# Identification of Structural Systems Using Particle Swarm Optimization

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### Abstract

Particle swarm optimization (PSO) is a new heuristic method that has yielded promising results for solving complex optimization problems. Its advantages are a simple structure, ease of use, quality of solution, and robustness. This paper utilizes the PSO algorithm for parameter estimation of structural systems, which could be formulated as a multi-modal numerical optimization problem with high dimension. Simulation results for identifying the parameters of multiple degree-of-freedom (DOF) linear and nonlinear structural systems are presented to demonstrate the effectiveness of the proposed method.

Keywords: particle swarm optimization; parameters estimation; identification

#### 1. Introduction

System identification is concerned with the derivation of mathematical models from experimental data. When given a data set, one typically applies a set of candidate models and chooses one based on a set of rules by which the models can be assessed (Ljung (1999)). System identification can be widely applied in civil engineering such as for health monitoring, nondestructive evaluation, and active control. Because of their wide applicability, system identification methods have been studied in civil engineering for various purposes. Currently, a wide range of analytical methods exist for linear or nonlinear system identification. Most common among these methods are the least squares method (Yang and Lin (2004); Tang et al. (2006); Yang et al. (2007)), the maximum likelihood method (Campillo and Mevel (2005)), the extended Kalman filter (Yang et al. (2005)), the  $H_{\infty}$  filter method (Sato and Qi (1998)), and the particle filter method (Li et al. (2004); Tang and Sato (2005)). Most of these methods require an initial guess to start the process. The problem can be very sensitive to the choice of these initial estimates, which makes them difficult to apply if no prior knowledge is available. The maximum likelihood method has proven suitable for problems with high noise, but requires a good initial guess. Studies have shown that the extended Kalman filter,  $H\infty$  filter, and particle filter methods give reasonably good results as long as the modeling method matches

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the physical system with reasonable accuracy, noise information is assumed, and the initial guess is good. Therefore, these parametric methods have common traits that tend to limit their applicability and success in dealing with complex systems in the real world.

Besides the traditional parametric identification methods, numerous nonparametric identification approaches in literature have been used in civil engineering applications, including structural control and health monitoring (Hung *et al.* (2003); Pei *et al.* (2005); Jiang and Adeli (2005); Tang *et al.* (2006)). However, these estimated nonparametric coefficients generally lack any physical meaning, and this makes it difficult to extract the physical characteristics of the system, such as mass, damping, or stiffness in a structural system, unless some of these are assumed a priori.

Some success has been achieved with various heuristic optimization algorithms such as genetic algorithms (GAs), evolution strategy (ES), simulated annealing (SA), and differential evolution (DE). These heuristic stochastic search techniques seem to be a promising alternative to traditional approaches. The SA and GA methods have been implemented to accurately describe the dynamic behavior of structures (Levin and Lieven (1998)). Cunha et al. (1999) used GAs to identify the elastic constants of composite materials. Franco et al. (2004) used ES to identify multiple DOF systems. GAs have been used to identify the damage severity of truss structures (Chou and Ghaboussi (2001)) and parameters of shear-type building structures (Koh et al. (2000); (2003)). Perry et al. (2006) presented a modified GA to identify structural systems. DE has been successfully applied in induction motor identification problems (Ursem and Vadstrup (2003))

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<sup>(</sup>Received April 7, 2009; accepted August 5, 2009)

and structural system identification (Tang and Xue (2008)).

As a novel evolutionary computation technique, particle swarm optimization (PSO) has gained much attention and has been widely used for solving complex optimization problems ever since Eberhart and Kennedy (1995) introduced the algorithm. It was inspired by the social behavior of flocking birds or schooling fish. PSO shares many similarities with evolutionary computation techniques such as GA and ES. The system is initialized with a population of random solutions and searches for optima by updating generations. Compared with GA, PSO has the following attractive characteristics. First, PSO has a memory; that is, the knowledge of good solutions is retained by all particles, whereas in GA, previous knowledge of the problem is destroyed once the population changes. Second, PSO has constructive cooperation between particles; that is, particles in the swarm share information. Third, PSO is easy to implement and there are few parameters to adjust.

Given the characteristics and advantages of PSO over the other optimization methods, PSO algorithms have become increasingly popular for solving complex, nonlinear, non-differentiable, and nonconvex optimization problems. In recent years, PSO has been successfully applied in different fields, mainly for optimization problems such as function optimization (Liang *et al.* (2006)), artificial neural network training (Meissner et al. (2006)), structural reliability assessment (Elegbede (2005)), optimal design (Perez and Behdinan (2007); He and Wang (2007); Li et al. (2007); Omkar et al. (2008)), system identification, and parameter estimation (Coelho and Krohling (2006); Voss and Feng (2001); Omkar and Mudigere (2007); Tang et al. (2007); Tang and Xue (2008)). Despite this, PSO has not seen wide use in civil engineering.

Numerous traditional approaches in the literature have tackled the problem of system identification in the field of civil engineering. However, it is difficult for these approaches to extract the physical characteristics of a structural system, such as mass, damping, or stiffness, unless some of these are assumed to be known a priori. Further, measuring the inputs and outputs of a real structural system tends to be complex and expensive. Thus, there is a significant interest in the development of an algorithm that uses as few measurements as possible to obtain the physical characteristics of the system, without a priori knowledge of the system.

In this study, a parameter estimation technique based on PSO is presented to overcome some of the difficulties encountered in the field, which could be formulated as multimodal numerical optimization problems with high dimension. Some numerical examples for identifying the parameters of linear and nonlinear structural systems are presented, from which the effectiveness and efficiency of PSO are investigated. The influence of the incomplete availability of measurements on PSO performance for system identification is also discussed.

## 2. Problem Statement

The identification problem can be understood as an optimization problem in which the error between an actual physical measured response of a structure and the simulated response of a parameterized model is minimized. To understand this in more detail, consider a general physical system

$$\mathbf{y}(k) = f(\mathbf{u}(k), \mathbf{\theta}) \tag{1}$$

where  $\mathbf{y} \in \mathbb{R}^{q}$  denotes system output,  $u \in \mathbb{R}^{p}$  denotes system input,  $\theta = (\theta_{1}, \theta_{2}, \dots, \theta_{n})$  are parameters to be estimated and  $k = 0, 1, \dots, T$  denotes the kth discrete time step.

To obtain successful identification, the candidate system,  $\hat{\mathbf{y}}(k) = f(\mathbf{u}(k), \hat{\mathbf{\theta}})$ , must be able to accurately reproduce the output of the physical system for any given input. Therefore, our interest lies in minimizing the predefined error norm of the outputs, e.g., the following mean square error (MSE) function.

$$F(\mathbf{\theta}) = \frac{1}{T} \sum_{k=1}^{T} \left\| \mathbf{y}(k) - \hat{\mathbf{y}}(k) \right\|^2$$
(2)

where  $\hat{\mathbf{y}}(k) = f(\mathbf{u}(k), \hat{\mathbf{\theta}})$  is the output of the model with estimated parameters and  $\|\cdot\|$  represents the Euclidean norm of the vectors. Formally, the optimization problem requires finding a vector  $\mathbf{\theta} \in \mathbb{R}^n$ , such that a certain quality criterion is satisfied—namely that the error norm  $F(\cdot)$  is minimized. The function  $F(\cdot)$ is commonly called a cost or objective function. Typically, an objective function that reflects the correctness of the solution is used in PSO. The problem of identification is thus treated as a linearly constrained multi-dimensional nonlinear optimization problem

$$\min F(\mathbf{\theta}), \boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_n)$$
  
s.t.  $\mathbf{\theta} \in \mathbb{R}^n | \theta_{\min,i} \le \theta_i \le \theta_{\max,i} \quad \forall i = 1, 2, \cdots, n$  (3)

where  $\theta_{\text{max}}$  and  $\theta_{\text{min}}$  denote the upper and lower bounds of the n parameters, respectively.

## 3. Particle Swarm Optimization (PSO) 3.1 The Basic PSO Algorithm

PSO is a population-based, cooperative search metaheuristic introduced by Kennedy and Eberhart in 1995. In PSO, candidate solutions of a population called particles coexist and evolve simultaneously based on knowledge sharing with neighboring particles. While flying through the problem search space, each particle generates a solution using a directed velocity vector. Each particle modifies its velocity to find a better solution (position) by applying its own flying experience (i.e., memory of the best position found in earlier flights) and the experience of neighboring particles (i.e., the best solution found by the population). The *d*th dimension of the *i*th particles update their positions  $x_i^d$  and velocities  $v_i^d$  as shown below:

$$v_i^d \leftarrow w * v_i^d + c_1 * r_1 * (pbest_i^d - \theta_i^d)$$

$$(4)$$

$$+c_2 * r_2 * (goest - \theta_i)$$

$$\theta_i^a \leftarrow \theta_i^a + v_i^a \tag{5}$$

where  $\mathbf{\theta}_i = (\theta_i^1, \theta_{i_1}^2, \dots, \theta_i^D)$  is the position of the *i*th particle and  $\mathbf{v}_i = (v_i^1, v_{i_1}^2, \dots, v_i^D)$  represents the velocity of particle *i*. **pbest**<sub>i</sub> = (*pbest*<sub>i</sub>, *pbest*<sub>i</sub>, *mbest*<sub>i</sub>) is the best previous position, yielding the best fitness value for the *i*th particle, and **gbest**<sub>i</sub> = (*gbest*<sub>i</sub>, *gbest*<sub>i</sub><sup>2</sup>, *mbest*<sub>i</sub>) is the best position discovered by the whole population.  $c_1$  and  $c_2$  are the acceleration constants reflecting the weighting of stochastic acceleration terms that pull each particle toward *pbest* and *gbest* positions, respectively.  $r_1$  and  $r_2$  are independent uniformly distributed random numbers in the range [0, 1]. *w* is the particle inertia weight. The inertia weight is used to balance the global and local search abilities. A large inertia weight is more appropriate for a global search, and a small inertia weight facilitates a local search.

# **3.2 Constriction Factors and Parameters**

The use of a constriction factor to insure convergence of the PSO has been introduced in (Clerc and Kennedy (2002)). Accordingly, the expression for velocity has been modified as:

$$v_i^d \leftarrow \chi(w * v_i^d + c_1 * r_1 * (pbest_i^d - \theta_i^d) + c_2 * r_2 * (gbest^d - \theta_i^d))$$
(6)

where the constriction factor  $\chi$  is defined as

$$\chi = \frac{2}{\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|} \tag{7}$$

$$\varphi = c_1 + c_2 > 4.0 \tag{8}$$

 $\chi$  controls the magnitude of the particle velocity and can be seen as a dampening factor. It provides the algorithm with two important features (Eberhart and Shi (2000)). First, it usually leads to faster convergence than standard PSO. Second, the swarm maintains the ability to perform wide movements in the search space, even if convergence is already advanced but a new optimum is found. Therefore, the constriction PSO has the potential to avoid being trapped in local optima while possessing a fast convergence—it was shown to have superior performance compared to a standard PSO.

## 3.3 Feasible or Physically Possible Parameter Space

Similar to the other stochastic search optimization algorithms, the search for an optimum in the whole  $R^n$  space could be performed in theory. However, this choice is not reasonable for a physical problem (Franco *et al.* (2004)). Note also that not all vectors in the search space might provide plausible systems.

In structural system identification using dynamic analysis, this problem arises when some parameter candidate sets evolved during the optimization process, representing unstable systems. In this paper, preserving a feasibility strategy is employed to deal with constraints. To find the optimum in feasible space, each particle searches the whole space but only tracks feasible solutions. All particles keep only feasible solutions in their memory. To accelerate this process, all the particles are initialized with a feasible solution. The procedure of pseudo-code of the modified PSO algorithm can be described as follows.

For each particle {

Do {
Initialize particle
} while particle is in the feasible

space



Choose the particle with the best fitness value of all the particles as the **gBest** 

For each particle {

Calculate particle velocity according to Eq.6; update particle position according to Eq.5 }

*} while* a stopping criterion is not met



Fig.1. 2-DOF Nonlinear Hysteretic Structure

### 4. Numerical Studies

To illustrate the effectiveness of the parameter estimation technique with the particle swarm optimization algorithm presented above, two different structural systems are considered. One is a 2-DOF nonlinear hysteretic structural system and the other is a 10-DOF system previously used by other authors to test other structural system identification methods.

## 4.1 Two-Degrees-of-Freedom Nonlinear Hysteretic System

To assess the effectiveness of the modified PSO on a more difficult, unknown mass, a nonlinear hysteretic system is considered. A 2-DOF nonlinear hysteretic shear-type structure system is shown in Fig.1. The dynamics of this structure are presented by the equation

$$M\ddot{\mathbf{x}}(t) + C\dot{\mathbf{x}}(t) + f(t) = u(t)$$
(9)

where M and C are  $2 \times 2$  mass and damping matrices

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{m}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{m}_2 \end{bmatrix} \quad \boldsymbol{C} = \begin{bmatrix} \boldsymbol{c}_1 + \boldsymbol{c}_2 & -\boldsymbol{c}_2 \\ -\boldsymbol{c}_2 & \boldsymbol{c}_2 \end{bmatrix}$$
(10)

*x*,  $\dot{x}$ , and  $\ddot{x}$  are the relative displacement, velocity, and acceleration vector to the ground, *u* is the input, and *f* =  $(f_1 - f_2, f_2)^T$ , and the restoring force vector is expressed by (Sato and Qi (1998))

$$\dot{f}_{1}(t) = k_{1}x_{1}(t) - \alpha_{1} \left| \dot{x}_{1}(t) \right| \left| f_{1}(t) \right|^{n_{1}-1} f_{1}(t) - \beta_{1}\dot{x}_{1}(t) \left| f_{1}(t) \right|^{n_{1}}$$

$$\dot{f}_{2}(t) = k_{2}(x_{2}(t) - x_{1}(t)) - \alpha_{2} \left| \dot{x}_{2}(t) - \dot{x}_{1}(t) \right| \left| f_{2}(t) \right|^{n_{2}-1} f_{2}(t) \qquad (11)$$

$$- \beta_{2}(\dot{x}_{2}(t) - \dot{x}_{1}(t)) \left| f_{2}(t) \right|^{n_{2}}$$

where  $\alpha_i$ ,  $\beta_i$ , and  $n_i$  (i = 1,2) are the nonlinear parameters.

Therefore, the system is fully described by the set of parameters

$$\mathbf{\Theta} = (m_1, m_2, k_1, k_2, c_1, c_2, \alpha_1, \alpha_2, \beta_1, \beta_2, n_1, n_2)$$
(12)

The properties of each story unit are:  $m_1 = 1$  kg,  $k_1 = 30$  kN/m,  $c_1 = 0.55$  kNs/m,  $\alpha_1 = 1$ ,  $\beta_1 = 2$ ,  $n_1 = 3$ ,  $m_2 = 0.8$  kg,  $k_2 = 24$  kN/m,  $c_2 = 0.5$  kNs/m,  $\alpha_2 = 2$ ,  $\beta_2 = 1$ ,  $n_2 = 2$ .

In this example, the mass distribution of the structure is supposed to be unknown a priori. It is assumed that the structure is excited by a known force (Niigata earthquake excitation (Japan, 2000)) and that the response of the structure, in terms of acceleration, is recorded at some given points. The influence of limited

Table 1. Simulation Results without Noise Corruption

availability of measurements on the performance of PSO for parameter estimation is discussed in this study. The following cases of data availability will be treated here as:

• Case 1: A full set of accelerations is available.

$$\mathbf{y}(t) = (\ddot{x}_1(t), \ddot{x}_2(t))$$
(13)

• Case 2: A partial set of accelerations is available.

$$\mathbf{y}(t) = \ddot{\mathbf{x}}_2 \tag{14}$$

The sampling time is 0.01 s and the time histories span a total of 10 s. The acceleration output measurement error norm is used as the fitness function. The upper bound of the search space is twice the actual value of the parameters, and the lower bound is one third of their actual values; thus,  $\theta_{max} = 2\theta^*$  and  $\theta_{min} =$  $\theta^*/3$ , where  $\theta^* = (1, 0.8, 30, 24, 0.55, 0.5, 1, 2, 2, 1,$ 3, 2). In simulation tests, the PSO parameters are set as follows: swarm size n = 30, maximum evolution generation Gen = 500 (stopping condition),  $c_1 = 2.05$ ,  $c_2 = 2.05$ .

The statistical simulation results of 20 independent runs for the example using PSO are shown along with the results obtained with the ES method for comparison. The input and output (I/O) data are polluted (in the cases considering noise) with Gaussian, zero mean, white-noise sequences, whose root mean square (RMS) value is adjusted to a certain percentage of the unpolluted time histories. The mean results of the parametric identification are summarized in Table 1. for clean signals and in Table 2. for input and output signals corrupted with 5% RMS noise. The results obtained using PSO show small errors, ranging from 0 to 3.1%, in the noise-free scenarios. The largest relative errors are usually observed in the nonlinear coefficients such as  $\alpha$ ,  $\beta$ , and *n*. In the noise-polluted cases, we can see that the errors are slightly higher, ranging from 0.4 to 7.8%. Although noise does not typically have a great impact on the estimation of mass and stiffness, it does

Parameters	True value	Case 1		Case 2	
		PSO	ES	PSO	ES
$m_{1}$	1	1.009(0.9)	1.014(1.4)	1.012(1.2)	1.017(1.7)
$m_2$	0.8	0.804(0.5)	0.806(0.8)	0.807(0.9)	0.811(1.4)
$k_1$	30	29.986(0.05)	30.071(0.3)	29.955(0.15)	30.270(0.9)
$k_2$	24	24.003(0.01)	24.060(0.2)	24.097(0.2)	24.249(1.0)
$c_1$	0.55	0.554(0.6)	0.562(2.1)	0.541(1.7)	0.522(5.1)
$c_2$	0.5	0.502(0.4)	0.493(1.4)	0.494(1.4)	0.523(4.5)
á	1	0.984(1.6)	0.955(4.5)	0.970(3.0)	0.963(3.7)
á <sub>2</sub>	2	2.016(0.8)	2.061(3.0)	2.037(1.8)	2.291(14.6)
â <sub>1</sub>	2	1.972(1.4)	1.940(3.0)	1.963(1.9)	1.953(2.4)
$\hat{a}_2$	1	1.014(1.4)	1.075(7.5)	1.023(2.3)	1.052(5.2)
$n_1$	3	3.024(0.8)	2.917(2.7)	2.955(1.5)	3.124(4.1)
$n_2$	2	1.977(1.2)	1.831(8.5)	1.938(3.1)	1.793(10.4)

Note: Relative errors of identification are in parentheses expressed in %.

Table 2. Simulation Results with 5% Noise Corruption

Parameters	True value	Case 1		Case 2		
		PSO	ES	PSO	ES	
<i>m</i> <sub>1</sub>	1	1.016(1.6)	1.024(2.4)	1.020(2.0)	1.029(2.9)	
$m_2$	0.8	0.809(1.1)	0.821(2.7)	0.813(1.6)	0.820(2.6)	
$k_{1}$	30	29.882(0.4)	30.547(1.8)	29.732(0.9)	30.284(1.3)	
$k_2$	24	24.151(0.6)	24.456(1.9)	24.288(1.2)	24.497(2.1)	
$c_1$	0.55	0.558(1.8)	0.581(5.7)	0.563(2.5)	0.601(10.2)	
$c_2$	0.5	0.507(1.2)	0.531(6.3)	0.486(2.9)	0.5318(6.1)	
á <sub>1</sub>	1	0.976(2.4)	1.205(20.5)	0.965(3.5)	0.773(22.7)	
á <sub>2</sub>	2	2.064(3.2)	2.190(9.5)	2.086(4.3)	2.251(12.5)	
â	2	1.960(2.0)	1.870(6.5)	1.923(3.8)	1.863(6.8)	
â2	1	1.059(5.9)	1.185(18.5)	1.067(6.7)	1.252(25.2)	
$n_1$	3	3.147(5.0)	3.185(6.1)	2.156(7.8)	3.514(17.1)	
$n_2$	2	1.954(2.3)	1.872(6.4)	1.901(4.5)	1.812(9.4)	

Note: Relative errors of identification are in parentheses expressed in %.

have one on the nonlinear coefficients. In the noise-free case, the results are comparable and the relative errors obtained in the estimation of the parameters are quite similar, with PSO performing slightly better than ES. As noise increases, however, PSO seems to still yield the same relative error levels, whereas ES appears more sensitive to the existence of noise, yielding very accurate results for noise-free cases but accruing more error as the noise level increases. The results show that ES has maximum errors of 14.6 and 25.2% for the



Fig.2. A Typical Simulation Result with Partial Measurements and 5% Noise



Fig.3. Fitness vs. Generations (Partial Measurements and 5% Noise)

noise-free and the noise-polluted cases, respectively. PSO has maximum relative errors of 1.6 and 7.8% for the noise-free and noise-polluted cases.

In general, for all cases studied, the results obtained by PSO are competitive with, or sometimes better than, those obtained with ES. The solutions obtained show even smaller errors, achieving a practically perfect identification in all cases. Note that the number of solution evaluations using PSO in the simulation tests is much less than that of ES in the literature—ES has evaluated 10,000 systems, while PSO has evaluated less than 200. So, it is concluded that PSO is more efficient than ES.

Typical estimation results for partial measurements and 5% noise are provided in Figs.2.-3. From these figures, it can be seen that objective values and parameter estimations converge quickly, and the modified PSO avoids being trapped in local optima. Also, while the unknown mass systems present a far greater challenge compared to systems where the mass is known, for most practical applications, the mass will be known at least approximately; thus, a smaller search space for mass may be adopted, resulting in better and faster identification.

# 4.2 Ten-Degrees-of-Freedom Shear-Type Building System

To compare the performance of the PSO methodology with other evolutionary computation methods that have been suggested in literature, such as GAs, a one-dimensional shear frame-type 10-DOF structural system similar to the previous example, with the structural properties given in Table 3., was analyzed. This system was used by Perry *et al.* (2006) to test a structural system identification algorithm called modified GA, which uses a search space reduction method (SSRM), and a modified GA based on migration and artificial selection (MGAMAS) strategy to provide a robust and reliable identification.

The dynamic equation of motion of an n-DOF structural system can be written as

$$M\ddot{\mathbf{x}}(t) + C\dot{\mathbf{x}}(t) + K\mathbf{x}(t) = \mathbf{u}(t)$$
<sup>(15)</sup>

$$\boldsymbol{C} = \alpha \boldsymbol{M} + \beta \boldsymbol{K}; \boldsymbol{\varsigma}_r = \frac{\alpha}{2\omega_r} + \frac{\beta\omega_r}{2}$$
(16)

Table 3. Structural Properties

Stiffness (kN/m)	
Levels 1–4	5000
Levels 5–8	4000
Levels 9–10	3000
Mass (kg)	
Levels 1–5	6000
Levels 6–10	4200
Natural period of vibration (s)	
First mode	1.321
Second mode	0.505

where M, C, and K are the mass, damping, and stiffness matrices, x is the displacement vector, and u is the input force vector. The mass of the structure is lumped at each floor level and Rayleigh damping matrix C (Eq. (16)), where modal damping ratio ( $\varsigma_n$ ) is set at 5% in the first two vibration modes (r = 1 and 2).

In this example, similar to other studies (Perry *et al.* (2006)), the damping parameters  $\alpha$  and  $\beta$  are assumed to be unknown. Input forces are applied at the 5th level of the structures as random white Gaussian noise, with the RMS of the force scaled to 1000 N. In the known mass case, acceleration measurements at floors 2, 4, 7, and 10 are available, whereas in the unknown mass case, acceleration measurements at floors 1, 2, 4, 6, 8, and 10 are available. The mass, stiffness, and damping matrices are all banded and constant over time, allowing for an efficient numerical procedure to be used. The search limits are taken as 0.5–2.0 times the exact values.

In simulation tests, the PSO parameters are set as follows: swarm size n = 30, maximum evolution generation Gen = 500 (stopping condition),  $c_1 = 2.8$ , and  $c_2 = 1.3$ . The statistical simulation results of 20 independent runs obtained using PSO are presented in Table 4., along with GA results directly cited from (Perry *et al.* (2006)) for comparison.

In general, for all cases studied, the results show small errors, ranging from 0.12 to 10.7%. The largest relative errors are usually observed in the damping coefficients. Due to the fact that the damping parameter has only a small contribution to the overall response, its value is generally poorly estimated. In the noisefree (known mass) case, results are comparable and the relative errors obtained in the parameter estimation are quite similar. In the noise-polluted case, average relative errors show that PSO seems to perform well compared to the modified GA. However, the largest relative errors of PSO are slightly higher than that of GA. A typical simulation result for the unknown mass system with 10% noise is shown in Fig.4. The convergence of objective value and parameter estimation is very fast.

In general, the performance of PSO is slightly

better than GA. It is simple, robust, and can find the optimum in almost every run. In addition, it has few parameters to set, and the same settings can be used for many different problems. Both GA and PSO are population-based algorithms. Thus, they are much better suited to the complex search spaces. Both GA and PSO, however, have their own set of strengths and weaknesses. The PSO algorithm is conceptually simple and can be implemented in a few lines of code. PSO also has memory, whereas in GA, if an individual is not selected, the information contained by that individual is lost. However, without a selection operator, PSO may waste resources on an individual that is stuck in a poor region of the search space. PSO group interaction enhances the search for an optimal solution, whereas GA has trouble finding an exact solution and is best at reaching a global region. GA and PSO hybrid strategies can be considered as a method of developing more effective and efficient searching strategies to overcome the weakness of a pure single algorithm (Esmin et al. (2006)).

### 5. Conclusions

A particle swarm optimization (PSO) method for structural system identification was presented. For an



Fig.4. A Typical Simulation Result with 10% Noise Corruption

Table 4. Simulation Results for Known and Unknown Mass Systems

identifiable structure, this methodology has no special requirements regarding the number and location of output measurements from the structure; even when all properties are unknown, the proposed strategy can still converge to accurate results, even in the presence of measurement noise, as illustrated by the numerical study.

The proposed PSO-based identification approach was successfully applied to the identification of nonlinear and linear structural systems, focusing on the effects of noise in the measurements and on the limited availability of measurements. Compared with other existing evolutionary computation techniques for structural system identification, the proposed PSO algorithm seems to perform as well as, or better than, the ES and modified GA approaches described in the literature, for reasonable levels of noise. Simulation results also show that the PSO method has a simpler procedure and higher computational efficiency.

#### Acknowledgments

This study was partially supported by the National Nature Science Foundation of China No. 50708076.

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	Known mass No noise			Unknown mass			
			5% n	5% noise		10% noise	
	PSO	GA	PSO	GA	PSO	GA	
Avg.error-m(%)			1.38	1.50	2.74	3.00	
Max. error-m(%)			4.83	3.79	7.11	6.81	
Avg. error-k(%)	0.12	0.43	1.29	1.60	3.00	2.98	
Max. error-k(%)	0.67	1.21	3.88	3.62	6.10	6.62	
Avg. error-c(%)	0.84	1.56	5.92	6.29	7.96	8.41	
Max. error-c(%)	2.01	2.76	8.52	11.4	10.7	15.3	

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