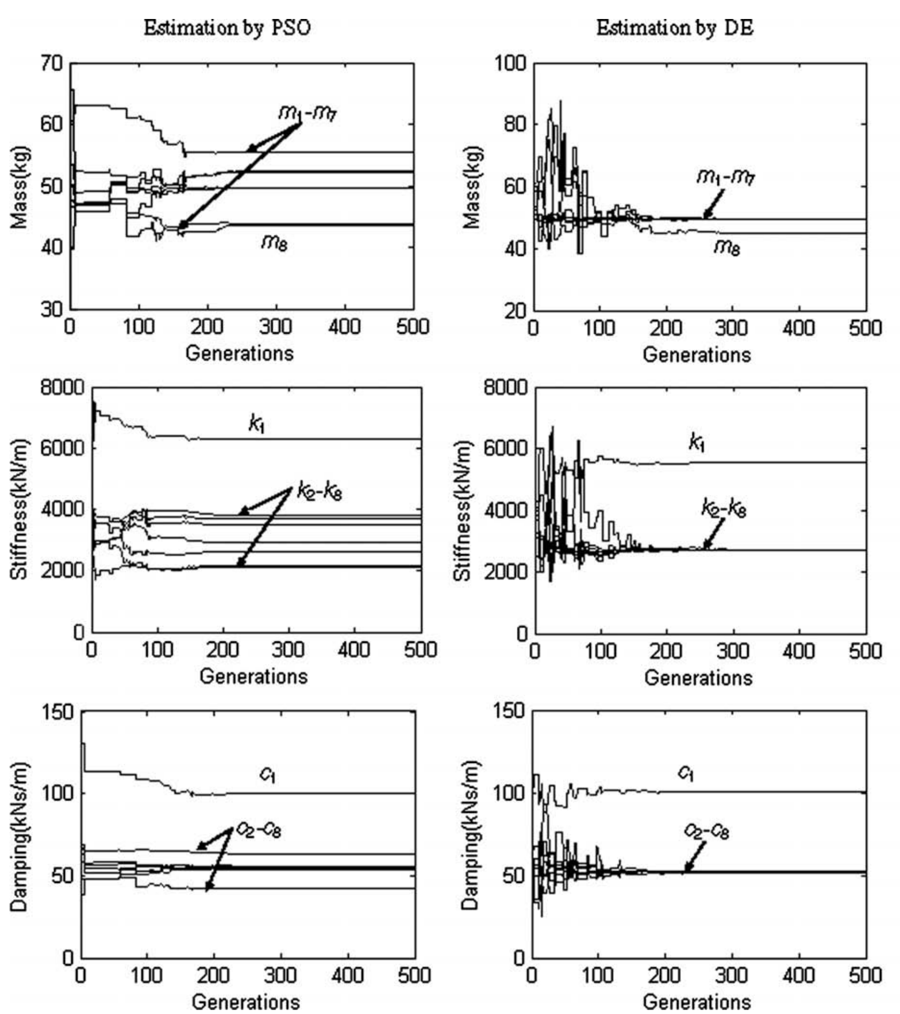

Fig. 6. Typical identification results for 8-DOF unknown mass system.

progressing rate (resulting in a straight line when plotted using a logarithmic y-axis). We may note that DE performs moderately better than the PSO.

In order to assess the effectiveness of the DE on more difficult, unknown mass systems, the same 8-DOF system is considered. The robustness of the strategy is demonstrated in the presence of 0% and 10% I/O noise with partial measurements. All identified average results over 20 runs are presented in Table 3.

As shown in Table 3, it is clear that the average and relative error results obtained by DE are better than those obtained by the PSO. The DE estimations of mass, stiffness and damping parameters are very good, even in the presence of large I/O noise. The mass, stiffness and damping parameters of the unknown mass system are identified with average error ranging from 0.4% to 2.6%, from 1.2% to 3.0% and from 2.4% to 5.6% under 0% noise respectively, and the corresponding errors ranging from 1.9% to 3.4%, from 2.2% to 4.0% and from 5.1% to 7.6% under 10% noise respectively. In addition, the DE approach is capable of locating the global optimum in all 20 runs. Unknown mass systems are highly multimodal problems. This problem is significantly more challenging than the known mass system. Nevertheless, the maximum error of DE in stiffness of only 4% under 10% noise is very good. On all of them, DE clearly performs better and it finds the global optimum in all cases. The DE seems to be more powerful in escaping local optima and in search for the global optimum on more complex unknown mass problems, and significantly improves the results on partial output scenario.

Fig. 6 depicts a typical DE search performance for the unknown mass system. It can be seen that the DE is able to reach the close vicinity of the final solution within the first 150 iterations. Fig. 7 shows the convergence graphs for this problem. DE converges to the optimum at an exponentially progressing rate. This figure highlights the large improvement that is achieved when using the DE.

Overall, DE is superior compared to the PSO algorithms in this study. It finds optimum in all cases. The calculation process is shown that the DE seems to be very robust over different trials on the test system in both solution accuracy and computational efficiency.

4.2. 20-DOF system

In order compare the performance of the DE methodology with other evolutionary computation methods that have been suggested in literature such as GAs, an one-dimensional shear frame type 20-DOF structural system similar to the above mentioned example with the structural properties as given in Table 4 is analyzed. This system was used by Perry et al. [10] to test a structural system identification algorithm denominated a modified GA, which involves a search space reduction method (SSRM) and a modified GA based on migration and artificial selection (MGAMAS) strategy to provide a robust and reliable identification.

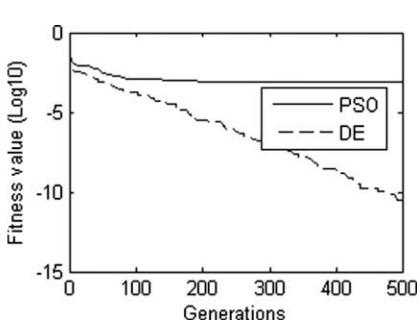

Fig. 7. Typical convergence characteristics of estimation for 8-DOF unknown mass system.

Table 4
Structural properties of 20-DOF system

Stiffness (kN/m)	
Levels 1–10	5000
Levels11–15	4000
Levels16–20	3500
Mass (kg)	
Levels 1–10	4000
Levels11–20	3000
Natural period of vibration (s)	
First mode	2.123
Second mode	0.797

The mass of the structure is lumped at each floor level and Rayleigh damping matrix $\boldsymbol{C}$ (Eq. (13)), where modal damping ratio (ς_r) is set as 5% in the first two modes of vibration ($r = 1$ and 2).

$$\boldsymbol{C} = \alpha \boldsymbol{M} + \beta \boldsymbol{K}; \quad \varsigma_r = \frac{\alpha}{2\omega_r} + \frac{\beta\omega_r}{2} \tag{13}$$

In this example, similar to other studies [10], the mass, stiffness and damping ratios are not known and have to be therefore estimated. Input forces are applied at the 5th level of the structure as random white Gaussian noise with the RMS of the force scaled to 1000 N. The input forces and noise pattern are freshly generated for each run to avoid any bias that might result from using the same inputs for all of the 20 runs. Acceleration measurements are obtained at the different floor levels as given in Table 5. The acceleration output measurements error norm is used as the fitness function. The search limits are taken as 0.5–2.0 times the exact values. The DE parameters are $F_1 = 0.95$, $F = 0.8$, CR = 0.85, maximum generations = 500 and population sizes = 400. The

Table 5
Location of acceleration measurements

System	Floor levels
Known mass	2, 4, 7, 10, 12, 14, 17, 20
Unknown mass	1, 2, 3, 4, 6, 8, 10, 12, 14, 16, 18, 20

Table 6
Results for 20-DOF known mass system (noise-free)

	SGA[a]	SSRM[b]	PSO	DE
Results				
Mean error-*k* (%)	8.33	0.52	0.71	0.41
Max. error-*k* (%)	31.28	1.60	3.37	1.29
Mean error-*c* (%)	15.81	0.64	2.24	0.53
Max. error-*c* (%)	28.97	1.21	8.31	1.45

[a,b]The results of SGA and SSRM are directly cited from [10].

Table 7
Results for 20-DOF unknown mass system

	5% Noise			10% Noise		
	SSRM[a]	PSO	DE	SSRM[a]	PSO	DE
Results						
Mean error-*m* (%)	1.51	3.6 1	1.42	3.00	7.06	3.29
Max. error-*m* (%)	4.02	10.81	3.56	10.40	16.27	11.21
Mean error-*k* (%)	1.38	3. 65	1.27	2.78	5.31	2.63
Max. error-*k* (%)	3.83	8.13	4.11	8.64	14.36	9.02
Mean error-*c* (%)	6.70	10.34	7.23	14.69	17.31	13.54
Max. error-*c* (%)	12.90	16.57	10.68	20.36	29.06	21.04

[a] The results of SSRM are directly cited from [10].

results (average and maximum error of mass, stiffness and damping properties) obtained with the usage of the DE are presented in Tables 6 and 7, along with the results obtained with a simple GA (SGA), PSO and the SSRM [10] for the sake of comparison. Fig. 8 depicts a typical DE search performance for the unknown mass system. Fig. 9 shows the convergence graphs for this problem.

In the noise-free known mass system, it is observed that DE is superior to both PSO and the SGA; results of the SSRM and DE are comparable and the relative errors obtained in the estimation of the parameters are quite similar, with the DE performing slightly better than the SSRM. One possible reason that DE works so well is that self-referential mutation operation is driven by differences between the parameter values of contemporary population members. This allows each parameter to self-tune, and gives an appropriate reduction in magnitude as the optimization proceeds and convergence is approached.

In the noise-polluted cases, the DE and SSRM performed better than the PSO at two levels of noise tested (5% and 10%). However, the DE seems to be slightly more sensitive to the existence of noise than the SSRM. It yields very accurate results for the noise-free case but accrues slightly higher error than the SSRM as the noise level increases. Nevertheless, the maximum errors in mass, stiffness of only 3.56% and 4.11% respectively under 5% noise are very good.

In general, the performance of DE is superior to the PSO algorithm tested. It is simple, robust, and it can find the optimum in almost every run. In addition, it has few parameters to set, and the same settings can be used for many different problems. The results also show that DE is almost as good as the SSRM in this study.

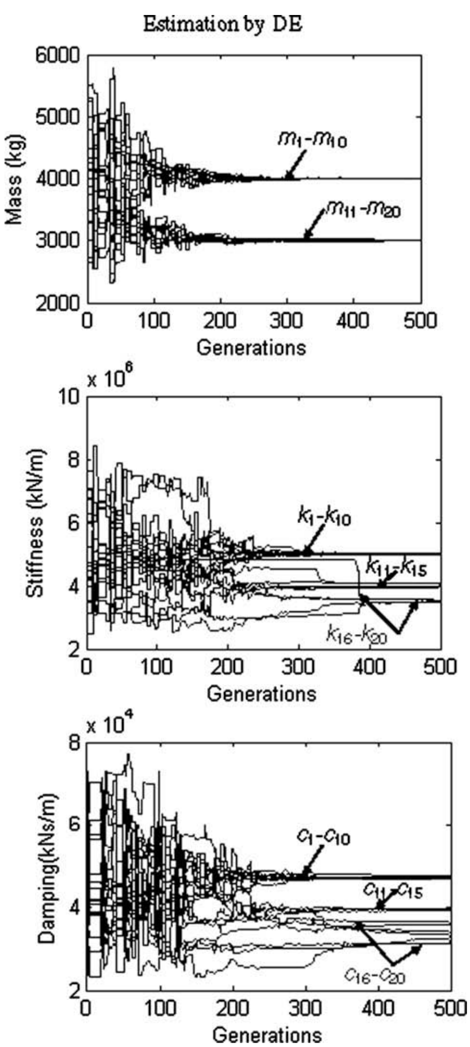

Fig. 8. Typical identification results for 20-DOF unknown mass system.

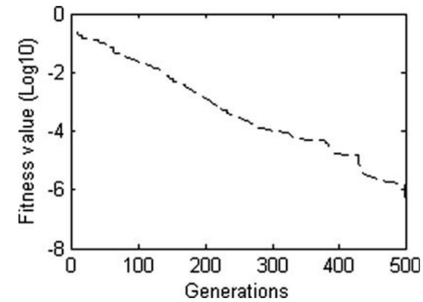

Fig. 9. Typical convergence characteristics of estimation for 20-DOF unknown mass system.

5. Conclusions

This paper has presented a differential evolution (DE) strategy for the problem of structural system identification. DE offers the advantage of incorporating a relatively simple and efficient form of self adapting mutation. DE is very easy to implement and requires hardly any parameter tuning compared to substantial tuning for PSO, original GA and some modified GAs. Comparative studies have been investigated to assess the applicability of the DE for structural parameters estimation. The results from our study show that DE is clearly and consistently superior compared to PSO for hard unknown mass problems, both in respect to precision as well as robustness of the results. The results also show that DE is almost as good as the SSRM in this study.

This proposed approach has no special requirements regarding the incomplete output measurements from the system. Even when the mass, stiffness and damping of the structure are unknown, the DE can still converge to accurate results as illustrated in the numerical study. The DE approach is a promising tool for parameters estimation of structural systems in the sense that it is an optimal method requiring no prior knowledge on the structure.

Acknowledgments

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