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Structural Health Monitoring

Structural Damage Detection Using Auxiliary Particle Filtering Method

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Structural damage identification is an important objective of health monitoring for civil infrastructures. Frequently, damage to a structure may be reflected by a change of some system parameters, such as a degradation of the stiffness. In this paper, an auxiliary particle filtering (APF) method is applied to track a dynamic system with sudden parameter changes. In the APF, the importance density is proposed as a mixture density that depends upon the past state and the most recent observations, and hence which has a good time-tracking ability that is more suitable for tracking the nonstationary system than the conventional particle filters. Simulation results for tracking the sudden parameter changes of nonlinear hysteretic structures are presented to demonstrate the application and effectiveness of the proposed technique in detecting the structural damages.

Keywords auxiliary particle filtering · nonstationary · damage identification · nonlinear

1 Introduction

In the field of civil engineering, real-time structural identification of dynamic system has been focused on the accurate prediction as well as structural health monitoring and damage assessment. System identification and damage detection based on measured vibration data have received intensive studies recently. Widely adopted approaches to addressing this problem are the Kalman filter (KF) and extended Kalman filter (EKF) over the past years [1–4]. Similar methods such as least-square estimation method (LSE) [5–10], H_∞ filter method [11], and unscented Kalman filter (UKF) method [12,13] have been developed in some useful forms for solving many practical problems in civil engineering. Nonetheless, when one applies the KF, EKF, or UKF to a complex system, a few implementation and numerical problems may arise [12]. The traditional Kalman filter assumes that the posterior density at every time step is Gaussian. However, if the true density is non-Gaussian in a nonlinear problem, then a Gaussian can never describe it well. The linearization process of the EKF can also introduce large errors which may lead to poor performance and estimation divergence of the filter for highly nonlinear problems. Despite the fact that the UKF apparently outperforms the EKF, UKF can encounter the ill-conditioned problem of the

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covariance matrix (though theoretically it is positive semi-definite) in practice.

A recently developed filtering technique, called particle filter (PF) [also called Monte Carlo (MC) filter, bootstrap filter, condensation, etc.] was proposed by Gordon et al. [14] and Kitagawa [15]. It is a useful tool to perform dynamic state estimation via Bayesian inference. It provides great efficiency and extreme flexibility to approximate any functional nonlinearity. Because the particle filter offers a general numerical tool to approximate the state a posterior density in nonlinear and non-Gaussian filtering problems with arbitrary accuracy, it has quickly become a popular tool in signal processing applications [16–18]. The PF method has been successfully used in the areas of nonlinear state estimation, such as radar tracking [19] and parameter identification [20,21]. The most common choice of importance density is the transition prior density function for particle filter, since it is intuitive and simple to implement. However, using the transition prior as the importance density suffers from the drawback of not using knowledge of the observations, and thus the posterior density estimated by the particles, only a few of which carry information, becomes inaccurate. The generic particle filter may not be appropriate for online damage detection, because structural damage must be nonstationary phenomenon.

A potential weakness of generic particle filters discussed above is that the particle-based approximation of filtered density is not sufficient to characterize the true density, due to the use of finite mixture approximation. To alleviate this problem, Pitt and Shephard [22] introduced the so-called auxiliary particle filtering (APF). In the APF, the proposal distribution is proposed as a mixture density that depends upon the past state and the most recent observations. The APF is a powerful nonlinear estimation technique and has been shown to be a superior alternative to the PF in a variety of applications in the areas of nonlinear state estimation, such as robot localization [23], human motion tracking [24], and visual tracking [25]. Despite the fact that the APF apparently surpasses the PF, the APF has not been used widely in the field of civil engineering.

In this paper, an APF method for structural damage identification is presented. Such an adaptive tracking technique yields a sparse approximation and gives a larger importance to more recent data in order to cope with the system parameter's variations. The proposed technique is capable of tracking the sudden changes of system parameters from which the event and severity of structural damage may be detected online. Simulation results demonstrate that the proposed method is suitable for tracking the changes of system parameters for hysteretic structures.

2 Bayesian Filtering

In a general discrete-time stochastic system model, the evolution of the state sequence $\{\boldsymbol{x}_k, k \in N\}$ of the system is given by

$$\boldsymbol{x}_k = \boldsymbol{f}(\boldsymbol{x}_{k-1}, \boldsymbol{v}_{k-1}) \tag{1}$$

where, $\boldsymbol{f}: \mathrm{R}^{n_x} \times \mathrm{R}^{n_v} \to \mathrm{R}^{n_x}$ is a possibly nonlinear function of the state $\boldsymbol{x}_{k-1}$, $\{\boldsymbol{v}_{k-1}, k \in N\}$ is an i.i.d. process noise sequence, n_x, n_v are dimensions of the state and process noise vectors, respectively, and N is the set of natural numbers. The objective of system is to recursively estimate from measurement

$$\boldsymbol{z}_k = \boldsymbol{h}(\boldsymbol{x}_k, \boldsymbol{n}_k) \tag{2}$$

where, $\boldsymbol{h}: \mathrm{R}^{n_x} \times \mathrm{R}^{n_n} \to \mathrm{R}^{n_z}$ is a possibly nonlinear function, $\{\boldsymbol{n}_k, k \in N\}$ is an i.i.d. measurement noise sequence, and n_z, n_n are dimensions of the measurement and measurement noise vectors, respectively. In particular, we seek filtered estimates of $\boldsymbol{x}_k$ based on the set of all available measurements $\boldsymbol{z}_{1:k} = \{\boldsymbol{z}_i\}_{i=1}^{k}$ up to time k.

The Bayesian filtering is to recursively calculate some degree of belief in the state $\boldsymbol{x}_k$ at time k, given the data $\boldsymbol{z}_{1:k}$ up to time k. Thus, it is required to construct the *pdf* $p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k})$. Our aim is to estimate recursively in time the *pdf* $p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k})$, which are given by two stages: prediction and update.

Assuming that $\boldsymbol{x}_k$ in system model [Equation (1)] is a Markov process of initial distribution $p(\boldsymbol{x}_0|\boldsymbol{z}_0) = p(\boldsymbol{x}_0)$ and $p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}, \boldsymbol{z}_{1:k}) = p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$. Supposed that the required *pdf* $p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1})$

at time $k-1$ is available, the prediction stage involves system model [Equation (1)] to obtain the prior *pdf* of the state at time k via the Chapman–Kolmogorov equation

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1}) = \int p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1})\mathrm{d}\boldsymbol{x}_{k-1} \quad (3)$$

where, the probabilistic model of the state evolution $p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ is defined by the system model [Equation (1)] and the known probability model for $\boldsymbol{v}_{k-1}$.

At time step k, a measurement $\boldsymbol{z}_k$ that is conditionally independent given the state $\boldsymbol{x}_k$ become available, and this may be used to update the prior density to obtain the required posterior density of the recurrent state via Bayes' rule

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{1;k}) = \frac{p(\boldsymbol{z}_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1})}{p(\boldsymbol{z}_k|\boldsymbol{z}_{1:k-1})} \quad (4)$$

where,

$$p(\boldsymbol{z}_k|\boldsymbol{z}_{1:k-1}) = \int p(\boldsymbol{z}_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1})\mathrm{d}\boldsymbol{x}_k \quad (5)$$

depends on the likelihood $p(\boldsymbol{z}_k|\boldsymbol{x}_k)$ defined by the measurement model [Equation (2)] and the known statistics of $\boldsymbol{n}_k$.

For linear Gaussian models, the integral of the recursion can be solved analytically with a finite dimensional representation leading to the Kalman filter recursion, where the mean and covariance matrix of the state are propagated. Generally, this recursive propagation of the posterior density is only a conceptual solution, and it cannot be determined analytically. Therefore, numerical approximations of the integral have been proposed. A recent important contribution is to apply simulation-based methods from mathematical statistics, the sequential MC methods, commonly referred to as particle filters.

3 Particle Filtering Methods

The particle filter is an attractive approach for implementing a recursive Bayesian filter to the problem of computing intractable posterior densities by MC simulations. The key idea is to represent the required posterior density function by a set of random samples with associated weights and to compute estimates based on these samples and weights. As the number of samples becomes very large, this MC characterization becomes an equivalent representation to the usual functional description of the posterior densities, and the particle filter approaches the optimal Bayesian estimate.

Let us introduce an arbitrary importance distribution $\pi(\boldsymbol{x}_{0:k}|\boldsymbol{z}_{1:k})>0$ whenever $p(\boldsymbol{x}_{0:k}|\boldsymbol{z}_{1:k})>0$, from which it is easy to get samples called importance sampling. Given N i.i.d. random particles $\left\{\left(\boldsymbol{x}^i_{0:k}\right)\right\}^N_{i=1}$ distributed according to $\pi(\boldsymbol{x}_{0:k}|\boldsymbol{z}_{1:k})$, an approximate MC estimate of the posterior density $p(\boldsymbol{x}_{0:k}|\boldsymbol{z}_{1:k})$ is given by

$$p(\boldsymbol{x}_{0:k}|\boldsymbol{z}_{1:k}) \approx \sum_{i=1}^{N} \tilde{w}^i_k \delta\left(\boldsymbol{x}_{0:k} - \boldsymbol{x}^{\,i}_{0:k}\right) \quad (6)$$

where the normalized importance weights are defined by

$$\tilde{w}^i_k = \frac{w^i_k(\boldsymbol{x}^i_k)}{\sum_{j=1}^{N} w^{\,j}_k(\boldsymbol{x}^{\,j}_k)} \quad (7)$$

where the importance weights are defined by

$$w^i_k = \frac{p(\boldsymbol{x}^i_{0:k}|\boldsymbol{z}_{1:k})}{\pi(\boldsymbol{x}^i_{0:k}|\boldsymbol{z}_{1:k})}. \quad (8)$$

The choice of importance distribution (proposal function) is one of the most critical design issues in importance sampling algorithms. The preference for proposal functions that minimize the variance of the importance weights is advocated by Doucet et al. [26]. The proposal distribution $\pi_{\mathrm{opt}}(\boldsymbol{x}_{0:k}|\boldsymbol{z}_{1:k}) = p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}, \boldsymbol{z}_k)$ minimizes the conditional variance of the importance weights, i.e., $\mathrm{var}_{\pi_{\mathrm{opt}}}[w^i_k|\boldsymbol{x}^i_{k-1}, \boldsymbol{z}_k] = 0$. Hence, with the assumptions of the states corresponding to a Markov process and the observations being conditionally independent of each other given the states, the important weights are recursively updated as

$$w^i_k = w^i_{k-1} p(\boldsymbol{z}_k|\boldsymbol{x}^i_{k-1}) \quad (9)$$

However, this proposal distribution suffers from certain drawbacks: first, it requires sampling from $p(\boldsymbol{x}_k|\boldsymbol{x}^i_{k-1}, z_k)$, which may be difficult, the other is calculation of the importance weights as specified in Equation (9) that requires evaluating the integral $p(z_k|\boldsymbol{x}^i_{k-1}) = \int p(z_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|\boldsymbol{x}^i_{k-1})\mathrm{d}\boldsymbol{x}_k$ that may be analytically intractable.

It should be pointed out that there is no universal choice for proposal distribution, which is usually problem dependent. A popular choice among practitioners is so-called prior transition distribution $\pi(\boldsymbol{x}_k|z_{1:k}) = p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ for its easy implementation, although it may be far from optimal, this choice of proposal distribution has been advocated by many researchers [14,15,27–30]. For this particular choice of importance distribution, it is evident that the weights are given by

$$w^i_k \propto w^i_{k-1}p(z_k|\boldsymbol{x}^i_k). \tag{10}$$

The weights given by the proportionality in (10) are normalized before the resampling stage. A generic algorithm of sampling importance resampling (SIR) particle filter using transition prior density as proposed distribution is given as follows.

For time steps $k = 0, 1, 2, \ldots$

1. Initialization: for $i = 1, \ldots, N$, sample $\boldsymbol{x}^i_0 \sim p(\boldsymbol{x}_0)$, set $w^i_0 = 1/N$.
2. Importance sampling: for $i = 1, \ldots, N$, draw samples $\boldsymbol{x}^i_k \sim p(\boldsymbol{x}_k|\boldsymbol{x}^i_{k-1})$.
3. Weight update: calculate the importance weights $w^i_k = p(z_k|\boldsymbol{x}^i_k)$ for $i = 1, \ldots, N$.
4. Normalize the importance weights: $\tilde{w}^i_k = w^i_k / \sum_{j=1}^N w^j_k$.
5. Resampling: generate N new particles $\boldsymbol{x}^j_k$ ($j = 1, \ldots, N$) from the set $\{\boldsymbol{x}^i_k\}^N_{i=1}$ according to the importance weights $\tilde{w}^i_k$.
6. Repeat steps 2–5.

The transition prior sampling method does have the advantage that the importance weights are easily evaluated and easy to sample from. However, using the transition prior as the importance sampling density is independent of measurement, the state space is explored without any knowledge of the observations z_k. This property is affected by the outlier problem [22], i.e., the posterior estimation may fail, specifically when model implausible observations occur, such as sudden parameter changes in the system. As shown in Figure 1, most of the particles drawn from the prior have a low likelihood or have little overlap with the prior in such situations, and thus the posterior density estimated by the particles, only a few of which carry information, becomes inaccurate. To achieve this, the proposed importance density should include the information from the observations.

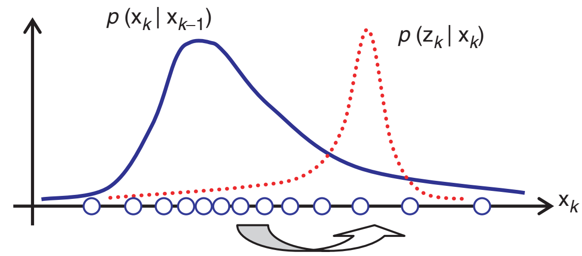

Figure 1 The APF proposal density allows us to move the particles in the prior to regions of high likelihood.

4 Auxiliary Particle Filtering

The APF was originally introduced by Pitt and Shephard [22]. This filter can be derived from the original particle filter framework by introducing an importance density $\pi(\mathbf{x}_k, i|\mathbf{z}_{1:k})$, which samples the pair $\{\boldsymbol{x}^j_k, i^j\}^N_{j=1}$, where, i^j refers to the index of the particle at $k-1$ from which $\boldsymbol{x}_k$ is predicted. The APF can be understood as a one-step ahead filter: the particle $\boldsymbol{x}^i_{k-1}$ is propagated to i^j in the next time step in order to assist the sampling from the posterior.

Using Bayes' rule, $p(\boldsymbol{x}_k, i|z_{1:k})$ can be expressed as

$$\begin{aligned} p(\boldsymbol{x}_k, i|z_{1:k}) &\propto p(z_k|\boldsymbol{x}_k, i)p(\boldsymbol{x}_k, i|z_{1:k-1}) \\ &= p(z_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|i, z_{1:k-1})p(i|z_{1:k-1}) \\ &= p(z_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|\boldsymbol{x}^i_{k-1})w^i_{k-1}. \end{aligned} \tag{11}$$

The APF operates by obtaining a sample from the joint density $p(\boldsymbol{x}_k, i|z_{1:k})$ and then omitting the indices i in the pair $(\boldsymbol{x}_k, i)$ to produce a sample $\{\boldsymbol{x}^j_k\}^N_{j=1}$ from the marginalized density $p(\boldsymbol{x}_k|z_{1:k})$.

Corresponding to Equation (11), the importance density used to draw the sample is defined to satisfy the proportionality

$$\pi(\boldsymbol{x}_k, i|\boldsymbol{z}_{1:k}) \propto p(z_k|\boldsymbol{\mu}_k^i)p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^i)w_{k-1}^i \tag{12}$$

where, $\boldsymbol{\mu}_k^i$ is a value associated with $p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^i)$ from which the i-th particle is drawn.

By writing

$$\pi(\boldsymbol{x}_k, i|\boldsymbol{z}_{1:k}) \propto \pi(i|\boldsymbol{z}_{1:k})\pi(\boldsymbol{x}_k|i, \boldsymbol{z}_{1:k}) \tag{13}$$

and defining

$$\pi(\boldsymbol{x}_k|i, z_{1:k}) = p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^i) \tag{14}$$

it follows from Equation (12) that

$$\pi(i|\boldsymbol{z}_{1:k}) \propto p(z_k|\boldsymbol{\mu}_k^i)w_{k-1}^i. \tag{15}$$

Thus, we can sample from $\pi(\boldsymbol{x}_k, i|\boldsymbol{z}_{1:k})$ by resampling with replacement from the sample set $\{\boldsymbol{x}_k^j, i^j\}_{j=1}^N$ that has an importance weight proportional to

$$w_k^j \propto w_{k-1}^{i^j} \frac{p(\boldsymbol{x}_k^j|\boldsymbol{x}_{k-1}^{i^j})p(z_k|\boldsymbol{x}_k^j)}{\pi(\boldsymbol{\mu}_k^{i^j}, i^j|z_{1:k})} = \frac{p(z_k|\boldsymbol{x}_k^j)}{p(z_k|\boldsymbol{\mu}_k^{i^j})}. \tag{16}$$

With choice of $\boldsymbol{\mu}_k^i \sim p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^i)$, the outline of the APF algorithm for online parameter estimation is given as follows:

For time steps $k = 0, 1, 2, \ldots$

1. Initialization: for $i = 1, \ldots, N$, sample $\boldsymbol{x}_0^i \sim p(\boldsymbol{x}_0)$, set $\boldsymbol{\mu}_k^i = \boldsymbol{x}_k^i$, $w_0^i = 1/N$.
2. For $i = 1, \ldots, N$, calculate $\boldsymbol{\mu}_k^i \sim p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^i)$.
3. For $i = 1, \ldots, N$, calculate the first-stage weights $w_k^i = w_{k-1}^i p(z_k|\boldsymbol{\mu}_k^i)$ and normalize weights $\tilde{w}_k^i = w_k^i / \sum_{j=1}^N w_k^j$.
4. Use the resampling procedure in SIR filter algorithm to obtain new $\{\boldsymbol{x}_k^j, i^j\}_{j=1}^N$.
5. For $j = 1, \ldots, N$, sample $\boldsymbol{x}_k^j \sim p(\boldsymbol{x}_k^j|\boldsymbol{x}_{k-1}^j, i^j)$, update the second-stage weights w_k^j according to Equation (16).
6. Repeat steps 2–5.

The APF is essentially a two-stage procedure: at the first stage, simulate the particles with large predictive likelihoods; at the second stage, reweigh the particles and draw the augmented states. Namely, the likelihood $p(z_k|\boldsymbol{\mu}_k^i)$ is used to select previous samples $\boldsymbol{x}_{k-1}^j$ that are likely to lead to current samples that are well matched to z_k. This is equivalent to making a proposal that has a high conditional likelihood *a priori*, thereby avoiding inefficient sampling. Because the proposal density includes information about the current observation, the estimation around the likely points should be more accurate in APF than in generic particle filter.

5 Numerical Examples

Consider an m degree of freedom (DOF) nonlinear hysteretic shear-type structure subject to ground excitation $\ddot{u}_g$, the equation of motion is

$$\boldsymbol{M}\ddot{x} + \boldsymbol{C}\dot{x} + \boldsymbol{f}(\dot{x}, x) = -\boldsymbol{M}\{\boldsymbol{I}\}\ddot{u}_g \tag{17}$$

where, $\boldsymbol{M}$, $\boldsymbol{C}$ are the mass and damping matrices; x, $\dot{x}$, and $\ddot{x}$ are the relative displacement, velocity, and acceleration vector to the ground; $\{\boldsymbol{I}\}$ is the identity of the $m \times 1$ column matrix; and $\boldsymbol{f}$ the restoring force vector expressed by the Bouc-Wen model [31]. In this case, the i-th component of the vector is

$$\dot{f}_i = k_i\dot{u}_i - \alpha_i|\dot{u}_i||f_i|^{n_i-1}f_i - \beta_i\dot{u}_i|f_i|^{n_i}, \quad i = 1, \ldots, m \tag{18}$$

where, $\dot{u}_i = \dot{x}_i - \dot{x}_{i-1}$ is the relative velocity between the $(i-1)$-th and i-th mass point; and c_i, k_i, α_i, β_i, and n_i are the damping, stiffness and the nonlinear parameters of the i-th mass point respectively.

Regarding the unknown parameters as state variables, one can define an augmenting state vector $\boldsymbol{X}$ as

$$\begin{aligned}\boldsymbol{X} = \left\{\ldots, \dot{u}_i, f_i, c_i, k_i, \alpha_i, \beta_i, \log_{10}^{n_i}, \ldots\right\}^T, \\ i = 1, \ldots, m\end{aligned} \tag{19}$$

in this state, to ensure positivity of the parameter n_{i}, $\log_{10}^{n_i}$ rather than n_{i} is included in the augmented state vector. Equations (16) and (17) can then be rewritten in the form of nonlinear state equations

$$\dot{X} = F(X) + v \tag{20}$$

where,

$$F(X) = \left\{ \begin{array}{c} \vdots \\ -\frac{c_i}{m_i}\dot{u}_i - \frac{f_i}{m_i} - \frac{(1-\delta_{im})(c_{i+1}\dot{u}_{i+1} + f_{i+1})}{m_i} - \ddot{u}_g \\ k_i\dot{u}_i - \alpha_i|\dot{u}_i||f_i|^{n_i-1}f_i - \beta_i\dot{u}_i|f_i|^{n_i} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{array} \right\},$$

$\delta_{im} = 0(i \neq m)$, $\delta_{im} = 1(i = m)$

and v is the process noise vector.

The observation equation here is expressed as

$$Z = HX + n \tag{21}$$

where, n is the observation noise vector, in which Z is the observation defined by

$$Z = \{\ldots, \dot{u}_i, \ldots\}^T, \quad i = 1, \ldots, m \tag{22}$$

and H is the measurement matrix given by

$$H = \begin{bmatrix} \ddots & & 0 \\ & h_i & \\ 0 & & \ddots \end{bmatrix}, \quad i = 1, \ldots, m \tag{23}$$

where

$$h_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{24}$$

Utilizing the APF technique in Equations (20) and (21), the state vector X_k can be estimated from the input $\ddot{u}_g$ and the observed output Z_k. Hence, the unknown parameters are estimated simultaneously.

5.1 SDOF Nonlinear Hysteretic System

Consider a single DOF (SDOF) nonlinear hysteretic Bouc-Wen system subject to the El Centro (NS, 1940) earthquake acceleration with the modified maximum amplitude of 25 cm/s^2. System parameters $m = 125.53$ kg, $c = 0.7$ kNs/m, $k = 24.5$ kN/m, $\alpha = 2$, $\beta = 1$, and $n = 2$ are chosen in the simulation. The structural responses sampling interval is 0.01 s.

To verify the time-varying tracking ability of the proposed technique, suppose that damage just occurs at $t = 6$ s, at which time the stiffness reduces abruptly from 24.5 to 19.6 kN/m, and the damping increases abruptly from 0.7 to 1.05 kNs/m. The initial estimate for the structural system parameters to be identified is $X \sim N\ (X_0, \sigma_0^2)$, where, $X_0 = \{0,\ 0,\ 1.05,\ 36.75,\ 1.5,\ 1.5,\ 0.2\}$, and $\sigma_0^2 = \text{diag}\ (0.01^2,\ 0.01^2,\ 0.07^2,\ 2.45^2,\ 0.15^2,\ 0.15^2,\ 0.02^2)$. The process and observation noises are defined by $v_k \sim N(0, Q)$ and $n_k \sim N(0, R)$, where, $Q = \text{diag}(0.001^2, 0.001^2, 0.007^2,\ 0.3^2,\ 0.02^2,\ 0.01^2,\ 0.03^2)$ and $R = 0.0025$. The total number of particle samples is $N = 200$.

For the purpose of exploring the identification robustness to noise, the measurement data are polluted (in the cases considering noise) with white-noise sequences, whose root mean-square (RMS) value is adjusted to be a certain percentage of the unpolluted time histories. The final parameter results (mean and standard deviation) of the last 400 estimated values for 1, 5, and 10% RMS noise levels are summarized in Table 1. In addition, a typical performance of PF and APF is provided in Figure 2. As shown in Table 1, the mean and standard deviation of the parameters obtained by the APF greatly outperform those obtained by the PF. It is observed from Figure 2 that the proposed method tracks the structural parameters and their changes online very well. From these simulation examples, we can see that the APF tracks the parameters more accurately than the PF and is also more robust to measurement noise level.

To evaluate the sensitivity of the APF to the initial values, three different sets of initial values are tested. Table 2 shows the different initial values of the structural parameters for this analysis. The final parameter results (mean and standard deviation) of the last 400 estimated values

Table 1 Estimation results for the SDOF nonlinear hysteretic system.

Noise level	*Method*	c	k	α	β	n
Exact value		1.05	19.6	2	1	2
1%	PF	0.841(0.258)	19.02(1.89)	2.047(0.095)	1.078(0.081)	2.087(0.073)
	APF	1.036(0.012)	19.46(0.31)	2.021(0.023)	0.986(0.017)	2.031(0.029)
5%	PF	0.537(0.472)	19.00(3.07)	2.056(0.134)	1.083(0.107)	2.095(0.099)
	APF	1.032(0.049)	19.31(1.84)	1.968(0.087)	0.987(0.069)	2.040(0.074)
10%	PF	0.501(0.607)	18.24(5.72)	2.077(0.198)	1.098(0.145)	2.127(0.192)
	APF	1.001(0.102)	19.12(2.69)	1.911(0.101)	0.972(0.081)	2.049(0.103)

Note: Standard deviations of identification are in parentheses.

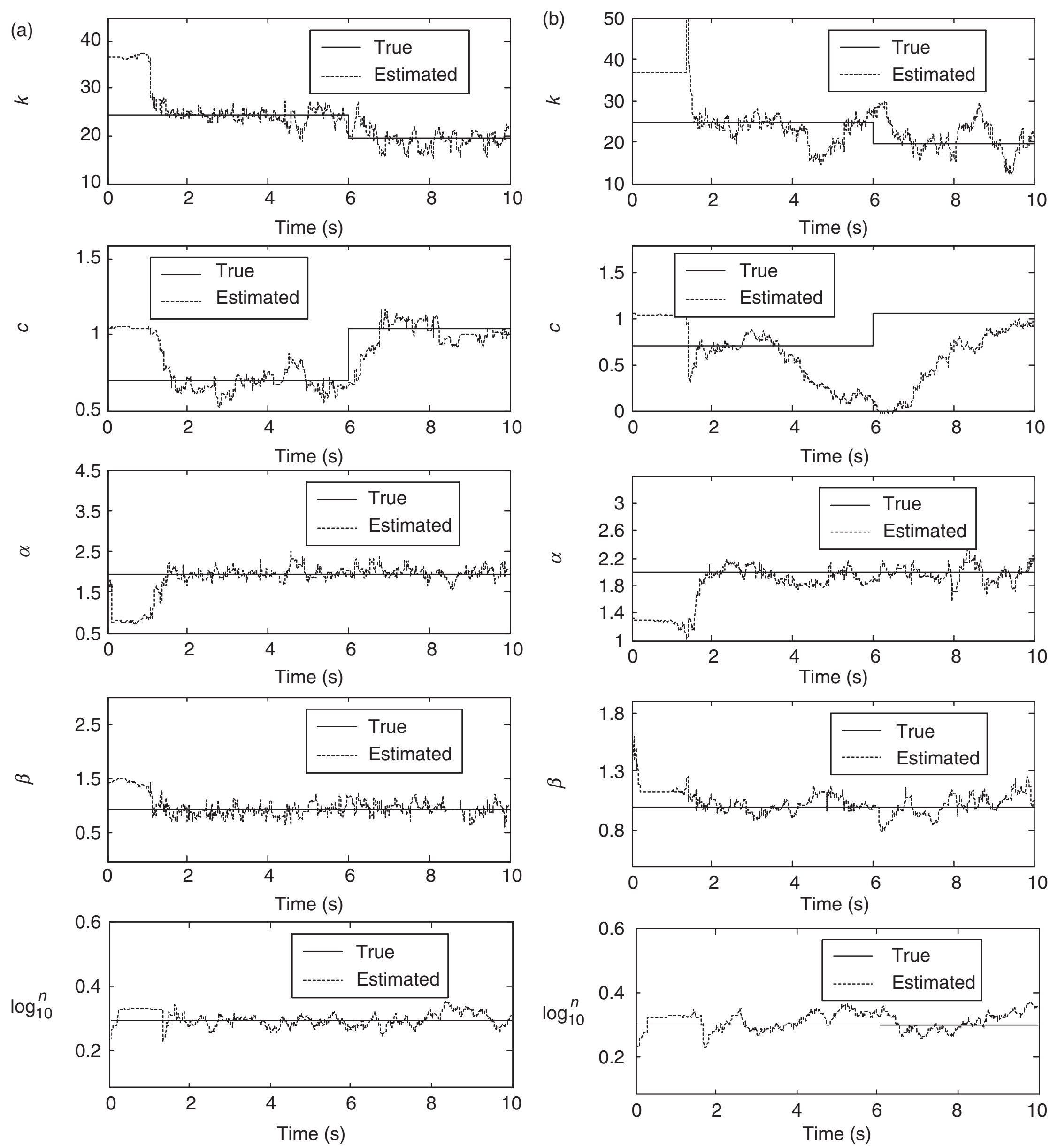

Figure 2 Identified parameters k, c, α, β, and $\log_{10}^{n}$ for a SDOF hysteretic structure with abruptly changed parameters: (a) auxiliary particle filter method, (b) particle filter method.

with different initial values are shown in Table 3. In Table 3, it is seen that the results obtained in the estimation of the parameters are quite similar, i.e., the APF is very robust to the initial values.

To further explore the tracking ability of the APF, the parameter changes (k and c) on the order of 3 and 5% of original values are conducted. For brevity time histories of the identified parameters are not shown here, however, the final parameter results (mean and standard deviation) of the last 400 estimated values for 1 and 10% noise levels are summarized in Table 4. It can be seen in Table 4 that the APF can track the parameter changes on the order of 3 and 5% of original values very well in the 1% noise level. However, the errors are quite higher in the 10% noise level. It should be noted that the tracking ability of the APF needs to be improved for a problem with very small order changes of the parameters and higher noise level of the measurements in the same time.

5.2 2-DOF Nonlinear Hysteretic System

To further explore the effectiveness of the APF algorithm, we consider a 2-story shear-beam building subject to the El Centro earthquake excitation. In this building, two inter-story force deflection relations, the Bouc-Wen model in Equation (17) is used. The properties of each story unit are: $m_1 = m_2 = 125.53$ kg, $c_1 = c_2 = 0.7$ kN s/m, $k_1 = k_2 = 24.5$ kN/m, $\alpha_1 = \alpha_2 = 2$, $\beta_1 = \beta_2 = 1$, $n_1 = n_2 = 2$. Suppose a damage just occurs in the 1st story unit at $t = 6$ s, at which time the stiffness in the first story unit k_1 reduces abruptly from 24.5 to 19.6 kN/m, and the damping c_1 increases abruptly from 0.7 to 1.05 kNs/m. Table 5 shows the initial conditions of the structural parameters for this analysis. The process and observation noises are defined by $\boldsymbol{v}_k \sim N(0, \boldsymbol{Q})$ and $\boldsymbol{n}_k \sim N(0, \boldsymbol{R})$, where, $\boldsymbol{Q} = \text{diag}(0.001^2,\ 0.001^2,\ 0.007^2,\ 0.3^2,\ 0.02^2,\ 0.01^2,$

Table 2 Different initial values ($X \sim N\ (X_0,\ \sigma_0^2)$) for the SDOF nonlinear hysteretic system.

Initial value		$\dot{u}$	f	c	k	α	β	$\log_{10}^{n}$
Case 1	X_0	0	0	1.05	36.75	1.5	1.5	0.2
	σ_0^2	0.01^2	0.01^2	0.07^2	2.45^2	0.15^2	0.15^2	0.02^2
Case 2	X_0	0	0	1.55	40.0	2.5	2.5	0.3
	σ_0^2	0.02^2	0.02^2	0.1^2	5.0^2	0.30^2	0.30^2	0.03^2
Case 3	X_0	0	0	0.95	30.0	0.5	0.5	0.1
	σ_0^2	0.2^2	0.05^2	0.2^2	3.0^2	0.45^2	0.25^2	0.01^2

Table 3 Estimation results for the APF method with different initial values.

Initial value	c	k	α	β	n
Exact value	1.05	19.6	2	1	2
Case 1	1.032(0.049)	19.31(1.84)	1.968(0.087)	0.987(0.066)	2.040(0.074)
Case 2	1.037(0.041)	19.40(1.77)	1.972(0.084)	0.990(0.067)	2.038(0.076)
Case 3	1.043(0.045)	19.34(1.80)	1.976(0.081)	0.984(0.064)	2.041(0.081)

Note: Standard deviations of identification are in parentheses.

Table 4 Estimation results for the APF method with different order changes.

Change order	*Noise level*	c	k	α	β	n
Exact value		0.721	23.94	2	1	2
3%	1%	0.718(0.011)	23.73(0.26)	2.019(0.024)	0.987(0.013)	2.023(0.025)
	10%	0.753(0.042)	23.07(1.91)	2.078(0.085)	0.961(0.068)	2.083(0.078)
Exact value		0.735	23.275	2	1	2
5%	1%	0.727(0.014)	23.21(0.31)	2.021(0.029)	0.988(0.015)	2.019(0.027)
	10%	0.775(0.067)	22.08(2.17)	2.094(0.117)	0.938(0.075)	2.107(0.101)

Note: Standard deviations of identification are in parentheses.

0.03^2, 0.001^2, 0.001^2, 0.007^2, 0.3^2, 0.02^2, 0.01^2, 0.03^2) and $\boldsymbol{R} = \mathrm{diag}(0.0025, 0.0025)$.

Based on the proposed tracking technique, the identified parameters for the 5% noise level, with the total number of particle samples $N = 600$, are presented in Figures 3(a) and 4(a), respectively. Also shown in Figures 3(b) and 4(b) are estimation results by particle filter for comparison.

Table 5 Initial values ($X \sim$ N (X_0, σ_0^2)) for the 2-DOF nonlinear hysteretic system.

X	$\dot{u}_1$	f_1	c_1	k_1	α_1	β_1	$\log_{10}^{n_1}$	$\dot{u}_2$	f_2	c_2	k_2	α_2	β_2	$\log_{10}^{n_2}$
X_0	0	0	1.05	36.75	0.86	0.65	0.2	0	0	0.85	29.4	0.55	0.65	0.2
σ_0^2	0.01^2	0.01^2	0.07^2	2.45^2	0.15^2	0.15^2	0.02^2	0.01^2	0.01^2	0.07^2	2.45^2	0.15^2	0.15^2	0.02^2

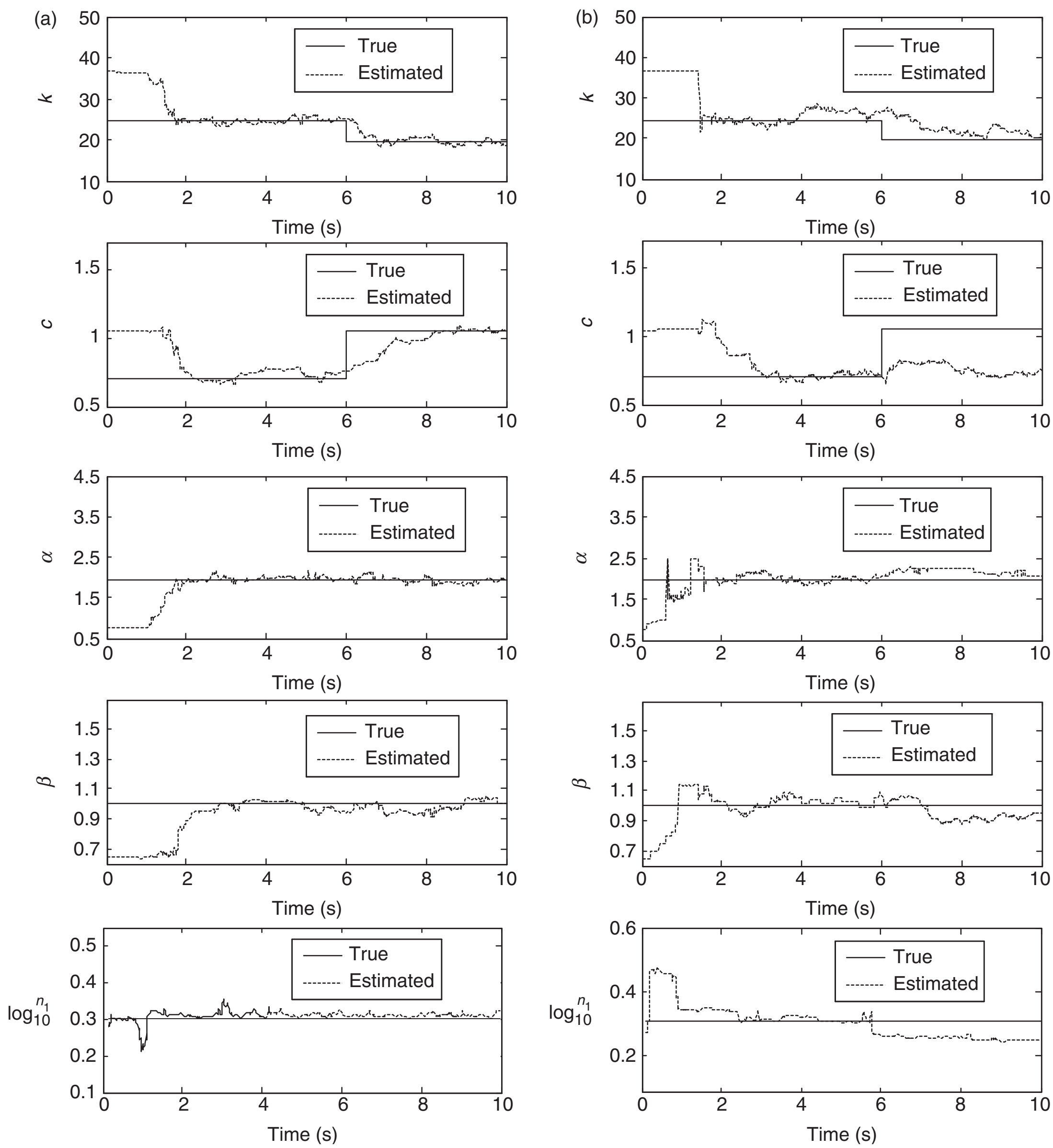

Figure 3 Identified parameters k, c, α, β, and $\log_{10}^{n}$ for a 2-DOF hysteretic structure with abruptly changed parameters (1st story): (a) auxiliary particle filter method, (b) particle filter method.

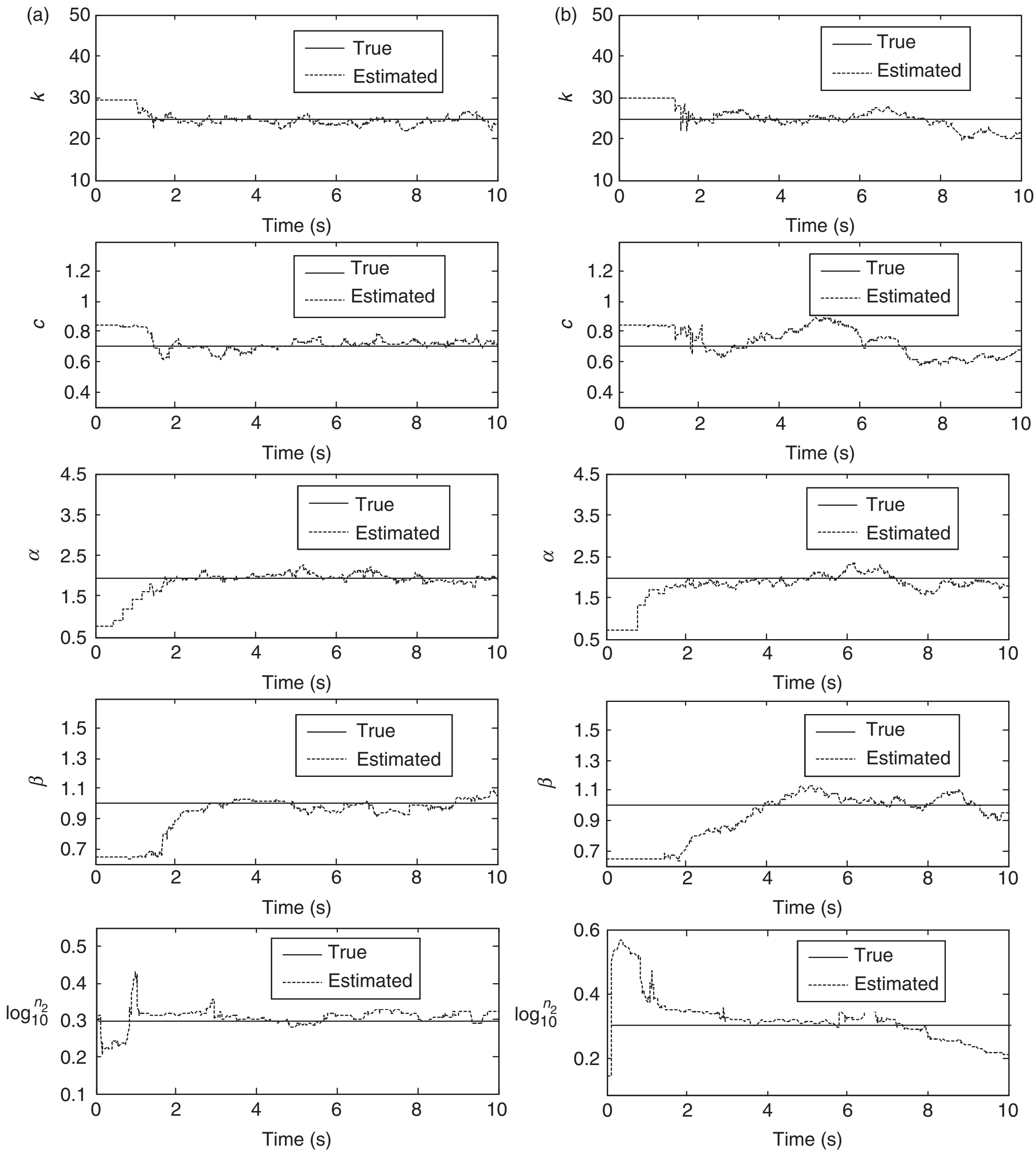

Figure 4 Identified parameters k, c, α, β, and $\log_{10}^{\eta}$ for a 2-DOF hysteretic structure with abruptly changed parameters (2nd story): (a) auxiliary particle filter method, (b) particle filter method.

It is observed from Figures 3 and 4 that the proposed method tracks the structural parameters and their variations very well. Also shown in these figures, the APF has a good time-tracking ability and is more suitable for tracking the nonstationary system than the conventional particle filters.

6 Conclusion

The auxiliary particle filtering technique has been proposed to identify online the structural parameters and their sudden changes due to damages for nonlinear hysteretic structures. It is shown that the proposed method consistently achieves a better level of accuracy for estimating and tracking the parameters and their abrupt changes than the traditional particle filter method. It has also been demonstrated that the APF is robust to the initial values. Numerical results indicate that the proposed approach is particularly suitable for tracking the sudden parameter changes from which the structural damage can be determined.

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