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## Big Bang-Big Crunch optimization for parameter estimation in structural systems

Hesheng Tang<sup>a,\*</sup>, Jin Zhou<sup>a</sup>, Songtao Xue<sup>a,b</sup>, Liyu Xie<sup>a</sup><sup>a</sup> Research Institute of Structural Engineering and Disaster Reduction, Tongji University, Shanghai 200092, China<sup>b</sup> Department of Architecture, Tohoku Institute of Technology, Taihaku-ku, Sendai, Miyagi 982-8577, Japan

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### ABSTRACT

A new approach to parameter estimation of structural systems using the recently developed Big Bang-Big Crunch (BB-BC) optimization is proposed, in which the parameter estimation is formulated as a multi-modal optimization problem with high dimension. The BB-BC method is inspired by one of the theories of the evolution of universe. The potentialities of BB-BC are its inherent numerical simplicity, high convergence speed, and easy implementation. The performances of the proposed method are investigated with simulation results for identifying the parameters of structural systems under conditions including limited output data, noise-polluted signals, and no priori knowledge of mass, damping, or stiffness. It is observed that BB-BC gives comparatively better results than existing methods. Moreover the method is computationally simpler.

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## 1. Introduction

In parameter estimation of structural systems, the basic parameters defining the physical system are estimated through noise corrupted measurement data. In general, the error surface of this problem that exists as a hyper-surface in the multi-dimensional parameter space. The hyper-surface potential is often highly multi-modal in the sense that it is a non-quadratic surface that possesses local minima, in addition to a global minimum that represents the optimal solution. Due to the frequent ill-conditioning and multi-modality of many of these problems, traditional local methods, like Levenberg–Marquardt or Gauss–Newton, may fail to identify the global solution and may converge to a local minimum. These local methods are performed by point-to-point search strategy. Such optimization methods work satisfactorily when the error surface contains no local minima. But most of the real life problems are multi-modal and also are distorted due to additive noise. A good initial guess of the parameter and gradient or higher-order derivatives of the objective function is generally required. There is always a possibility to fall into a local minimum rather than the global minimum. In order to surmount these difficulties, there is a distinct need for using global optimization methods, which do not converge to local minima and thus expected to provide accurate estimates of the system parameters.

In recent years, heuristic computational intelligence methods belonging to the global optimization category have proven to be promising tools to solve many multi-modal optimization problems. These have been found to be powerful methods in domains where local methods have not been proved to be effective. Among the most important global heuristic optimization methods, such as genetic algorithms (GAs), evolution strategy (ES), simulated annealing (SA), ant colony

\* Corresponding author. Tel.: +86 21 65982390; fax: +86 21 65983410.  
E-mail address: thstj@tongji.edu.cn (H. Tang).

optimization (ACO), particle swarm optimization (PSO) and differential evolution (DE) have been successfully applied to a variety of optimization problems such as wire routing, scheduling, traveling salesman, image processing, engineering design, parameter fitting, computer game playing, knapsack problems, and transportation problems [1,2].

The heuristic intelligence optimization algorithms are highly adaptive methods originated from the laws of nature and biology. Unlike the tradition methods, one of the important characteristics of these methods is their effectiveness and robustness in coping with uncertainty, insufficient information, and noise. The importance of using these methods for parameter estimation in structural systems has been increasingly recognized in recent years. Koh et al. [3–6] applied GAs to solve the global system identification problem in shear-type building structures. Perry et al. [7] presented a modified GAs to identify structural systems. Cunha et al. [8] used GAs to identify the elastic constants of composite materials. Barbieri et al. [9] applied GAs to identify the physical parameters of sandwich beams. Wang [10] used a hybrid GA to identify structural systems. Chou and Ghaboussi [11] introduced GAs to identify damage severity of trusses. Tang et al. [12,13] and Omkar and Mudigere [14] introduced PSO to linear and non-linear system identification. DE has been successfully applied in structural system identification [15]. Franco et al. [16] used ES to identify multiple degree of freedom (DOF) systems. Jeong and Lee [17] proposed an adaptive simulated annealing genetic algorithm for system identification. Levin and Lieven [18] applied SA method to optimize a finite element model for describing the dynamic behavior of structures.

As a novel heuristic computational intelligence method, Big Bang-Big Crunch (BB-BC) algorithm has gained much attention and wide applications for solving complex optimization problems since Erol and Eksin [19] introduced the algorithm in 2006. It is a global optimization method that relies on one of the theories of the evolution of the universe, namely, the Big Bang-Big Crunch theory. The algorithm generates random points in the Big Bang phase and shrinks those points to a single representative point via a center of mass or minimal cost approach in the Big Crunch phase. After a number of sequential Big Bangs and Big Crunches, where the distribution of the randomness within the search space during the Big Bang becomes smaller and smaller about the average point computed during the Big Crunch, the algorithm converges to a solution. It is reported to be capable of quick convergence even in long, narrow parabolic shaped flat valleys or in the existence of several local minima [19,20].

BB-BC optimization method has several advantages over other evolutionary methods: the inherent numerical simplicity of the algorithm with relatively few control parameters, quick convergence, and easy implementation. BB-BC algorithm has been shown to outperform enhanced and classic genetic algorithms for many benchmark optimization functions [19]. The results in [20] indicate that the BB-BC design procedure may provide more computationally efficient designs than either the ACO or GA approaches. For its simple structure, easy use, convergence property, quality of solution, and robustness, recently BB-BC has been successfully applied to a variety of optimization problems, such as function optimization [19], fuzzy system control [21], automatic target tracking [22], and optimal design [20].

In the realm of structural engineering, identification of structural systems with unknown mass, stiffness, and damping properties—a challenging problem rarely considered due to the difficulty encountered in many identification methods with regards to separating effects of mass and stiffness properties [6]. Further, measuring the inputs and outputs of a real structural system tends to be complex and expensive. Thus, there is a significant interest in the development of a robust and efficient method that uses as few measurements as possible to obtain the physical characteristics of the system, without a priori knowledge of the system. The novel heuristic technique presented in this paper aims to ensure the proper solution of these problems by adopting a global optimization approach, while keeping the computational effort under reasonable values. The BB-BC optimization method presents as peculiar characteristic the fact that working with a population of variable size, unlike the most popular heuristics, which consider a single solution at a time as the SA or a population of solutions as the GA or ACO. In this study, the BB-BC optimization method was applied to a set of challenging parameter estimation problems of structural systems with no priori knowledge, insufficient information, and noise, outperforming very significantly the other methods previously used for these problems.

## 2. Big Bang-Big Crunch (BB-BC) algorithm

The BB-BC algorithm [19] relies on one of the theories of the evolution of the universe, the so-called Big Bang-Big Crunch Theory. In the Big Bang phase, energy dissipation produces a random disordered state of particles, whereas, in the Big Crunch phase, randomly distributed particles are drawn into an order. Randomness can be seen as equivalent to the energy dissipation in nature while convergence to a local or global optimum point can be viewed as gravitational attraction. Since energy dissipation creates disorder from ordered particles, the random nature of the Big Bang (energy dissipation) is used as a transformation from an ordered state (a convergent solution) to a disordered or chaotic state (new set of candidate solutions).

The initial Big Bang is similar to the other evolutionary methods in respect to creating an initial population randomly over the entire search space. In this phase, the candidate solutions are spread all over the search space in a uniform manner. The Big Bang phase is followed by the Big Crunch phase. In this phase, the contraction operator takes the current positions of each candidate solution in the population and its associated fitness function value and computes a center of 'mass'. The center of mass is the weighted average of the candidate solution positions with respect to the inverse of the

fitness function values computed as

$$x_c = \frac{\sum_{i=1}^N \frac{1}{f_i} x_i}{\sum_{i=1}^N \frac{1}{f_i}} \quad (1)$$

where  $x_c$  (contraction factor) is position of the center of mass;  $x_i$  is the position of candidate  $i$  in an  $n$ -dimensional search space;  $f_i$  is a fitness function value of candidate  $i$ ; and  $N$  is the population size in Big Bang phase. The fittest individual can be selected as the center of mass.

The positions of the candidate solutions for the next iteration of the Big Bang are normally distributed around the center of mass,  $x_c$ , using the following:

$$x_i^{new} = x_c + \sigma \quad (2)$$

where  $x_i^{new}$  is the new candidate position  $i$  of next iteration; and  $\sigma$  is the standard deviation of a standard normal distribution. In BB-BC algorithm, the standard deviation is related to a subset of search space, which decreases inversely with each succeeding iteration, via the following formula:

$$\sigma = \frac{r\alpha(x_{max} - x_{min})}{I} \quad (3)$$

where  $r$  is the random number from a standard normal distribution;  $\alpha$  is the parameter limiting the size of the search space;  $x_{max}$  and  $x_{min}$  are the upper and lower limits on the values of the candidates; and  $I$  is number of Big Bang iteration.

In order to improve the computational efficiency and performance of the general BB-BC algorithm, an elitist strategy introduced by Camp [20] is applied in this BB-BC parameter estimation algorithm. The studies in [20] indicated that there is a significant improvement in the quality of the solutions and the computational efficiency of the original BB-BC algorithm [19]. In this algorithm, positions of new candidate solutions at the beginning of each Big Bang are normally distributed around a new point located between the center of mass,  $x_c$ , and the best global solution,  $x_{best}$ , using the following:

$$x_i^{new} = \beta x_c + (1 - \beta) x_{best} + \frac{r\alpha(x_{max} - x_{min})}{I} \quad (4)$$

where  $\beta$  is the parameter controlling the influence of the global best solution  $x_{best}$  on the location of new candidate solutions. In some sense, the weighted average of  $x_{best}$  and  $x_c$ , controlled by  $\beta$ , may be viewed as equivalent to an elitist strategy, wherein the best solution is allowed to influence the direction of the search over many iterations of the technique, and therefore potentially improve the overall search performance. Through empirical studies, Camp [20] has observed that the optimal solution can be provided by  $\alpha=1$  and  $\beta=0.2$  for the truss design problems. In our experiments, computational results show that  $\alpha=1$  and  $\beta=0.7$  provided the best estimation results.

### 3. BB-BC based parameter estimation

The parameter estimation problem can be understood as an optimization problem in which the error between an actual physical measured response of a structure and the simulated response of a parameterized model is minimized. To understand this in more detail, consider a general physical system with input  $u$  and output  $y$ . Let  $y(t_i)$  for  $i=1, \dots, T$  denotes the value of the actual system at the  $i$ th discrete time step. Suppose that a parameterized model that is able to capture the behavior of the physical system is developed and this model depends on a set of  $n$  parameters, i.e.,  $x = (x_1, x_2, \dots, x_n)^T \in R^n$ . Let  $\hat{y}(t_i)$  for  $i=1, \dots, T$  denotes the value of the candidate model, i.e., the identified system at the  $i$ th discrete time step. In order to obtain a successful identification, the candidate system must be able to accurately reproduce the output of the physical system for any given input. Therefore, our interest lies in minimizing the error norm of the outputs. In this study, the error norm of all the simulated outputs of the identified system with respect to those measured from the actual system, defined as:

$$f(x) = \frac{1}{T} \sum_{i=1}^T \|y(t_i) - \hat{y}(x, t_i)\|^2 \quad (5)$$

where  $\|\cdot\|$  represents the Euclidean norm of vectors. Formally, the parameter identification problem requires finding a set of  $n$  parameters  $x^* \in R^n$ , so that a certain quality criterion is satisfied, namely that the error norm  $f(\bullet)$  is minimized. In these terms, an estimation problem can be expressed as a linearly constrained multi-dimensional optimization problem, namely

$$\begin{aligned} & \text{Minimize } f(x), \quad x = (x_1, x_2, \dots, x_n)^T \\ & \text{s.t. } x \in S, \quad S = \{x : x_{min,i} \leq x_i \leq x_{max,i}, \quad \forall i = 1, 2, \dots, n\} \end{aligned} \quad (6)$$

where  $f(x)$  is a cost function or objective function, which maps decision variable  $x$  into the objective space  $f: R^n \rightarrow R$ ;  $S$  is the  $n$ -dimensional feasible search space;  $x_{max}$  and  $x_{min}$  denote the upper bounds and the lower bounds of the  $n$  parameters,

respectively. In parameter estimation problem, typically a fitness function is used, which reflects the goodness of the solution. Since our problem is one of the minimization, a fitter solution will be characterized with a lower value of the cost function. Therefore, the fitness function can be defined as the negative or inverse of the cost function, i.e.,  $-f$  or  $1/f$ . Minimization of  $f$  is then equivalent to maximize the fitness  $-f$  or  $1/f$ .

In this study, the BB-BC parameter estimation algorithm follows the general procedures developed by Erol and Eksin [19] and improved by Camp [20]. The parameter estimation of structural system is a batch type optimization problem. The steps involved in the BB-BC based algorithm for identification are as follows:

Step 1: Form an initial generation of  $N$  candidates in a random manner. Respect the prescribed limits of the search space in Big Bang phase.

Step 2: Calculate the fitness function values of all the candidate solutions using Eq. (5).

Step 3: Find the center of mass according to Eq. (1). Best fit individual can be chosen as the center of mass instead of using Eq. (2).

Step 4: Calculate new candidates according to Eq. (4) around the center of mass by adding or subtracting a normal random number whose value decreases as the iterations elapse.

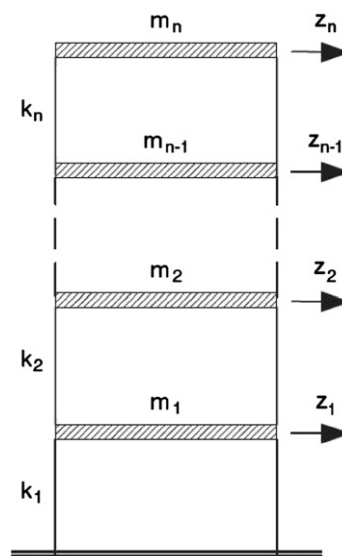
Step 5: Return to Step 2 until stopping criteria has been met.

#### 4. Numerical studies

To verify the proposed BB-BC algorithm in the parameter estimation of structural systems, numerical simulations of multiple DOF systems are carried out in this study. The structural system considered is two-dimensional shear frame-type structures with properties as given in Table 1. The structure consists of rigid beams and flexible columns, effectively reducing the motion to a single translational degree of freedom at each floor level as shown in Fig. 1.

**Table 1**  
8-DOF structural properties.

Stiffness (kN/m)	
Levels 1	5.529E+03
Levels 2–8	2.723E+03
Mass (kg)	
Levels 1–7	49.48
Levels 8	45.06
Damping ratio	
$\zeta_1$	0.5
$\zeta_2$	0.5



**Fig. 1.**  $n$ -DOF structure.

The dynamic equation of motion of the structural system can be written as

$$M\ddot{z}(t)+C\dot{z}(t)+Kz(t)=u(t) \tag{7}$$

where  $M$ ,  $C$ , and  $K$  are the mass, damping and stiffness matrices,  $z$  is the displacement vector, and  $u$  is the input force vector. The mass of the structure is lumped at each floor level and Rayleigh damping matrix  $C$  (Eq. (8)), where modal damping ratio ( $\zeta_r$ ) is set as 5% in the first two modes of vibration ( $r=1$  and 2).

$$C = \alpha M + \beta K; \quad \zeta_r = \frac{\alpha}{2\omega_r} + \frac{\beta\omega_r}{2} \tag{8}$$

Therefore, the system is fully described by the set of parameters

$$x = (m_1, \dots, m_n, k_1, \dots, k_n, \zeta_1, \zeta_2) \tag{9}$$

It is assumed that the structure is excited by known forces,  $u(t)$ , and that the response of the structure, in terms of accelerations,  $\ddot{z}(t)$ , is recorded at some given points ( $q$  outputs). Let  $z_j^M(t_i)$  for  $j=1,2, \dots, q$  and for  $i=1, \dots, T$  denote the value of  $j$ th output of the actual system at  $i$ th discrete time step, and let  $\hat{z}_j(t_i)$  also for  $j=1,2, \dots, q$  and  $i=1, \dots, T$  denote the value of the  $j$ th output of the identified system at the  $i$ th time step. At this point, the stacked vectors of all available output records for the actual and identified systems can be written as

$$y^M = \left[ \begin{matrix} \ddot{z}_1^M(t_1) & \ddot{z}_1^M(t_2) & \dots & \ddot{z}_1^M(t_T) & \ddot{z}_2^M(t_1) & \dots & \ddot{z}_q^M(t_1) & \dots \end{matrix} \right] \tag{10}$$

$$\hat{y} = \left[ \begin{matrix} \hat{z}_1(t_1) & \hat{z}_1(t_2) & \dots & \hat{z}_1(t_T) & \hat{z}_2(t_1) & \dots & \hat{z}_q(t_1) & \dots \end{matrix} \right] \tag{11}$$

The error norm of all the simulated outputs of the identified system with respect to those measured from the actual system is computed according to Eq. (2). The output  $y(t_i)$  is a non-linear function of  $x$  because the  $\dot{z}(t_i)$  and  $z(t_i)$  are unknown. It is evident that output error norm is a non-linear function of  $x$  and, hence, the mean square output error is not a quadratic function and therefore it can have multiple minima. The BB-BC algorithm therefore has been employed in this paper in updating those parameters of the structural system so that the parameter estimates will be optimal.

Firstly, we consider parameter estimation of an 8-DOF structural system under conditions including limited input/output data, noise-polluted signals, and no priori knowledge of mass, damping, or stiffness of the system. The mass, stiffness, and damping parameters are all banded and constant over time allowing for an efficient numerical procedure to be used. The search limits are taken as 0.5–2.0 times the exact values. The fourth-order Runge–Kutta method is employed to carry out the simulation of the structural response to a given excitation. It is assumed that the structure is excited by the Niigata earthquake (Japan, 2000) and that the response of the structure, in terms of accelerations, is recorded at some given points. The influence of limited availability of measurements on the performance of BB-BC for parameters estimation is also discussed in this study. In the “full output” scenario, measurements at all floors are available, whereas in the second “partial output” scenario, only floors 1, 3, 5 and 7 are available. The input and output (I/O) data are polluted (in the cases considering noise) with Gaussian, zeromean, and white noise sequences, whose root mean square (RMS) value is adjusted to be a certain percentage of the unpolluted time histories. The robustness of the strategy is demonstrated in the presence of 10% I/O noise. In simulation tests, the BB-BC parameters are set as follows:  $\alpha=1$ ,  $\beta=0.7$ , population size=100, and maximum generations=500. All identified results (average results over 20 runs) are presented in Tables 2 and 3.

As shown in Table 2, it is clear that the BB-BC estimations of mass, stiffness, and damping parameters of the known mass system are very good, even in the presence of large I/O noise and using partial measurements. In the full measurement scenarios, the stiffness and damping ratio parameters are identified with average error showing 0% and ranging from 0.01% to 0.08% under 0% noise pollution, and the corresponding errors ranging from 0.02% to 0.12% and 0.64%

**Table 2**  
Estimation results for known mass system.

Actual	No noise				10% noise				
	Full measurements		Partial measurements		Full measurements		Partial measurements		
	estimated	error(%)	estimated	error(%)	estimated	error(%)	estimated	error(%)	
$k_1$	5529	5528.92	0.00	5529.43	0.01	5526.14	0.05	5523.38	0.12
$k_2$	2723	2723.04	0.00	2722.92	0.00	2725.13	0.08	2727.21	0.27
$k_3$	2723	2723.04	0.00	2722.69	0.01	2724.33	0.05	2725.06	0.19
$k_4$	2723	2722.97	0.00	2721.9	0.04	2720.8	0.08	2716.52	0.13
$k_5$	2723	2722.96	0.00	2722.74	0.01	2721.48	0.06	2717.3	0.10
$k_6$	2723	2722.97	0.00	2721.95	0.04	2720.88	0.08	2718.99	0.04
$k_7$	2723	2723.02	0.00	2723.35	0.01	2723.47	0.02	2723.19	0.12
$k_8$	2723	2723.03	0.00	2723.74	0.03	2726.17	0.12	2735.59	0.57
$\zeta_1$	0.03	0.03003	0.08	0.03048	1.61	0.03137	4.58	0.03285	9.51
$\zeta_2$	0.05	0.05001	0.01	0.05011	0.22	0.05032	0.64	0.05066	1.33

**Table 3**  
Estimation results for unknown mass system.

Actual	No noise					10% noise			
	Full measurements		Partial measurements			Full measurements		Partial measurements	
	estimated	error(%)	estimated	error(%)		estimated	error(%)	estimated	error(%)
$k_1$	5529	5528.72	0.01	5528.03	0.02	5427.37	1.84	5377.62	2.74
$k_2$	2723	2722.9	0.00	2723.98	0.04	2770.77	1.75	2808.4	3.14
$k_3$	2723	2722.57	0.02	2723.87	0.03	2742.03	0.70	2845.06	4.48
$k_4$	2723	2722.2	0.03	2720.48	0.09	2756.91	1.25	2793.84	2.60
$k_5$	2723	2721.81	0.04	2719.43	0.13	2727.18	0.15	2693.38	1.09
$k_6$	2723	2721.96	0.04	2720.49	0.09	2749.14	0.96	2760.01	1.36
$k_7$	2723	2722.57	0.02	2721.22	0.07	2708.91	0.52	2838.27	4.23
$k_8$	2723	2723.02	0.00	2724.37	0.05	2750.59	1.01	2822.63	3.66
$m_1$	49.48	49.4809	0.00	49.4857	0.01	49.2043	0.56	50.4199	1.90
$m_2$	49.48	49.4737	0.01	49.466	0.03	49.7395	0.52	50.4943	2.05
$m_3$	49.48	49.4669	0.03	49.4389	0.08	49.7298	0.51	50.0423	1.14
$m_4$	49.48	49.4681	0.02	49.4576	0.05	49.5674	0.18	50.4328	1.93
$m_5$	49.48	49.4696	0.02	49.446	0.07	49.7626	0.57	50.487	2.04
$m_6$	49.48	49.4713	0.02	49.4641	0.03	49.8374	0.72	50.3599	1.78
$m_7$	49.48	49.4729	0.01	49.4658	0.03	49.8352	0.72	50.0844	1.22
$m_8$	45.06	45.0531	0.02	45.0443	0.03	45.4564	0.88	45.9312	1.93
$\xi_1$	0.03	0.03031	1.02	0.03077	2.55	0.03239	7.97	0.03353	11.80
$\xi_2$	0.05	0.05008	0.15	0.05018	0.37	0.05222	4.51	0.05286	5.72

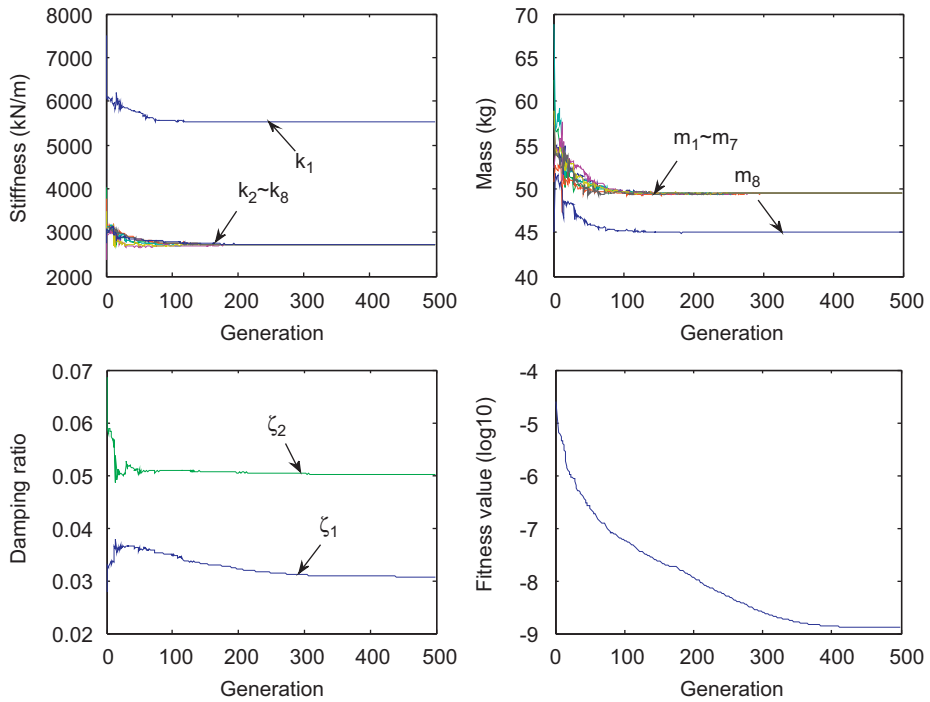
to 4.58% under 10% noise, respectively. In the partial measurement scenarios, the results of Table 2 show that the errors are slightly higher, ranging from 0% to 0.04% for 0% noise in the stiffness parameters, ranging from 0.04% to 0.57% for 10% noise, and quite higher in the damping ratio parameters, ranging from 0.22% to 1.61% for 0% noise, and ranging from 1.33% to 9.51% for 10% noise. Nevertheless, the maximum error of BB-BC in stiffness and damping of only 0.57% and 9.51% under 10% noise are very good.

In the unknown mass cases, the results of Table 3 show that the errors are slightly higher, ranging from 0% to 0.08%, 0% to 0.13% and 0.15% to 2.55% for the mass, stiffness and damping parameters, respectively, and quite higher in the noise-polluted scenarios, ranging from 0.18% to 4.48%, 0.15% to 2.05%, and 4.51% to 11.8% for the mass, stiffness and damping parameters respectively. The largest relative errors are usually observed in the damping coefficients, which are far from perfect. Due to the fact that the damping parameter has only a small contribution to the overall response, its value is generally poorly estimated. Estimation of the unknown mass systems with insufficient information and noise presents a far greater challenge compared to systems where the mass is known, for it is a highly multi-modal problem. Nevertheless, the results of Table 3 show that the maximum errors of BB-BC in mass, stiffness and damping ratio of only 2.05%, 4.48% and 11.8% in the scenario with 10% noise pollution respectively, are very good.

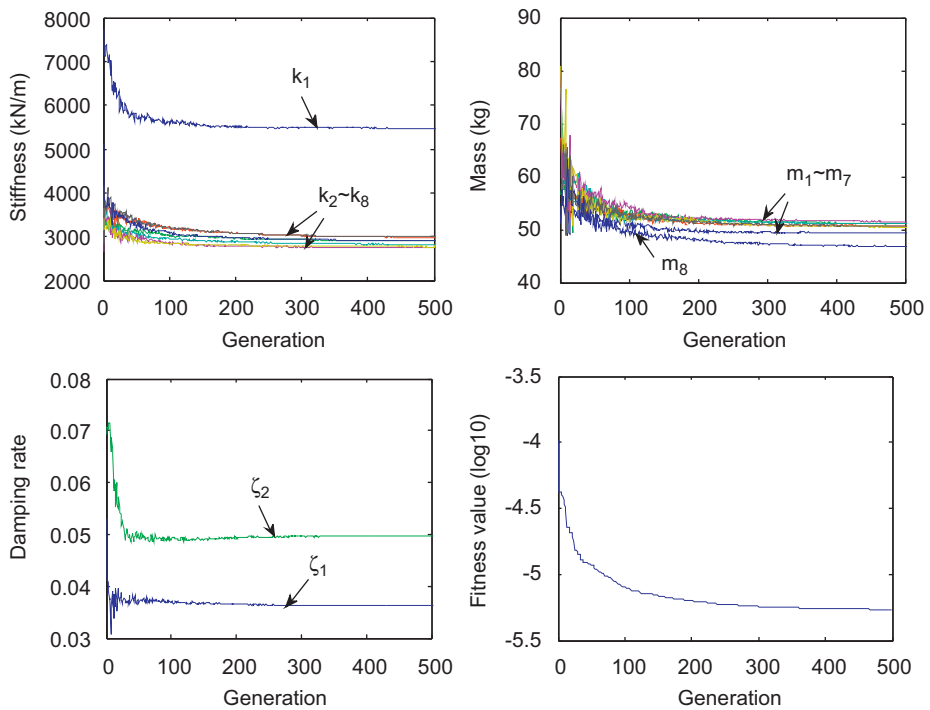
To illustrate the evolution process of the parameters, the values of all parameters and objective have been plotted in Fig. 2 and 3 for the noise-free and noise-polluted scenarios of unknown mass system, respectively. In the non-polluted case, the mass, stiffness, and damping parameters approach their respective actual values showing a quite low error after generating 200. In the noise-polluted scenario, these parameters asymptotically approach values that are close to their actual values but contain slightly higher errors. The convergence of the objective values for these problems indicates that the BB-BC optimization algorithm has a high convergence speed and good global search capability.

Secondly, in order to compare the performance of the BB-BC methodology with other evolutionary computation methods that have been suggested in the literature, such as GA and PSO, a one-dimensional shear frame-type 10-DOF structural system, with the properties given in Table 4, is analyzed. This system was used by Perry et al. [7] to test a structural system identification algorithm denominated as a modified GA, which involves a search space reduction method and a modified GA based on migration and artificial selection strategy to provide a robust and reliable identification. The PSO algorithm (described in [23]) is a biologically-inspired algorithm motivated by a social analogy. The representation of the optimization problem is similar to the encoding methods used in GAs. Instead of genes, the variables are called dimensions that create a multi-dimensional hyperspace. “Particles” fly in this hyperspace and try to find the global minima/maxima, their movement being governed by a simple mathematical equation.

In this example, similar to other studies [7], the mass, stiffness, and damping ratios are not known and have to be therefore estimated. Input forces are applied at the 5th level of the structure as random white Gaussian noise with the RMS of the force scaled to 1000 N. The input forces and noise pattern are freshly generated for each run to avoid any bias that might result from using the same inputs for all of 20 runs. Acceleration measurements are obtained at different floor levels: acceleration measurements at floors 2, 4, 7, and 10 are available in the known mass case, and acceleration measurements at floors 1, 2, 4, 6, 8, and 10 are available in the unknown mass case. The acceleration output measurements error norm is used as the fitness function. The search limits are taken as 0.5–2.0 times the exact values. The BB-BC parameters are  $\alpha=1$ ,  $\beta=0.7$ , and population size=100. In order to compare the accuracy that can be achieved in a given time, the total number of



**Fig. 2.** Typical results of the evolution of the physical parameters and convergence of the objective value for unknown mass system in noise-free corruption and partial information scenario.



**Fig. 3.** Typical results of the evolution of the physical parameters and convergence of the objective value for unknown mass system in 10% noise corruption and partial information scenario.

function evaluations is fixed for each algorithm. The total evaluations refer to the number of times the system time history simulation is carried out and is set at 20000. For different algorithms, one generation (iteration) has different number of evaluations. The results (average and maximum errors of mass, stiffness, and damping properties) obtained with the usage



**Table 4**  
10-DOF structural properties.

Stiffness (kN/m)	
Levels 1–4	5000
Levels 5–8	4000
Levels 9–10	3000
Mass (kg)	
Levels 1–5	6000
Levels 6–10	4200
Natural period of vibration (s)	
First mode	1.321
Second mode	0.505

**Table 5**  
Estimation results for known and unknown mass systems with 20,000 evaluations.

(%)	Known mass			Unknown mass					
	No noise			5% noise			10% noise		
	GA	PSO	BB-BC	GA	PSO	BB-BC	GA	PSO	BB-BC
Avg. error- <i>m</i>				1.60	1.38	1.52	2.98	2.74	2.62
Max. error- <i>m</i>				3.62	4.83	2.04	6.62	7.11	4.61
Avg. error- <i>k</i>	0.43	0.12	0.15	1.50	1.29	1.47	3.00	3.00	2.63
Max. error- <i>k</i>	1.21	0.67	0.52	3.79	3.88	2.13	6.81	6.10	3.52
Avg. error- <i>c</i>	1.56	0.84	0.59	6.29	5.92	1.82	8.41	7.96	2.83
Max. error- <i>c</i>	2.76	2.01	1.76	11.43	8.52	3.14	15.29	10.7	4.81

**Table 6**  
Estimation results of BB-BC with 10,000, 30,000, and 50,000 evaluations.

Number of evaluations (%)	Known mass			Unknown mass					
	No noise			5% noise			10% noise		
	10,000	30,000	50,000	10,000	30,000	50,000	10,000	30,000	50,000
Avg. error- <i>m</i>				2.63	1.44	1.32	3.55	2.61	2.61
Max. error- <i>m</i>				2.94	1.96	1.94	4.72	4.56	4.55
Avg. error- <i>k</i>	0.19	0.15	0.12	2.86	1.42	1.3	3.48	2.26	2.25
Max. error- <i>k</i>	0.62	0.48	0.45	3.02	1.86	1.73	3.86	3.34	3.33
Avg. error- <i>c</i>	0.87	0.55	0.53	2.06	1.42	1.28	3.01	2.79	2.79
Max. error- <i>c</i>	2.25	1.74	1.73	3.63	2.32	2.22	5.12	4.72	4.72

of the BB-BC are presented in Table 5, along with the results obtained with PSO and modified GA [7] for the sake of comparison. In order to assess the rate of convergence of the BB-BC method, 10000, 30000, and 50000 evaluations are also considered and the results of the parametric identification are summarized in Table 6. All identified results presented in Tables 5 are average results over 25 runs.

As shown in Table 5, it is clear that the estimation results of the BB-BC show small errors for all cases. This is seen in the results where the maximum error is only 1.76% and 4.81% for the noise-free and noise-polluted cases, respectively. The largest relative errors are usually observed in the damping coefficients. In the noise-free (known mass) case, results are comparable and the relative errors obtained in the parameter estimation are quite similar. In the noise-polluted case, average relative errors show that BB-BC seems to perform well compared to the modified GA as well as the PSO algorithm. The results show that the BB-BC can give better results than the other method in the same time. Carrying out the identification procedure for a larger number of evaluations can help to reduce the errors. However, it was shown in Table 6 that the existence of noise does not let the parameters approach a zero-error plateau, even after 50,000 evaluations. In addition, a typical BB-BC search performance with 500 maximum generations for the noise-free scenario of known mass system is provided in Fig. 4, and the unknown mass system with 10% noise scenario is provided in Fig. 5. From these figures, it can be seen that the mass, stiffness, and damping parameters asymptotically approach their respective actual values showing a quite low error after generating 150, and the BB-BC converges to the optimum at an exponentially



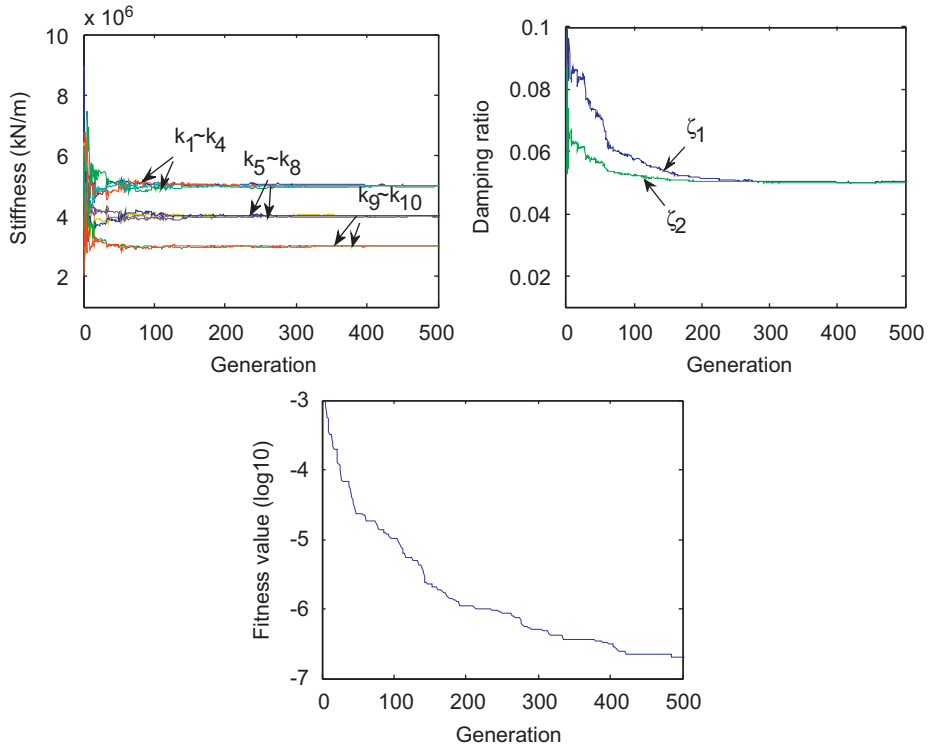


Fig. 4. Typical results of the evolution of the physical parameters and convergence of the objective value for known mass systems in noise-free scenario.

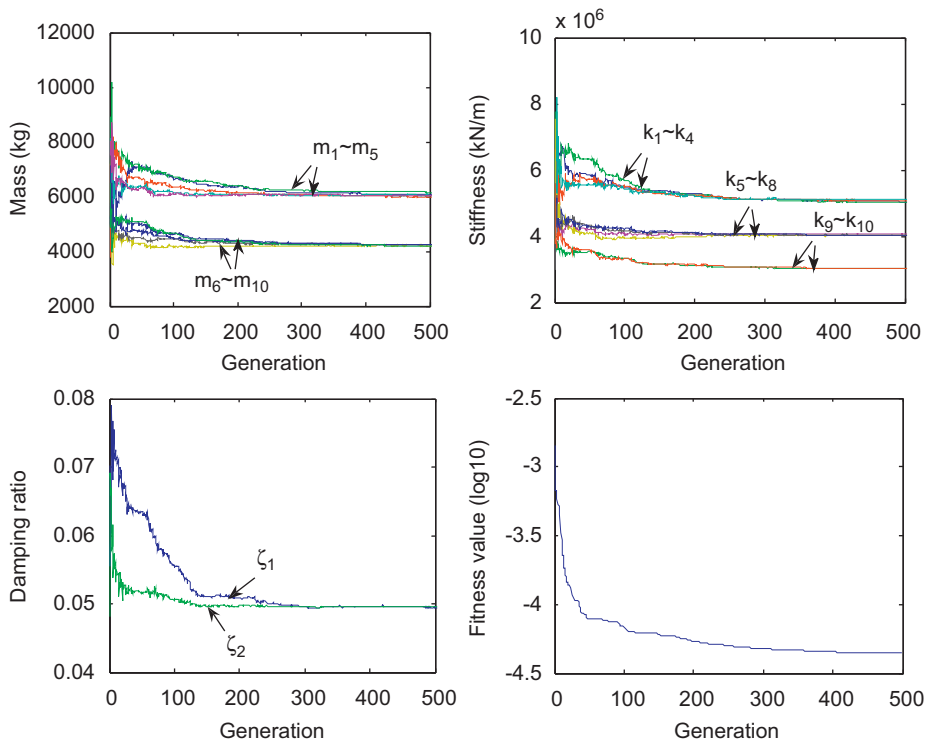


Fig. 5. Typical results of the evolution of the physical parameters and convergence of the objective value for unknown mass systems in 10% noise corruption scenario.

progressing rate. In general, for all cases studied, the results (mean and maximum errors) obtained by BB-BC are competitive with, or sometimes better than, those obtained with GA and PSO. The solutions obtained show even smaller errors, achieving a practically perfect identification in all cases. GA, PSO, and BB-BC are all population-based algorithms. Thus, they are much better suited to the complex search spaces. However, these three algorithms have their own strengths and weaknesses. GA usually suffers a lot from convergence speed and execution time. BB-BC owns mechanism of group interaction that enhances the search for an optimal solution by contraction factor between the best solution and the mass center. It has been reported [19] that BB-BC method finds the optimum solution in finite steps, while the amelioration is infinite as the steps increase, and ultimately leads to exact solution. A rigorous study of the influence of the initialization of the individuals is not intended in this paper. However, several tests were made using different initializations. In all cases, convergence to the right optimum was obtained without problems for BB-BC. It is able to reproduce the same results consistently over many trials, whereas the performance of PSO is far more dependent on the randomized initialization of the individuals. As a result, PSO must be executed several times to ensure good results. Both BB-BC and PSO algorithms are conceptually simple and can be implemented in a few lines of code. But BB-BC has fewer parameters (only two parameters) to set, and the same setting can be used for many different problems, which make the utilization of BB-BC simple and high efficiency.

## 5. Conclusions

This paper has presented a new technique using the BB-BC optimization method for parameter estimation of structural systems. Comparative studies have been investigated to assess the applicability of the BB-BC for structural parameter estimation. It is clear from the results that the proposed method can obtain higher quality solutions with better computation efficiency than the GA and PSO methods. The numerical simulation results show that a satisfactory optimal performance can be achieved by the proposed method even when all system's properties are unknown, and the simulated data are incomplete and corrupted by noise.

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