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Identifiability of linear superstructures under feedback—Taking base-isolated structures as example

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SUMMARY

The identifiability condition for substructural identification in free-vibration systems is investigated by spectral analysis and parametric methods in the framework of closed-loop systems. Substructures governed by linear and nonlinear feedback laws are both considered. The feedback law mechanism is shown to have greater influence on identifiability than does the model structure or the identification method. Copyright © 2008 John Wiley & Sons, Ltd.

KEY WORDS: closed loop; identifiability; free vibration; substructural identification; feedback

1. INTRODUCTION

For identification of free-vibration systems, many methods have been developed to identify the dynamical characteristics of the overall structure, such as the eigensystem realization algorithm [1]. Some situations, however, only require information about a substructure. For example, in the case of a base-isolated structure, the superstructure is lightly damped and can be viewed as a nearly linear system, while the base isolator is heavily damped, hysteretic, and nonlinear. The modal frequencies and damping ratios of the overall system have strong amplitude dependence [2,3]. This means that these dynamic properties vary with the intensity of excitation even if the system is intact. Consequently, the changes in these properties cannot indicate the status of a nonlinear system. Since some substructural parts of the overall structure remain in the linear domain, the conventional indices of these substructures, defined as the changes in the dynamic

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properties, are still valid for damage indication and detection. Therefore, the damping ratios and frequencies of the substructures are required for local damage detection.

The structural engineering community takes for granted that substructural identification of a superstructure is equivalent to the case in which a structure is subject to ground motion, if the base story of the superstructure is considered as the ground. Structural engineers assume that identification will never fail if the input and output signals are known. Substructural identification of the superstructure sometimes fails, however, in the case of free vibration. This cannot be explained by the theory of open-loop identification, even though the input and output signals of the substructure are both known. Therefore, investigating the identifiability condition requires new insight, with substructural identification moved into the framework of closed-loop systems.

In this paper, we take the example of a base-isolated structure decomposed into two substructures: a superstructure and a base isolation layer. The superstructure is considered as a linear system. The base isolation layer can be either linear or nonlinear. We first investigate the linear case, in which the isolation layer is represented by a linear model with stiffness k_b and damping coefficient c_b, as illustrated in Figure 1. Here, x is the displacement relative to the ground, $\ddot{x}_g$ is the ground acceleration, and the superscript a refers to the absolute coordinates.

The motion equation of the overall structure is expressed as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{r}\ddot{x}_g \tag{1}$$

where **K**, **C**, and **M** are the stiffness, damping, and mass matrices, respectively, and **Mr** is the diagonal vector of **M**.

After decomposing the structure into the superstructure and the isolation layer, these two subsystems compose a complete closed-loop system, as shown in Figure 2. The superstructure is taken as the plant, while the isolation layer is the regulator. The subscripts s and b denote the superstructure and the isolation layer, respectively. The ground acceleration functions as the reference signal, a persistent excitation of any order, and is the input to the overall system. The output of the plant is contaminated by unmeasured noise sources. We assume that there are no process disturbances between the plant and the controller, and that the unmeasured noise does

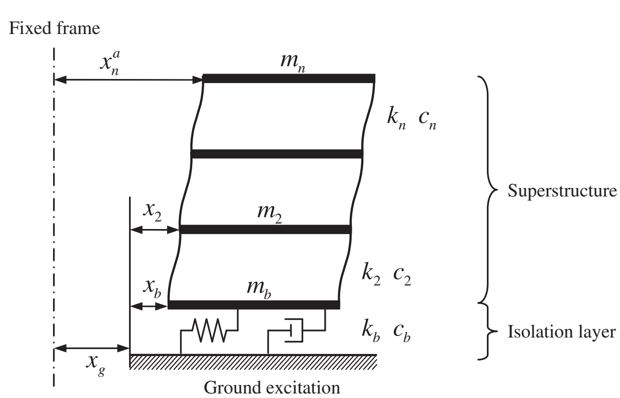

Figure 1. Structure model.

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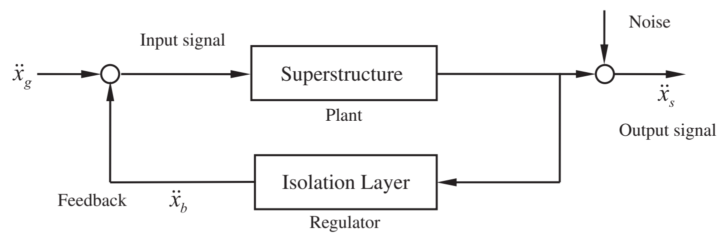

Figure 2. Subsystems in a closed-loop scheme.

not affect the controller, meaning that there is no correlation between the noise and the input. The noise sources are modeled within output signals; therefore, they are located outside the feedback loop. The isolation layer generates the feedback, which is the relative acceleration of this layer with respect to the ground. The feedback added via the reference signal is equal to the absolute acceleration of the layer exciting the plant.

The rest of this paper is organized as follows. Section 2 addresses the basic concepts of closed-loop identification. Section 3 investigates the identifiability problem for linear systems under linear feedback laws, by examining spectral analysis and parametric methods. We also derive the identifiability condition for parametric methods. Section 4 briefly explains the identifiability of linear systems under nonlinear feedback laws. In Section 4, we apply the identifiability condition to study the identification of a superstructure by numerical simulation. Finally, the last section summarizes our main results.

2. BASIC CONCEPTS OF CLOSED-LOOP IDENTIFICATION

The essential concept in closed-loop identification is identifiability, which means that there exists an identified model $M(\theta)$ that can describe the true system S when the number of measurements tends to infinity. In the case of a closed-loop system, as shown in Figure 3, the input and the unmeasurable noise, which is inside the feedback loop, are correlated whenever the feedback controller exists. This is why several methods that can be applied in open loops fail when applied to closed-loop data.

The identifiability problem of linear systems under linear feedback was first investigated by Akaike [4] by using spectral analysis, showing that under pure feedback conditions spectral analysis fails to yield informative results for the plant. Box and MacGregor [5] concluded an identical result by using correlation methods, which are not applicable to the causality of true systems. Ljung *et al.* [6] explored the same problem by direct, parametric approaches and proved that by shifting between different linear regulators it is always possible to achieve identifiability for pure feedback systems. The required number of regulators depends only on the numbers of inputs and outputs. Söderström *et al.* [7] then included noise sources in the regulator and external input signals for a general configuration. Ng *et al.* [8] derived the identifiability conditions for joint input–output approaches, which require that there be no correlation between the noise in the forward and reverse paths. The presence of delays in either the plant or the regulator is necessary to avoid an algebraic loop [6,7]. This can be relaxed to a condition relating to the absence of algebraic loops in closed-loop systems [9]. Wang *et al.* [10] used a fast-sampling direct approach to lift these restrictive identifiability conditions for a closed-loop

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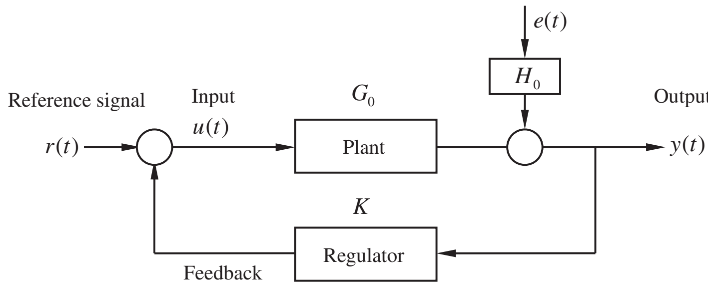

Figure 3. A closed-loop system.

system without external signals. Finally, Forssell and Ljung [11] and Gustavsson *et al.* [12] provided comprehensive surveys of closed-loop identifiability.

In the structural engineering community, the identifiability of substructural identification has seldom been considered. It is vitally important to determine under what conditions it is possible to obtain reliable, identified results for closed-loop systems. Generally speaking, the result of identification depends on the following items [12]:

(a) system;
(b) feedback structure;
(c) model structure;
(d) identification method;
(e) experimental conditions.

2.1. System

Consider a linear time-invariant dynamical system in a discrete-time representation:

$$S : y(t) = G_0(q)u(t) + v(t), \quad v(t) = H_0(q)e(t) \tag{2}$$

where $y(t) \in R^p$ is a p-dimensional output signal, $u(t) \in R^m$ is an m-dimensional input signal, $e(t) \in R^p$ is a sequence of independent random variables with zero mean and covariance matrix $Ee(t)e^{\mathrm{T}}(t) = \Lambda > 0$, and $G_0(q)$ and $H_0(q)$ are rational transfer function matrices, with $H_0(q)$ being an inversely stable, monic filter. In this paper, q denotes the forward shift operator, e.g. $q^{-1}u(t) = u(t-1)$.

2.2. Feedback structure

Assume that this system is operated under a linear feedback law:

$$u(t) = r(t) + K(q)y(t) \tag{3}$$

where $r(t)$ is a q-dimensional reference signal assumed to be independent of the noise v, which is either an additional measurable signal or a noise disturbance in the regulator output, and $K(q)$ is a linear, time-invariant regulator of appropriate dimensions. The feedback structure plays a vital role in the identifiability conditions of a closed-loop system. The number of regulators or the complexity of the regulator can influence the identifiability of a closed-loop system.

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2.3. Model

To identify the open-loop system S, we consider a model set:

$$M : y(t) = G_\theta(q)u(t) + H_\theta(q)\epsilon(t), \quad \theta \in \Theta \subset R^d \tag{4}$$

where $\varepsilon(t)$ is a sequence of independent, random vectors with zero mean values and covariances $\tilde{\Lambda}$, $G_\theta(q)$ and $H_\theta(q)$ are the dynamics model and the noise model, respectively, and both are appropriate rational transfer function matrices depending on a real-valued parameter vector θ. When θ varies within a feasible region, Equation (4) represents a family of models, sometimes called a model structure. We assume that G is causal and H is both monic and causal.

2.4. Identification method

The procedure to determine the parameter vector θ is called the identification method. For a closed-loop system, identification methods can be classified into three main groups [12]:

(a) The direct approach: Ignore the feedback and identify the system directly by measuring the input and output, exactly as if it was an open-loop system.
(b) The indirect approach: If the regulator is known, then the closed-loop system as a whole can be identified. The corresponding open-loop system is determined through the knowledge of the regulator.
(c) The joint input–output approach: Jointly consider both the input and the output as the output from a system driven by some extra input or noise. The corresponding open-loop system is identified by estimating the characteristics of this augmented system.

Each group includes several different methods, such as correlation and spectral analysis, the parametric identification method [13], and the subspace identification method [14]. If there is a time delay in either the system or the regulator, and if the regulator noise is independent of the system noise, then the direct and joint input–output approaches are equivalent for determining identifiability. Furthermore, the indirect approach has no advantage over direct identification in terms of either identifiability or accuracy. Therefore, we adopt the direct identification approach in this paper.

2.5. Experimental conditions

Basically, the experimental conditions consist of the sampling rate and the length of the experiment, which describe how the input is determined. An experimental system can be operated in an open loop, or the experimental conditions can be determined by the feedback of a given regulator. Traditionally, the conditions include the regulator characteristics. In this paper, however, we exclude these characteristics from the experimental conditions and instead regard the regulator characteristics as a dependent item, called the feedback structure.

2.6. Identifiability definition

The property of identifiability is related to the consistency of parameter estimation. There are several definitions on different levels, as defined by Ljung and coworkers [12]. For a certain

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model structure, experimental conditions, and identification method, a system is said to be system identifiable (SI) if an identified model $M(\theta)$ converges to the true system S when the number of measurements tends to infinity. If a system is SI for all possible model structures, then it is said to be strongly system identifiable (SSI).

3. IDENTIFIABILITY CONDITIONS FOR LINEAR FEEDBACK LAWS

3.1. Nonparametric methods

As a classical, well-established method, spectral analysis was first used by Akaike [4] to study the identifiability problem with nonparametric identification methods on closed-loop data. The frequency response function can be obtained from the spectrum analysis:

$$G(\omega) = \frac{\Phi_{uy}(\omega)}{\Phi_u(\omega)} \tag{5}$$

where $\Phi_u(\omega)$ and $\Phi_{uy}(\omega)$ are the spectrum of input u and the cross-spectrum between input u and output y, respectively. Another formulation of $G(\omega)$ is the following:

$$G(\omega) = \frac{G_0(e^{i\omega})\Phi_r(\omega) + K(e^{i\omega})|H_0(e^{i\omega})|^2\sigma_e^2}{\Phi_r(\omega) + |K(e^{i\omega})|^2|H_0(e^{i\omega})|^2\sigma_e^2} \tag{6}$$

where $\Phi_r(\omega)$ is the spectrum of the reference signal r. When the reference signal exists, the frequency response gives a weighted average of the true process frequency response and the frequency response of the controller's inverse. If there is no persistent excitation signal, meaning that $\Phi_r(\omega) = 0$, then nonparametric methods identify only the inverse of the feedback controller:

$$G(\omega) = \frac{1}{K(e^{i\omega})} \tag{7}$$

This shows that spectral analysis will not yield information about the plant if applied to a pure feedback operation. Box and MacGregor [5] concluded an identical result by using correlation methods. Nonparametric methods fail to yield informative results because the causality of true systems cannot be implied by these methods. Instead, these methods only identify the best correlation relationship between the input and output, which is represented by the feedback law.

This result can also be explained from the viewpoint of modal vibration theory. The free-vibration response, which is determined by the initial conditions, contains frequency content only at the modal frequencies of the whole system. Furthermore, strictly speaking, the auto- or cross-spectrum of the response is nonzero at modal frequencies and zero at other frequencies. Therefore, spectral analysis cannot obtain the frequency response function of the substructure or plant in terms of the input and output vibration signals in the frequency domain.

3.2. Parametric methods

Söderström *et al.* [7] generalized the identifiability conditions for a system by including the noise sources in the regulator and the external input signals. The regulator is assumed to shift among r different feedback laws:

$$u(t) = r(t) + K_i(q)y(t), \quad 1 \leqslant i \leqslant r \tag{8}$$

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Each case applies during a nontrivial period of the total time of an experiment. Here, we introduce some abbreviations to facilitate concise description. We denote $G = G_0(q)$, $\hat{G} = G_\theta(q)$, $K_i = K_i(q)$, and so forth. Equations (8) and (2) then give

$$y(t) = GK_i y(t) + Gr(t) + He(t) \tag{9}$$

The input can be expressed as

$$u(t) = [K_i(I - GK_i)^{-1}G + I]r(t) + K_i(I - GK_i)^{-1}He(t)$$

We also introduce the following notation for the feedback law K_i:

$$P_i = (I - GK_i)^{-1}G$$

Now, we consider direct identification. The residual $\varepsilon(t)$ is given as

$$\begin{aligned}\epsilon(t) &= \hat{H}^{-1}[y(t) - \hat{G}u(t)]\\ &= \hat{H}^{-1}[P_i - \hat{G}(K_iP_i + I)]r(t) + \hat{H}^{-1}(I - \hat{G}K_i)(I - GK_i)^{-1}He(t)\end{aligned} \tag{10}$$

Since K_i is causal and the input $u(t)$ is independent of $e(t)$, the minimum variance prediction error $\varepsilon(t)$ is asymptotically given as

$$\epsilon(t) \equiv e(t)$$

Therefore, Equation (10) implies the following:

$$\begin{cases}\hat{H}^{-1}[P_i - \hat{G}(K_iP_i + I)] = 0,\\ \hat{H}^{-1}(I - \hat{G}K_i)(I - GK_i)^{-1}H = 1,\end{cases} \quad 1 \leqslant i \leqslant r \tag{11}$$

The first equation can be rewritten in this form:

$$(\hat{H}^{-1}\hat{G} - H^{-1}G)(-K_iP_i - I) + (\hat{H}^{-1} - H^{-1})P_i = H^{-1}(GK_iP_i + G - P_i) = 0$$

The second equation is simplified as:

$$-(\hat{H}^{-1}\hat{G} - H^{-1}G)K_i + (\hat{H}^{-1} - H^{-1}) = 0$$

Thus, we rewrite Equation (11) in matrix form:

$$\begin{aligned}&[\hat{H}^{-1} - H^{-1} \quad H^{-1}G - \widehat{H}^{-1}\hat{G}\hat{H}]\begin{bmatrix} P_i & I \\ K_iP_i + I & K_i\end{bmatrix}\\ &= [\,\hat{H}^{-1} - H^{-1} \quad H^{-1}G - \hat{H}^{-1}\hat{G}\,]\begin{bmatrix} I & 0 \\ K_i & I\end{bmatrix}\begin{bmatrix} P_i & I \\ I & 0\end{bmatrix} = [\,0 \quad 0\,]\end{aligned} \tag{12}$$

Because the matrix $\begin{bmatrix} P_i & I \\ I & 0\end{bmatrix}$ is nonsingular, we finally have

$$[\,\hat{H}^{-1} - H^{-1} \quad H^{-1}G - \hat{H}^{-1}\hat{G}\,]R_r = [\,0 \;\; 0\,]$$

$$R_r = \begin{bmatrix} I_1 & \cdots & I_1 & 0 & \cdots & 0 \\ K_1 & \cdots & K_r & I_2 & \cdots & I_2\end{bmatrix} \tag{13}$$

where I_1 has order $n_y|n_y$, 0 has order $n_y|n_r$, K_i has order $n_u|n_y$, and I_2 has order $n_u|n_r$. Thus, R_r is a matrix of order $n_y{+}n_u|r(n_y{+}n_r)$. Here, r is the number of regulators, and n_y, n_u, and n_r are the

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numbers of plant outputs, plant inputs, and reference signals, respectively. If the identification result converges to the true model, then we have

$$\hat{H} = H, \quad \hat{G} = G$$

This implies that

$$\hat{H}^{-1} - H^{-1} = 0$$

$$H^{-1}G - \hat{H}^{-1}\hat{G} = 0 \tag{14}$$

This requires that R_r be nonsingular.

In conclusion, the identifiability condition for a linear system under linear feedback can be summarized as follows: For a linear multivariate system, if there is a reference signal, it must be a persistent excitation of any finite order. Assume that there is a time delay in either the system or the regulator, such that $G(0)K_i(0) = 0$. Then, the system is SSI if and only if

$$\text{rank}(R_r) = n_y + n_u \tag{15}$$

A necessary condition for Equation (15) to hold is that

$$r \geqslant (n_y + n_u)/(n_y + n_r) \tag{16}$$

The presence of a delay in either the plant or the regulator, given by $G(0)K_i(0) = 0$, avoids algebraic relations between the input and output necessary to guarantee identifiability. Here, this is represented strictly, but Schoen [9] relaxed this classic delay-structure condition for the identifiability of closed-loop systems.

3.3. *Identifiability conditions of superstructures*

We can now apply the derived identifiability condition to the case of superstructures for two situations: ground excitation and free vibration.

Case 1: In the case of ground excitation, the reference signal is of the same dimension as the input signal, i.e. $n_u = n_r$, and the regulator is not replaceable, i.e. $r = 1$. Then, the identifiability condition becomes

$$\text{rank}\begin{bmatrix} I_1 & 0 \\ K_1 & I_2 \end{bmatrix} = n_y + n_u \tag{17}$$

Therefore, no matter what the feedback might be, the necessary condition from Equation (16) always holds if there is ground motion. In other words, if the input and output of the plant are given, the superstructure in the case of an earthquake is SSI.

Case 2: The case of free vibration means a situation with pure linear feedback laws; therefore, $n_r = 0$. Then, the identifiability condition becomes

$$\text{rank}\begin{bmatrix} I_1 & \cdots & I_1 \\ K_1 & \cdots & K_r \end{bmatrix} = n_y + n_u \tag{18}$$

Therefore, the necessary condition from Equation (16) for the free-vibration case is given as

$$r \geqslant 1 + n_u/n_y$$

This means that even if no reference signal exists, the presence of at least two linear feedback laws guarantees the system identifiability of the superstructure.

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If the identifiability condition is not satisfied, the system is not SSI. It can, however, be SI for certain model structures. Söderström *et al.* [15] investigated the identifiability of a pure feedback system without external inputs for certain model structures and derived the necessary and sufficient condition for identifiability. This condition requires that the order of the regulator be higher than that of the plant for identifiability to hold.

When the regulator is operated by a nonlinear feedback law, Equation (6) cannot clarify the identifiability of the plant, because the description of the regulator in the frequency domain is invalid. In parametric approaches, if the feedback law is nonlinear, it can be viewed as a different linear regulator during every short time segment. Therefore, the complexity of the nonlinear regulator guarantees the identifiability of the plant in the case of pure feedback, and in the case with external signals, as well.

4. NUMERICAL SIMULATION

In this simulation, we considered a four-story structure. The part of the structure above the base layer was defined as the superstructure. The superstructure was of the shear type and assumed to remain within its elastic range. The stiffness and mass could vary from floor to floor, as illustrated in Figure 4. The damping coefficient matrix was proportional to the stiffness matrix as $\mathbf{C} = \alpha\mathbf{K}$, with $\alpha = 0.001$. Tables I and II list the modal information for the superstructure and the overall structure, respectively. The time step for the simulation was 0.01 s. The structure was placed in a state of free vibration by setting its initial displacement. The acceleration response was contaminated by 1% white noise (i.e. the standard deviation of the noise was 1% of the standard deviation of the response).

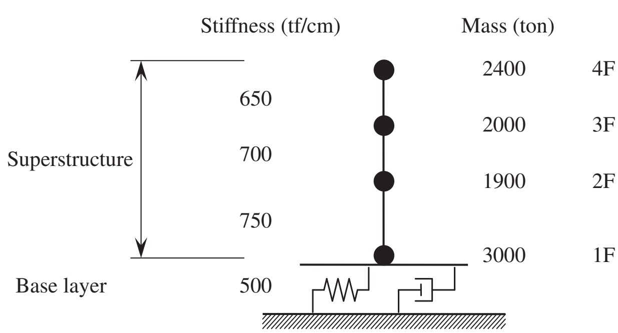

Figure 4. Simulation model for a four-story building.

Table I. Modal information for the superstructure (fixed base).

	Frequency (Hz)	Damping ratio
First	1.2762	0.0040
Second	3.6125	0.0113
Third	5.3489	0.0168

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Table II. Modal information for the overall structure.

	Frequency (Hz)	Damping ratio
First	0.9113	0.0029
Second	2.4803	0.0078
Third	4.0756	0.0128
Fourth	5.5001	0.0173

Autoregressive moving average models with exogenous inputs (ARMAX models) include disturbance dynamics that can flexibly describe a disturbance as a moving average of white noise. The model is given as follows:

$$y(t) + a_1 y(t-1) + \cdots + a_{n_a} y(t-n_a) = b_1 u(t-1) + \cdots + b_{n_b} u(t-n_b) + e(t) + c_1 e(t-1) + \cdots + c_{n_c} e(t-n_c) \qquad (19)$$

with

$$A(q) = 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}$$

$$B(q) = b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b}$$

$$C(q) = 1 + c_1 q^{-1} + \cdots + c_{n_c} q^{-n_c}$$

where, a_i, b_i, and c_i are the coefficients of the AR, X, and MA part, respectively. n_a, n_b, and n_c are model orders of each part. This model can also be rewritten as

$$A(q)y(t) = B(q)u(t) + C(q)e(t) \qquad (20)$$

$A(q)y(t)$ is the autoregression part of the output, $B(q)u(t)$ describes a process of exogenous inputs, and $C(q)e(t)$ represents the disturbance dynamics, which is the moving average of a stationary white noise $e(t)$. In addition, this corresponds to Equation (2) with

$$G(q,\theta) = \frac{B(q)}{A(q)}, \quad H(q,\theta) = \frac{C(q)}{A(q)} \qquad (21)$$

For further information about the model estimation, readers can refer to Ljung's book [13].

Thus, an ARMAX model is used for identification, using the acceleration at the first floor for the input signals and the acceleration at the fourth floor for the outputs signals, as shown in Figure 5. When the regulator (i.e. the base story) was governed by a linear feedback law, the parametric method obviously failed to identify the superstructure, as illustrated in Figure 6.

To ensure the nonlinearity of the regulator, a cubic hardening stiffness was added to the base layer: $F_n = \beta k_{\mathrm{b}} x_{\mathrm{b}}^3$ ($\beta = 10\,000$), where x_{b} is the displacement relative to the ground. As shown in Figure 7, the result identified by the ARMAX model was consistent with that of the analytical model.

5. CONCLUSION

We have explored the identifiability condition for substructural identification in the cases of free vibration. As explained by Akaike [4], spectral analysis in the frequency domain cannot obtain

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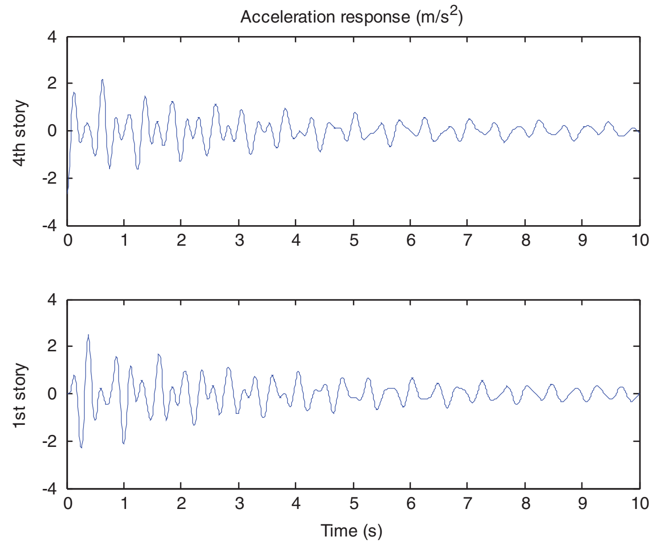

Figure 5. Acceleration response (initial displacement: 0.01 m at the fourth story, 0 elsewhere).

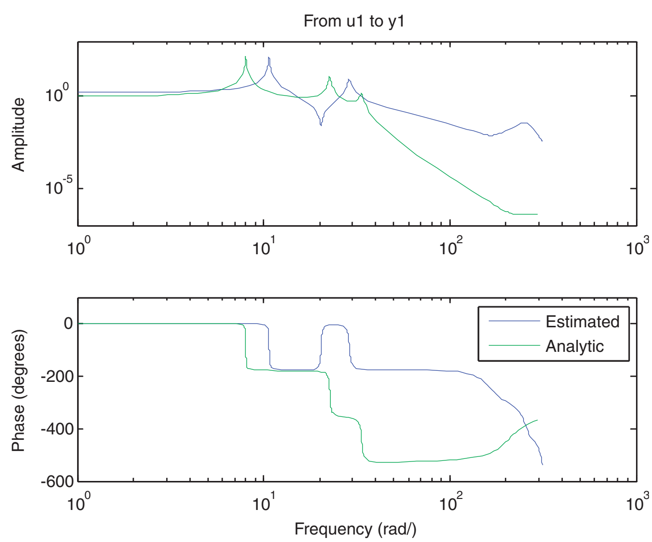

Figure 6. Parametric method for the linear case.

the identifiability of a superstructure. If the base layer is governed by a linear feedback law, the identifiability of the superstructure is lost unless there are at least two regulators. By making the regulators nonlinear, identifiability can be regained with parametric methods. In a free-vibration field test, the identification of a linear substructure under linear feedback laws can be guaranteed

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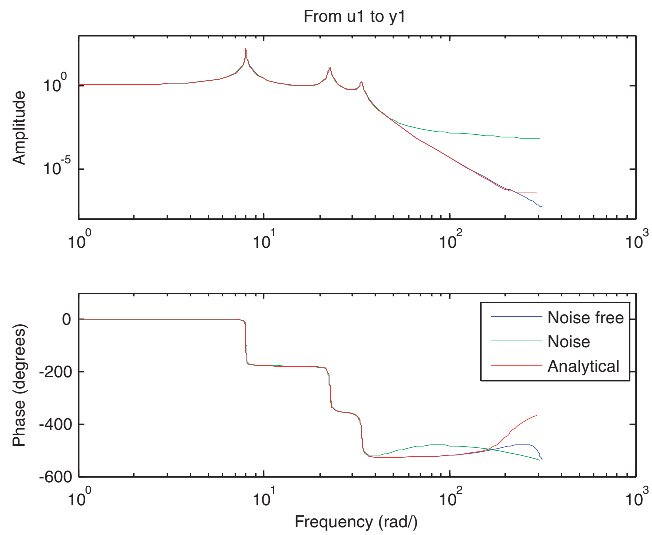

Figure 7. Parametric method for the nonlinear case.

if a nonlinear device is attached to the rest of the structure. The feedback, determined by the characteristics of the regulator, has a greater influence on the identifiability than does the model structure or the identification method.

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