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Article *in* Advances in Structural Engineering · April 2016

DOI: 10.1177/1369433216643250

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Article

Advances in Structural Engineering

Advances in Structural Engineering
1–17

DOI: 10.1177/1369433216643250
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Multi-objective differential evolution for truss design optimization with epistemic uncertainty

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Abstract

A robust multi-objective optimization method for truss optimum design is presented. In the robust design, materials and loads are assumed to be affected by epistemic uncertainties (imprecise or lack of knowledge). Uncertainty quantification using evidence theory in optimum design subject to epistemic uncertainty is undertaken. In addition to a functional objective, an evidence-based plausibility measure of failure of constraint satisfaction is minimized to formulate the robust design into a multi-objective optimization problem. In order to alleviate the computational difficulties in the evidence theory-based uncertainty quantification analysis, a combined strategy of differential evolution-based interval optimization method and parallel computing technique is proposed. A population-based multi-objective differential evolution optimization algorithm is designed for searching robust Pareto front. Two truss structures with shape and sizing optimum design problems are presented to demonstrate the effectiveness and applicability of the proposed method.

Keywords

differential evolution, epistemic uncertainty, evidence theory, multi-objective optimization, uncertainty quantification

Introduction

Many real-world engineering design problems create lots of uncertainties in analysis simulations or manufacturing process. These uncertainties may arise from the inherent variability (also referred to as variability, irreducible uncertainty or aleatoric uncertainty) of a physical system, such as uncertain geometric parameters and operating conditions, or from a lack of knowledge (also called reducible uncertainty, incertitude uncertainty, or epistemic uncertainty), such as those due to unknown physical phenomena (Oberkampf et al., 2001). In real-life applications, both kinds of uncertainties are often present. Most optimization algorithms treat them the same, while there is a distinct difference between both types. The quantification for the aleatoric uncertainties is relatively straightforward to perform. Techniques such as Monte Carlo method are frequently used. A probability distribution can be represented by its moments, or more recently, by techniques such as Karhunen–Loève (Perrin et al., 2013) and polynomial chaos expansions (Schoefs et al., 2011). In comparison to the quantification of aleatory uncertainty, the quantification of epistemic uncertainty has proved more challenging. Epistemic uncertainty is not well characterized by probabilistic approaches, because it might be difficult to infer any statistics for lack of knowledge.

Currently, reliability-based design optimization (RBDO) and robust design optimization (RDO) represent two major paradigms for design uncertainty. RBDO is to optimize the design which is reliable with small chance of failure under an acceptable level (Deshpande et al., 2010; Mathakari et al., 2007), whereas RDO is to optimize design that makes the system performance insensitive to various variations (Beyer and Sendhoff, 2007). Moreover, with the requirement of reliability and robustness, RDO and RBDO can be combined into a hybrid single-objective algorithm named reliability-based robust design optimization (RRDO), which searches for robust optima while obeying reliability type of constraints (Tang et al., 2012; Yao et al., 2011). Recently, as opposed to the single optimal result obtained by the single-

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objective version, a multi-objective optimization design which maintains control over structural reliability through the inherited RBDO constraint, but upgrades robustness to being pursued through an optimization objective (together with structural cost or weight) has been widely studied in the literature (Mourelatos and Liang, 2006), which produces a set of trade-off optimal structural design between quality and robust for designer to make decision. Therefore, this multi-objective strategy for robust design is studied in this work. It is noted that the most important step in uncertainty-based design optimization is to quantify the uncertainty. In these methods, the uncertainty is generally characterized using probability theory. As discussed above, probability theory is not a good choice to quantify the epistemic uncertainty. In such a case, the usual probabilistic methodologies cannot be used in the optimization design problem involving epistemic uncertainties and an alternative approach which can effectively quantify the epistemic uncertainties and include them in the optimization procedures to meet the prescribed reliability and robustness levels is required.

In recent years, some of the promising non-probabilistic theories have been developed to handle the uncertainty, such as possibility theory (Dubois et al., 1988; Moller et al., 1999; Zhou and Mourelatos, 2008), convex interval analysis (Du, 2012; Majumder and Rao, 2009), fuzzy set theory (Jahani et al., 2014), and evidence theory (Dempster, 1967; Hu and Luo, 2015; Shafer, 1976). A comparative study conducted by Klir and Smith (2001) and Bae et al. (2004) indicated that evidence theory is more general and flexible than the traditional probability and possibility theories, and it is also more intuitive and reasonable of using belief and plausibility (lower and upper probability bounds) to measure the likelihood of events instead of a single probabilistic value when the given information is not precise. Thus, several researchers employed evidence theory to quantify the uncertainty and formulate RBDO in many areas. Tonon et al. (2000) adopted evidence theory to quantify the parameter uncertainty in rock engineering and then carried out a reliability-based design of tunnels. Bae et al. (2003) applied evidence theory to handle the imprecise data situation and whereby formulate evidence theory-based design optimization (EBDO) of an aircraft wing structure. Agarwal et al. (2004) suggested using evidence theory to perform a design optimization for multi-disciplinary systems. Mourelatos and Zhou (2006) implemented the evidence theory to resolve the design optimization of pressure vessel under epistemic uncertainty. Alyanak et al. (2008) adopted a gradient projection technique to conduct EBDO for aircraft wing structures. Huang et al. (2013) developed possibility and evidence-based reliability analysis and design optimization. Meanwhile, the RDO using evidence theory was also formulated by many researchers. Vasile (2005) and Croisard et al. (2010) applied evidence theory to model uncertainty rooted in the preliminary design of space missions and translated this design into a multi-objective problem of maximizing the belief together with optimizing the mission goals. Zuiani et al. (2012) developed evidence-based robust design of deflection actions for near-earth objects. Hou et al. (2013) presented a robust optimization strategy for micro Mars entry probe design, in which evidence theory is used to quantify the uncertainties. (Srivastava and Deb, 2011; Srivastava et al., 2013) proposed a bi-objective approach for pressure vessel design optimization using evidence theory, considering a minimized plausibility of failure as an additional objective.

Despite the fact that evidence theory can handle epistemic uncertainty effectively, the use of evidence theory has barely been explored in engineering application. One of the major difficulties may be its high computational time. Unlike in probability theory, uncertainty variable is represented by many discontinuous sets instead of a smooth and continuous explicit probability density function, the belief or plausibility calculation need propagate uncertainty through a discrete basic belief assignment (BBA) structure, which involves the evaluation of the maximum and minimum values of the limit state function over all possible set. In order to overcome the difficulty of intensive computational cost, some promising techniques have been developed. Because most of the structural designs involve repeating patterns in their analytical finite element model (FEM) of structures, it is becoming a common practice to use surrogate model instead of FEMs in most of the structural optimization problems. Bae et al. (2004) created a multi-point approximation at a certain point on the limit state surface to improve the efficiency of reliability analysis. Agarwal et al. (2004) proposed an efficient reliability analysis through a sequential approximate strategy. Mourelatos and Zhou (2006) used local surrogate models of the active constraints to keep computational cost low. Bai et al. (2012) compared three meta-modeling techniques for evidence theory-based reliability analysis. Salehghaffari et al. (2012b) used the radial basis function (RBF) meta-modeling approach and Latin hypercube sampling (LHS) to reduce the computational burden of uncertainty propagation and EBDO problem. Hou et al. (2013) employed artificial neural network (ANN)-based surrogate model to simulate the actual limit state surface. Zhang et al. (2014) proposed a high precise response surface approach to compute the actual structural response. Mukhopadhyay et al. (2015) proposed an efficient surrogate model-based random sampling-high dimensional model representation (RS-HDMR) to replace the actual finite element; Yao et al. (2013) used the sequential optimization and mixed uncertainty analysis (SOMUA) to decompose the traditional nested optimization into less consumed problem and applied RBF neural network to approximate the analysis function. Although the surrogate model is an effective alternative to the analytical FEM of structures, it is difficult to carry out and may bring error into design results for the complex structures. Jiang et al. (2013) introduced a non-probabilistic reliability index approach to reduce the number of focal elements with extreme analysis; Guo et al. (2008) and Bai et al. (2013) proposed the interval analysis technique to reduce the computation number of the limit state function. Su et al. (2012) proposed a differential evolution (DE) algorithm (Storn and Price, 1995, 1997) based interval optimization method for computing bounds in mixed aleatory–epistemic uncertainty quantification (UQ). Srivastava et al. (2013) used graphical processing unit (GPU) parallel computing to speed up the EBDO, which is a preferable alternative to solve the computational difficulties without sacrificing the accuracy.

In the past decades, a large number of nature-inspired computing algorithms have been proposed to address optimization problems. Among them, some meta-heuristic search algorithms with population-based framework have shown satisfactory capabilities to handle high-dimensional optimization problems, such as genetic algorithm (GA) (Adeli and Cheng, 1993, 1994a, 1994b; Lin and Ku, 2014; Sarma and Adeli, 2000), evolutionary algorithms (EAs) (Back, 1996; Bartz-Beielstein et al., 2014; Zhao et al., 2008), simulated annealing (SA) (Locatelli, 2000; Miettinen et al., 2006), artificial immune system (AIS) (Farmer et al., 1986), ant colony optimization (ACO) (Putha et al., 2012), particle swarm optimization (PSO) (Plevris and Papadrakakis, 2011; Shi and Eberhart, 1998), and DE (Storn and Price, 1995, 1997).

As a novel evolutionary computation technique, DE has gained much attention and wide applications for solving complex optimization problems in the recent years. DE proposed by Storn and Price (1995) is a stochastic parallel direct search evolution strategy optimization method that is fairly fast and reasonably robust. These are precisely the characteristics of DE that makes it attractive to extend it to solve multi-objective optimization problems. Several researchers have studied the extension of DE to solve multi-objective optimization problems without requirement of any gradient and achieved good results (Abbass, 2002; Abbass et al., 2001; Adeyemo and Otieno, 2009; Ali et al., 2012; Madavan, 2002; Robič and Filipič, 2005; Xue et al., 2003). Naturally, multi-objective optimization based on DE developed by Robič and Filipič (2005) is appropriate to be used in evidence-based bi-objective problem.

In this work, due to the limitation of sufficient information requirement in probability-based robust optimization, an evidence theory-based robust design for epistemic uncertainty in civil engineering is proposed and is formulated into multi-objective optimization design, the optimization objective is to minimize the weight of structure for economical purposes, while at the same time minimize the maximum of failure plausibility for considering structural reliability and robust. A differential evolution for multi-objective optimization (DEMO) method for this multi-objective optimum design involving epistemic uncertainties is developed. In order to alleviate the computational difficulties in the evidence theory-based UQ analysis, a combined strategy of DE-based interval optimization method with parallelization technique is proposed to enhance the computational efficiency for calculating failure plausibility.

The article is organized as follows. Section "Fundamentals of evidence theory" describes the fundamentals of evidence theory. The evidence theory-based multi-objective RDO problem and parallel computing technique are described in section "Evidence-based multi-objective design optimization," while the next section deals with the implementation of the robust design problem using DEMO. Numerical simulation of truss structures with shape and sizing optimum design under uncertainty is given in section "Examples," followed by concluding remarks in section "Conclusion."

Fundamentals of evidence theory

Evidence theory, also called belief functions theory, is a theoretical frame work for reasoning with partial and unreliable information. It was first introduced by Dempster (1967). Later, Shafer (1976) developed Dempster's work and established the basis of evidence theory. In particular, it offers the possibility to explicitly represent doubt and conflict. We give here some of the basic notions of the theory and refer the reader to Helton et al. (2007) for details.

Evidence theory starts by defining a Frame of Discernment Ω that indicates "a set of mutually exclusive elementary propositions." In many engineering applications of the evidence theory, an uncertain parameter u is expressed by various intervals, these intervals can be scattered, nested, or partially overlapped with the distribution of the obtained data. In this case, $u \in [a, b]$ is an elementary proposition; thus, an element of the power set is 2^{Ω} (all the possible subset propositions of Ω). The degree of confidence in a particular proposition of 2^{Ω} is quantified by the corresponding BBA function m that satisfies the three axioms $m(\phi) = 0$, $m(E) \neq 0$ for $E \in 2^{\Omega}$, and $\sum_{E \in 2^{\Omega}} m(E) = 1$. An element $E \in 2^{\Omega}$ for which $m(E) > 0$ is named as a focal element.

When multiple parameters are considered uncertain, the joint frame of discernment is composed of the Cartesian products of uncertain intervals, and then the BBA value of each element of joint frame of discernment is also the Cartesian product of BBA value assigned on the corresponding interval. For example, only two uncertain parameters u_1 and u_2 are given, and corresponding intervals are located in $[a_1, b_1]$ and $[a_2, b_2]$; then, the BBA structure of this problem is defined as

$$\begin{aligned} m(u_1, u_2) &\in [a_1, b_1] \times [a_2, b_2] \\ &= m(u_1 \in [a_1, b_1]) * m(u_2 \in [a_2, b_2]) \end{aligned} \tag{1}$$

Due to a fundamental lack of information to construct precise portable document format (PDF), it seems more reasonable to make use only of this available information to produce two uncertain measures, the Belief (Bel) and the Plausibility (Pl) functions (Figure 1) that represent the lower and upper probability of an event to happen, as opposed to a single value in traditional probability theory, these two measures are defined as follows

$$\mathrm{Bel}(A) = \sum_{E \subseteq \mathrm{A}} m(E) \quad \text{for all } E \subseteq \Omega \tag{2}$$

$$\mathrm{Pl}(A) = \sum_{E \cap A \neq \phi} m(E) \quad \text{for all } E \subseteq \Omega \tag{3}$$

Uncertainty is measured as the gap between plausibility and belief with probability of proposition A bounded as $\mathrm{Bel}(A) \leqslant \mathrm{P}(A) \leqslant \mathrm{Pl}(A)$. $\mathrm{Pl}(A)$ represents the maximal degree of belief supporting the subset A. It is important to note that Pl boils down to a probability measure when m is a Bayesian BBA and to a possibility measure when the focal elements are nested. Probability and possibility measures are thus recovered as special cases of belief functions.

Sometimes, the available evidence can come from different sources. Such bodies of evidences can be aggregated using the rule of combination. Dempster's rule of combination is one of the most popular rules of combination used. It is given by the following formula

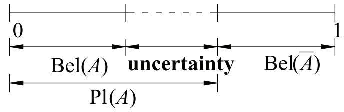

Figure 1. Belief (Bel) and Plausibility (Pl).

$$m(B) = \frac{\sum_{A \cap C = B} m_1(A) m_2(C)}{1 - \sum_{A \cap C = \phi} m_1(A) m_2(C)} \quad \text{for all } B \neq \phi \tag{4}$$

where $\sum_{A \cap C = \phi} m_1(A) m_2(C)$ can be viewed as contradiction or conflict among the information given by the independent knowledge sources. Some improved rules of combination are discount and combine method, Yager's modified Dempster's rule, Inagaki's unified combination rule, Zhang's center combination rule, Dubois and Prade's disjunctive consensus rule, and mixing or averaging rule. More detail about the rule of combination can be found in Sentz and Ferson (2002).

Evidence-based multi-objective design optimization

Formulation of the evidence theory-based multi-objective design optimization

For a generic RBDO problem of a structure, the general formulation is defined as follows

$$\begin{aligned} &\underset{\boldsymbol{d}, \boldsymbol{u}^N}{\text{minimize}} \ f(\boldsymbol{d}, \boldsymbol{u}^N) \\ &\underset{\boldsymbol{d}, \boldsymbol{u}}{\text{maximize}} \ R_j = \Pr[g_j(\boldsymbol{d}, \boldsymbol{u}) \geqslant 0] \quad j = 1, \ldots, J \end{aligned} \tag{5}$$

where f indicates the objective function related to economy, while R reflects the reliability of design by means of calculating the probability of satisfying all J constraints. $\boldsymbol{d}$ is the vector of deterministic design variables, $\boldsymbol{u}$ is the vector of uncertain parameters, which is characterized probabilistically, and $\boldsymbol{u}^{\mathbf{N}}$ denotes the nominal value of uncertain parameter. $g()$ is the limit state function of structure, $g \geqslant 0$ indicates that structure is safe.

As stated in above formulation, the complete probabilistic information about uncertainty parameter must be accurately known in the traditional RBDO, Unfortunately, in practical engineering, it is almost cost-prohibitive to obtain sufficient information or precise knowledge for constructing the probability distribution of uncertainty parameters; in this case, the above RBDO formulation is not preferable and therefore, evidence theory is proposed to achieve this design optimization under such uncertainty.

Usually, the failure domain for each constraint is much smaller than the safe domain over the joint frame of discernment at the optimum. As a result, the calculation of the plausibility of failure will be much more efficient than the belief calculation in safe region, and hence, Mourelatos and Zhou (2006) suggested that the plausibility measure $\mathrm{Pl}(g < 0)$ is preferred instead of using the equivalent belief measure $\mathrm{Bel}(g \geqslant 0)$. Then, the design problem discussed in equation (5) can be reformulated to be an EBDO as follows

$$\begin{aligned}&\underset{\boldsymbol{d},\boldsymbol{u}^N}{\text{minimize}}\ f(\boldsymbol{d},\boldsymbol{u}^N)\\&\underset{\boldsymbol{d},\boldsymbol{u}}{\text{minimize}}\ \text{Pl}_{\max}=\max_{j=1}^{J}(\text{Pl}[g_j(\boldsymbol{d},\boldsymbol{u})<0])\end{aligned} \tag{6}$$

It should be noted that the robust optimization (equation (6)) requires a nested optimization loop; the UQ is nested within the bi-objective optimization outside loop. More specifically, the purpose of UQ is to calculate the plausibility of failure $\text{Pl}_{\max}$, and evidently, it is a computation bottle-neck in practical implementation of EBDO. Therefore, an efficient approach which makes plausibility calculation less-consuming is needed.

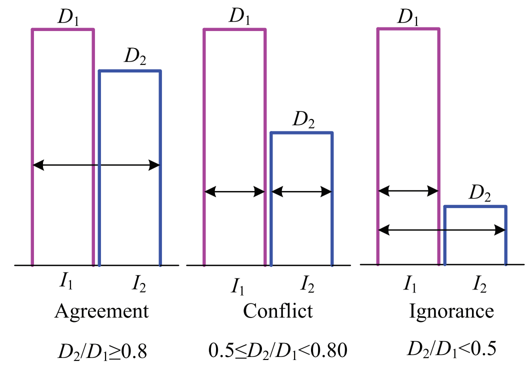

Figure 2. Three relationships of uncertain intervals.

Calculation of plausibility

The calculation of plausibility of failure involves three necessary steps: uncertainty representation, propagation, and measurement. In this section, the three steps are briefly described.

Uncertainty representation. For the purpose of plausibility calculation, the first step is the uncertainty representation of parameters using evidence theory, in which separate belief structures for each uncertain parameter should be constructed. In this work, we adopt a general methodology as described previously by (Salehghaffari and Rais-Rohani, 2012; Salehghaffari et al., 2012a) to obtain necessary information from available data and express the uncertain variables in the mathematical framework of evidence theory.

According to Salehghaffari et al. (2012a), two principle steps are involved in this methodology: (1) representation of uncertain parameters in several intervals through drawing bar charts using all available data or directly from expert opinions and (2) identification of three relationships between all adjacent intervals and construction of the associated BBA structure. To further illustrate this, assuming that D_1 and D_2 represent the number of data points within two adjacent intervals I_1 and I_2, respectively, and $D_1 > D_2$, three relationships of two adjacent intervals can be identified as agreement ($D_2/D_1 \geqslant 0.8$), conflict ($0.5 \leqslant D_2/D_1 < 0.8$), as well as ignorance ($D_2/D_1 < 0.5$) (Figure 2), the corresponding belief structure and BBA value for these three relationships are calculated by equations (7) to (9), respectively.

$$m(\{I\}=\{I_1,I_2\})=\frac{D_1+D_2}{D_T} \tag{7}$$

$$m(\{I_1\})=\frac{D_1}{D_T},\quad m(\{I_2\})=\frac{D_2}{D_T} \tag{8}$$

$$m(\{I_1\})=\frac{D_1}{D_T},\quad m(\{I_1,I_2\})=\frac{D_2}{D_T} \tag{9}$$

where D_T denotes the total number of data points; following this approach, a reasonable BBA structure of uncertain parameter is constructed based on available data and knowledge, and a more detailed illustration of uncertainty representation in intervals with assigned BBA value is referred in Salehghaffari et al. (2012a).

Uncertainty propagation. As discussed above, uncertain parameters are represented by multiple intervals instead of common explicit function; hence, uncertainty propagation is required to determine BBA structure of system response, that is to say the maximum and minimum responses for each joint proposition of uncertain parameters at a design point are needed to calculate.

$$\begin{aligned}&[g_{i,\min},g_{i,\max}]=[\min g(u_i),\ \max g(u_i)]\\&\text{subject to } \underline{u}_i \leqslant u_i \leqslant \bar{u}_i \quad i=1,2,\ldots,m\end{aligned} \tag{10}$$

where u_i is the ith element of the joint frame of discernment, m is the number of all the possible joint focal elements; it can be inferred from equations (3) and (6) that the minimum value of the constraints $g()$ is enough to identify whether the joint focal element contributes to the calculation of plausibility of failure, so we just need to search for minimum value of the constraints $g()$ at the process of uncertainty propagation. However, the computations of problem (equation (10)) are still cost-prohibitive, due to the fact that m is increased exponentially with respect to the number of uncertain parameters in complex structural system. To alleviate this computational burden, a DE-based computational strategy for the propagation representation of epistemic uncertainty combined parallelization technique is proposed herein. We will illustrate that, using Figure 3, only one uncertain parameter is considered.

Parallel technique, in the literal sense, is a program of converting original sequential task into parallel task

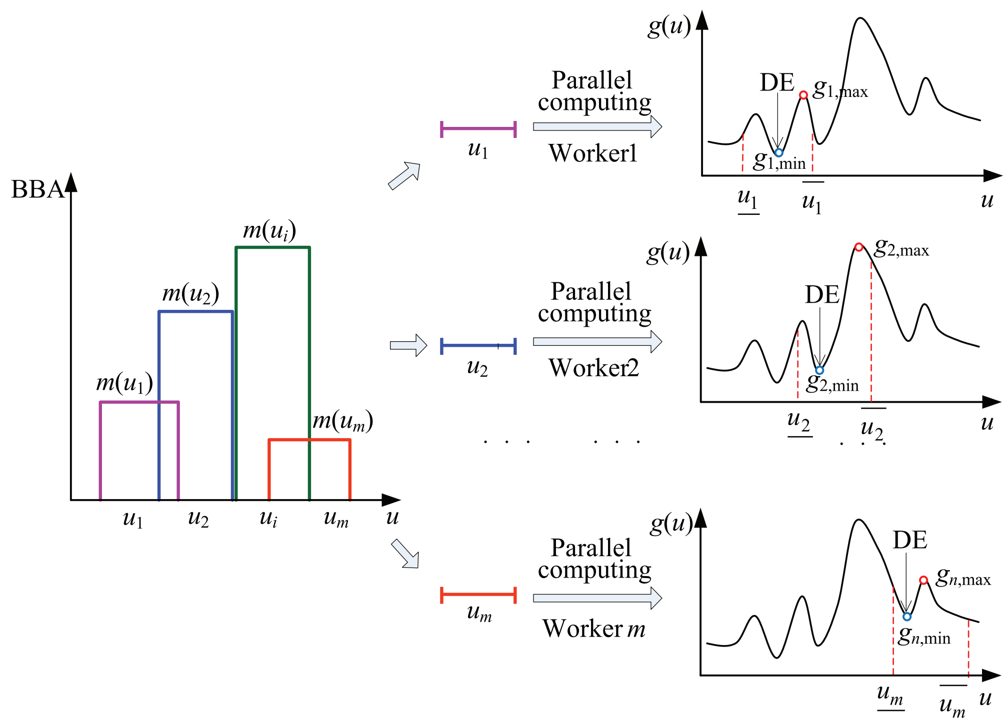

Figure 3. Uncertainty propagation using parallel computing and DE algorithm.

by means of distributing task to be computed in several workers, this will make CPU utilized effectively during the parallel computation and thus, its computational time will be saved significantly. It can be seen from Figure 3 that the evaluation of the structural response bound among different input intervals can be done independently and simultaneously. Therefore, it is suitable to adopt the parallel computation to distribute high time-consuming evaluation in several workers. Furthermore, the strategy for parallelization is also used in other two aspects: each design point can be evaluated in parallel and each constraint can be evaluated in parallel.

Based on above inspiration of the parallel strategy to reduce the computation time, the parallel computing toolbox exploited by MathWorks is introduced in this article. The parallel computing toolbox can solve computationally and data-intensive problems on multiple processors with local workers (with for-loops), take advantage of Graphical Processing Units (GPUs), and scale up to a cluster (with MATLAB Distributed Computing Server). As for the task–parallel problem in Figure 3, the function of parallel for-loops which execute code loop in parallel and require few program modifications is a well-suited framework to convert the original plausibility calculation to parallel calculation. The other features will be applied in future work.

In reality, the parfor loop is an implementation of the master–worker pattern. In an interactive session, the client MATLAB acts as the master, while the MATLAB worker processes on the cluster receive work from the master. A rough schematic of the process for a simple parfor loop is shown in Figure 4 assuming that $m = 10$ and four available workers are provided. Depending on the availability of workers, the plausibility calculation may be divided differently, each worker picks up different pieces of focal elements as shown in Figure 4. As a result, the original task is converted from serial MATLAB applications to parallel MATLAB applications with few code modifications. More detailed information about the mechanics of parfor loop can be found in Sharma and Martin (2009).

After implementing the parallel technique, a subtask of finding the minimum bound values of the limit state function in each worker should be undertaken. There are many methods for solving this minimized problem such as the sampling method, the vertex method, and the optimization method. Unfortunately, the sampling method or the traditional optimization method might be infeasible in complex structure, and the result of the vertex method is valid only for convex system. In this work, the global DE optimization method for computing minimum bound is presented. DE initially is proposed by Storn and Price (1997) for optimization problems over continuous spaces. In recent years, it has emerged as one of the simple and efficient techniques for solving global optimization

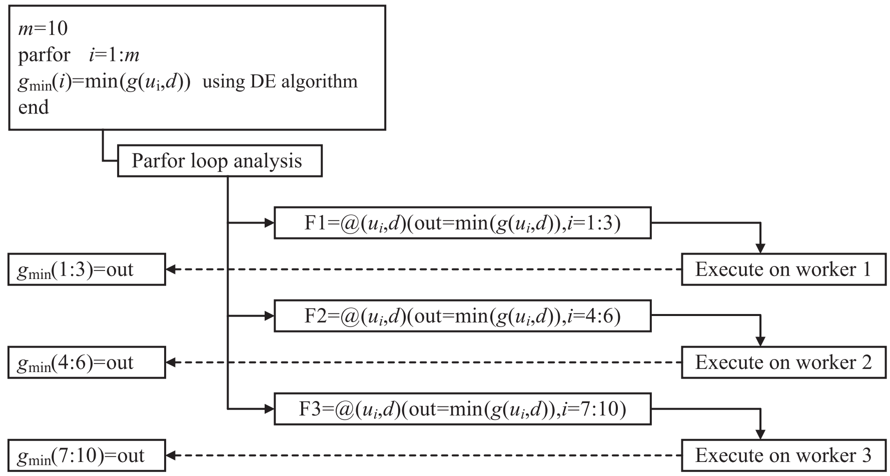

Figure 4. Schematic of task parallelism using parfor.

problems with advantage of great robustness and fast convergence (Arya and Choube, 2013; Huang et al., 2007). Also, the characteristics of derivative-free and capability of handling discrete plausibility values make DE method to be a good choice for such an interval bound subtask (Su et al., 2012). The procedure of uncertainty propagation using the DE strategy combined with parallel technique is as follows:

Step 1: For multiple uncertain parameters, the BBA structure of the m-dimensional joint frame of discernment including the all possible joint intervals is constructed according to equation (1).
Step 2: CPU parallelize technique is introduced to distribute the bound calculation over all joint focal element into several workers, the number of workers is dependent on the computer's hardware.
Step 3: At each worker, global DE algorithm is adopted to complete the subtask of computing the minimum value of the constraint function in each joint interval which is distributed in each worker, this procedure is repeated until all the workers are evaluated.
Step 4: Finally, the minimum values obtained by parallel technique are assembled together to form the belief structure of the constraint function.

Uncertainty measurement for plausibility calculation. Once the BBA structure of limit state function is constructed, the plausibility of failure is calculated as follows

$$\begin{aligned} \mathrm{Pl}(g \leqslant 0) &= \sum\nolimits_{u \cap U_f \neq \phi} m(u) \\ U_f &= \{u_f : g(d, u_f) \leqslant 0\} \end{aligned} \tag{11}$$

where $u \cap U_f \neq \phi$ means that the joint focal element u can be entirely or partially within the failure domain $g() < 0$. That is to say if the minimum value of the limit state function in a joint focal element is negative, then this focal element will contribute to the calculation of plausibility. Otherwise, it will not contribute to the calculation of plausibility, then the BBA values of all the focal element identified in failure region are summed up to obtain plausibility of failure. After that, the maximum value of plausibility of failure over all constraints is selected as the second objective to be minimized according to equation (6).

Implementation of the EBDO using DEMO

DEMO methodology is an extension version of the DE that combines the basic operation of DE to solve multi-objective optimization problems. DE is a stochastic population-based search method, proposed by Storn and Price (1997) for solving non-linear, high-dimensional and complex computational optimization problems. As a novel evolutionary computation technique, DE resembles the structure of an EA but differs from traditional EAs in its generation of new candidate solutions and by its use of a "greedy" selection scheme. Moreover, to enable diversity of solution, the mutation and crossover operation are used, similar with GA with setting variants and crossover constant. The characteristics together with other factors of DE make it a fast and robust algorithm as an alternative to EA.

DEMO combines the DE's advantages with the superior mechanisms of Pareto-dominate ranking and crowding distance sorting, this makes DEMO to have the powerful ability of finding a widely distributed trade-off optimal set with simple operation; thus, it is preferred to introduce DEMO to solve the EBDO (equation (6)) as mentioned in section "Evidence-based multi-objective design optimization." The main procedure of DEMO includes initialization of population, Pareto-dominance selection, non-dominate sorting, crowding distance computation and sorting, performing DE operations, and reiterating the search to close the true Pareto-optimal solutions. In this section, the DEMO approach is briefly described. A detailed survey of the DEMO family of algorithms can be found in Robič and Filipič (2005).

First, some DEMO parameters are required to input such as crossover constant, maximum generation, bound constraints, and so on, and an initial NP-sized population signifying design variables is generated randomly within its bound. By performing mutation and crossover operations on the current population also called parent, another NP member called candidate is created. Then evaluating two populations and using the concept of dominance to select whether the candidates replace the parent or not, if the candidate dominates the parent, the candidate is put into new population, otherwise, the candidate is discarded. This step is repeated until all NP number of population is executed and a population of the size between NP and 2·NP is obtained.

After that, the process of non-dominated sorting is used to assign rank to the selected population and the crowding distance metric with the same rank is needed to be estimated. According to the above two operators, the individual with higher rank and the smaller distance is rejected; this truncation maintains only the best NP individual into the next generation. If the termination criterion is satisfied, the current non-dominated solution set is considered to be optimal; otherwise, go back to generate the new design variables by mutation and crossover operations and proceed the optimization, the more detailed explanation about non-dominated ranking and crowding distance metric can be found in Deb et al. (2000). Consequently, a set of trade-off optimal solution, close to true Pareto front about discrete plausibility and other performance objectives, can be obtained to instruct decision-makers to decide which solution is selected in the final design according to their requirement or preference.

The procedure of DEMO to solve the evidence-based multi-objective optimization design is shown in Figure 5.

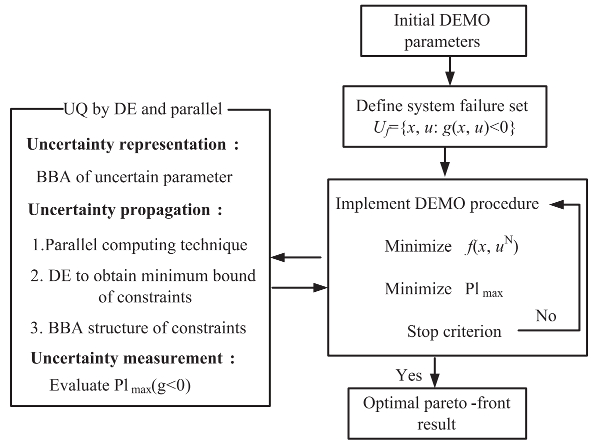

Figure 5. Implementation of DEMO to optimization design under epistemic uncertainty.

Examples

Optimization of truss and frame structures is a popular topic in mechanical, civil, and structural engineering due to the complexity of problems and benefits to industry. In this study, two typical truss design including size and shape optimizations under epistemic uncertainty are presented to demonstrate the proposed evidence-based multi-objective optimization design. The two objectives of these examples are to minimize the structural weight and plausibility of failure. Moreover, comparisons are made with probability-based reliable design and deterministic design results.

10-bar planar truss

The 10-bar plane truss with the node and element numbering shown in Figure 5 is one of the most classical optimization design problems. This problem has previously been presented as a deterministic problem (Soh and Yang, 2000; Yang and Soh, 1995; Zheng et al., 2006). The detailed information are as follows: two vertical loads of 444.5 kN are applied on nodes 2 and 4 simultaneously, the modulus of elasticity E is 206.7 GPa and material density ρ is 2.768 g/cm^3, the maximum allowable tensile and compressive stress of all member are 172.25 MPa, and the maximum allowable displacement at node 2 is 50.8 mm. The variable of sizing is defined by the cross-sectional areas of each member, and only nodes 1 and 3 are permitted to move. Thus, there are 14 independent design variables which include 10 sizing variables and 4 configuration variables. The bound on the sizing variables is 45.2–6451.6 mm^2, and side constraints for configuration variables are 10,160 mm $< x_1 <$ 20,320 mm and 5080 mm $< x_3, y_1, y_3 <$ 12,700 mm.

In this example, the uncertainties are assumed to exist in the load (P_1, P_2) and modulus of elasticity (E). Due to cost-prohibitive for obtaining the sufficient data information in practical engineering, only 50 samples are considered in this example. According to this limited available data, we use the methodology as explained in section "Uncertainty representation" to

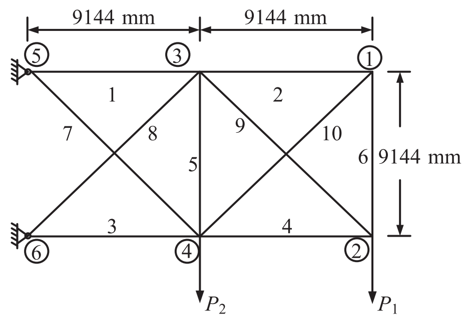

Figure 6. 10-bar planar truss structure.

represent the uncertain parameters in evidence-based BBA structure as shown in Figure 6. Conventionally, it is common to use traditional probability theory to qualify the uncertain parameter in this situation; thus, the probabilistic representation is also obtained for comparison with evidence theory. It can be seen from Figure 6 that uncertain parameters seem to follow the Gaussian distribution, and its approximate expected value and variance are calculated by statistical technique, also indicated in Figure 6.

Due to uncertainty in the material property and load, this weight minimization problem is now converted to a bi-objective problem to obtain a robust design as explained in section "Evidence-based multi-objective design optimization," where minimizing the plausibility of failure is considered as an additional objective; thus, the corresponding bi-objective design based on evidence theory can be formulated as follows

$$\begin{aligned}
&\text{find} \quad \boldsymbol{x} = \{A_1, A_2, \ldots, A_{10}, x_1, y_1, x_3, y_3\} \\
&\min \quad f = \sum_{i=1}^{10} \rho A_i L_i \\
&\min \mathrm{Pl}_{\max} \\
&\mathrm{Pl}_{\max} = \max(\mathrm{Pl}[g_{\sigma,d}(\boldsymbol{x},\boldsymbol{u}) < 0]) \\
&g_\sigma(\boldsymbol{x},\boldsymbol{u}) = 172.25 - \max|\sigma_k(\boldsymbol{x},\boldsymbol{u})| \quad k = 1, 2, \ldots, 10 \\
&g_d(\boldsymbol{x},\boldsymbol{u}) = 50.8 - \max\left|d_{jl}(\boldsymbol{x},\boldsymbol{u})\right| \quad j = 1, 2, \ldots, 6;\ l = 1, 2, 3 \\
&\boldsymbol{u} = \{E, P_1, P_2\}
\end{aligned} \tag{12}$$

where $\boldsymbol{x}$ and $\boldsymbol{u}$ are vector of design variables and uncertain parameters, respectively; g_σ and g_d are the limit states with respect to stress and displacement failure modes, respectively; L_i is the length of the ith member; d_{jl} is the displacement of the jth joint at the l-direction.

The above problem is solved using DEMO with a population size of 140 for 500 generations, and this program is executed on an Intel(R) Core (TM) i7-4770 CPU @3.40 GHz processor system. DE strategy combined with parallel technique is implemented. According to the number of CPU core, six available workers are constructed to proceed the parallel computing in the calculation for the failure plausibility. For comparison purposes, we also employ Monte Carlo simulation (5000 sample points) to achieve the RBDO with approximate PDFs for uncertainty parameters as shown in Figure 7. The feasible Pareto fronts of two methods are presented in Figure 8, and four optimal geometry results of the truss selected from the evidence-based feasible Pareto-optimal set are plotted in Figure 9.

It should be noted in Figure 8 that the optimal front result obtained by evidence theory is a stairs-type Pareto front which is discontinuous and jumped due to the discrete nature of the BBA structure. On the contrary, the probabilistic RBDO result is continuous and smooth. In addition, an overwhelming majority of optimal points obtained by evidence theory locate in the right of probability results (Figure 8). It is obvious that EBDO method sacrifices its weight for avoiding the wrong optimum results by the situation that the real probability distribution is always unknown in practical engineering.

In order to verify the bi-objective optimal results, a single-objective optimal design subject to constraint $\mathrm{Pl}_{\max} < \mathrm{P}_{\lim} = 0.05$ is considered here. The corresponding bi-objective optimal results are selected from the feasible Pareto-optimal set, which is also marked using dot line in the inset plot in Figure 8. Table 1 summarizes the EBDO results and compares them with deterministic results given by Soh and Yang (2000) and RBDO results. As shown in Table 1, both weight and $\mathrm{Pl}_{\max}$ of bi-objective optimal design are very close to that of single-objective optimal design. It is shown that the single-objective is a special case of bi-objective design.

Additionally, the bi-objective of evidence-based optimum obtains the less weight of 1008 lb than single-objective design of 1018 lb under the same failure plausibility of 0.044; it can be seen that the bi-objective not only achieves better performance in optimization design under uncertainty, but also provides many optimal designs for designer to choose. Furthermore, comparing the evidence-based bi-objective design with the RBDO, the optimal weight of bi-objective EBDO is more heavier than RBDO result; however, this redundant weight resulted from increasing the sizing variable and reassigning the shape variable will gain more reliable to guarantee the safety of the optimal structure under epistemic uncertainty and probability assumption error. That is to say, EBDO method is more robust compared to RBDO. The deterministic result is an invalid solution in this uncertainty situation.

In order to verify the effective of the parallel technique, Table 1 also lists the computational times of the different design optimization algorithms. The proposed evidence-based bi-objective optimization design

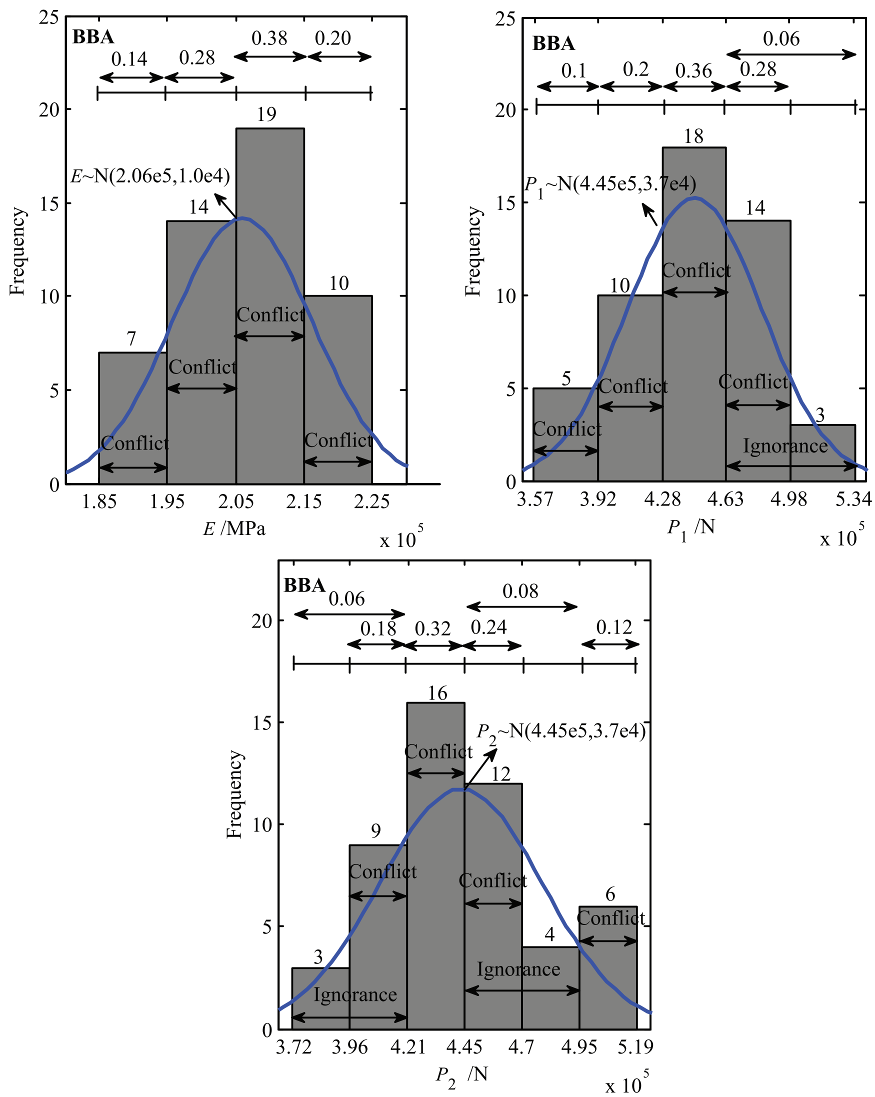

Figure 7. Uncertain quantification of E, P_1, and P_2.

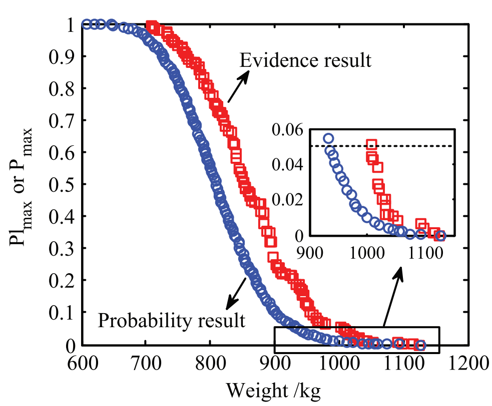

Figure 8. Comparison of the feasible Pareto front results obtained by evidence theory and probability theory.

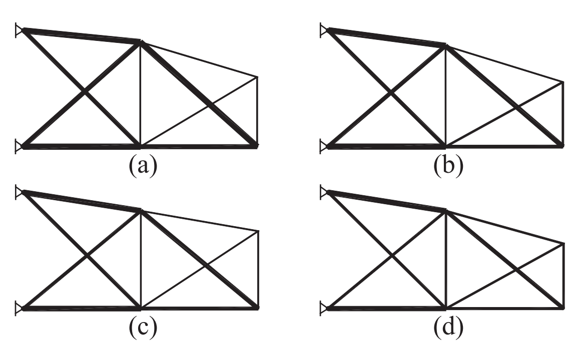

Figure 9. Geometry of 10-bar truss obtained in the feasible Pareto-optimal set: (a) Pl_{max} = 0.044, (b) Pl_{max} = 0.224, (c) Pl_{max} = 0.62, and (d) Pl_{max} = 0.92.

Table 1. Comparison results of 10-bar truss.

Variable	Evidence-based optimum		Reliable optimum		Deterministic optimum
	Bi-objective	Single-objective	Bi-objective	Single-objective	
A_1 (mm^2)	6446.4	6397.3	6445.5	6416.4	5935
A_2 (mm^2)	78.1	327.6	75.7	62.4	58.1
A_3 (mm^2)	6437.9	6410.6	6383.2	6450.0	5445
A_4 (mm^2)	5064.3	4728.1	4166.6	4707.2	3026
A_5 (mm^2)	62.5	251.7	64.8	124.2	48.4
A_6 (mm^2)	47.3	323.8	78.0	58.0	116.1
A_7 (mm^2)	4365.0	4105.7	4166.3	4041.1	3710
A_8 (mm^2)	5032.2	5722.2	4306.1	3770.3	4097
A_9 (mm^2)	6438.4	6145.8	6120.0	6448.6	4923
A_{10} (mm^2)	60.2	179.3	61.3	71.3	129
y_1 (mm)	5095.7	5205.8	5303.3	5112.3	7633
y_3 (mm)	8275.6	8188.7	8081.5	7803.9	8466
Weight (kg)	1008.8	1018.0	938.9	938.4	801.8
Computing time (s)	16,120 (parallel) 57,020 (non-parallel)	21,500	51,836	41,282	–
$Pl(g_\sigma<0)$	0.044	0.044	0.043	0.04	–
$Pl(g_d<0)$	0.042	0.042	0.049	0.049	–

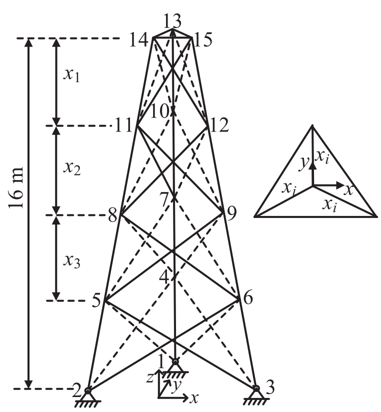

Figure 10. 39-bar tower structure.

takes 8060 s, compared with non-parallel of 57,020 s, about 3.5 times of computational time is speeded up by parallel technique. Moreover, the proposed method also consumes less time than single-objective using serial method and RBDO evaluation. So, the proposed parallel technique can alleviate the computational difficulty of multi-objective EBDO significantly and then make EBDO promising to be applied in real-world design.

39-bar truss tower

The weight minimization of 39-bar triangular tower, shown in Figure 10, was previously analyzed using various deterministic optimization methods (Canyurt and Hajela, 2005; Thierauf and Cai, 1997; Wang et al., 2002). The detail information is listed as follows: the Young's modulus E is 2.1×10^6 MPa, material density ρ is 7800 kg/m^3. The load consists of three horizontal forces, $P_1 = P_2 = P_3 = 25$ kN imposed on nodes 13, 14, and 15 in the positive y-direction.

Table 2. Sizing variables of 39-bar tower.

Sizing variables	Elements defined
A_1	(1,4), (2,5), (3,6)
A_2	(4,7), (5,8), (6,9)
A_3	(7,10), (8,11), (9,12)
A_4	(10,13), (11,14), (12,15)
A_5	Rest of the element

There are six geometric variables: x_1, x_2, and x_3 represent the z-coordinates of tower. x_4 is the distance between the centroid and three nodes 4, 5, and 6 of equilateral triangle defined in x-y plane. x_5 represents the distance between the centroid and three nodes 7, 8, and 9 of equilateral triangle defined in x-y plane, and x_6 is defined by the triangle with the nodes 10, 11, and 12. Moreover, there are five sizing variables shown in Table 2. A more complete description may be found in Canyurt and Hajela (2005).

In this problem, three loads P_1, P_2, and P_3 and Young's modulus E are considered uncertain. Similar to the previous example, only 50 sample points are assumed to be provided. Figure 11 shows the mathematical framework of evidence theory for uncertain parameters E, P_1, P_2, and P_3 by means of intervals with associated BBA values, the approximate probability distribution estimated by statistical technique just based on limited data points is also presented in Figure 11.

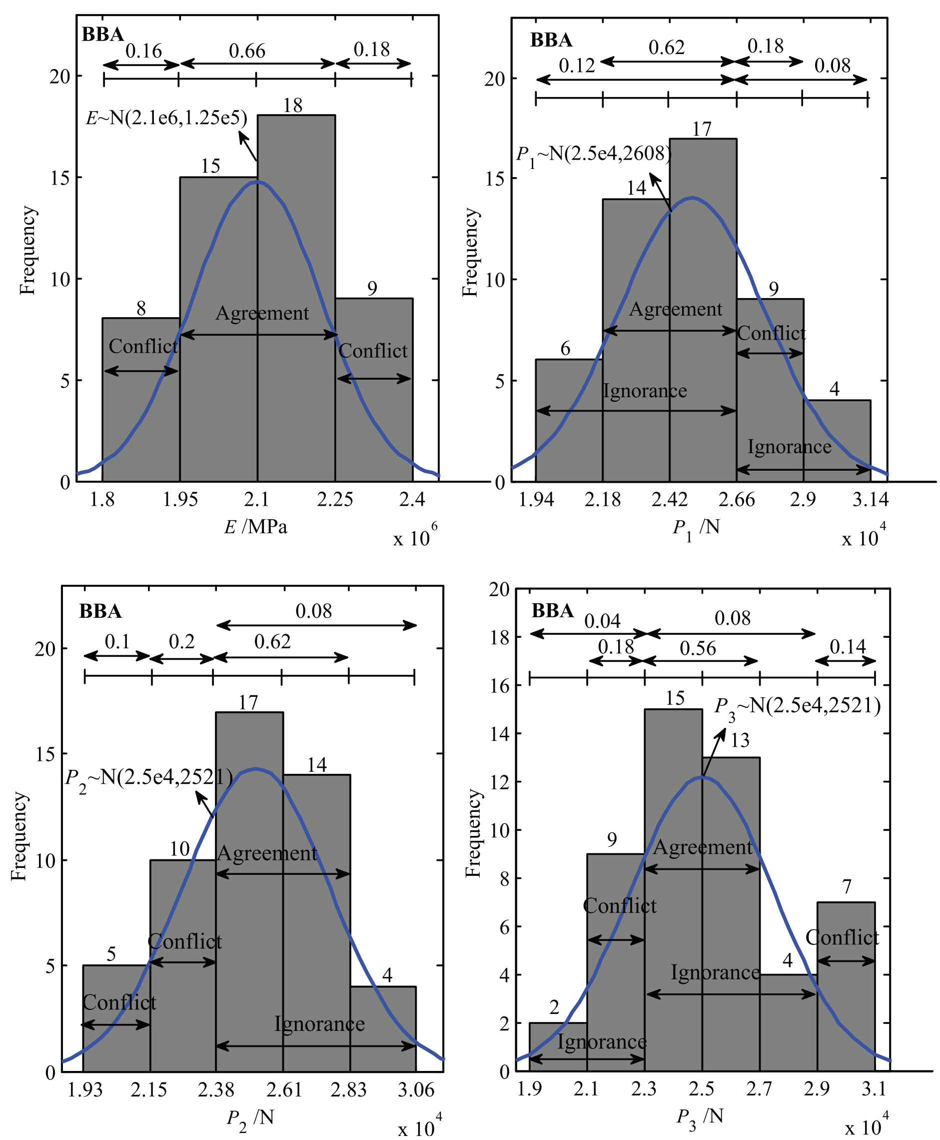

Figure 11. Uncertain quantification of E, P_1, P_2, and P_3.

As a result, the original problem is converted to the bi-objective problem as before, where minimizing the plausibility of failure is treated as the second objective, the corresponding bi-objective design based on evidence theory can be formulated as follows

$$\begin{aligned}
&\text{find} \quad \boldsymbol{x} = \{x_1, x_2, \ldots, x_6, A_1, A_2, \ldots, A_5\} \\
&\min \quad f = \sum_{i=1}^{39} \rho A_i L_i \\
&\min \mathrm{Pl}_{\max} \\
&\mathrm{Pl}_{\max} = \max(\mathrm{Pl}[g_{\sigma,d}(\boldsymbol{x}, \boldsymbol{u}) < 0]) \\
&\quad g_\sigma(\boldsymbol{x}, \boldsymbol{u}) = 150 - \max|\sigma_k(\boldsymbol{x}, \boldsymbol{u})| \quad k = 1, 2, \ldots, 39 \\
&\quad g_d(\boldsymbol{x}, \boldsymbol{u}) = 3 - \max|d_{jy}(\boldsymbol{x}, \boldsymbol{u})| \quad j = 14, 15 \\
&\quad \boldsymbol{u} = \{E, P_1, P_2, P_3\}
\end{aligned} \tag{13}$$

where g_σ and g_d are the limit states with respect to stress and displacement failure modes. This problem is solved using a population size of 150 for 300 generations of DEMO algorithm, the plausibility calculation is distributed into six workers in parallel computing.

In this example, to facilitate understanding, a step-by-step procedure of implement of bi-objective evidence-based design in 39-bar tower is shown as follows:

Step 1: Generate an initial 150 sized population signified design variables $\boldsymbol{x}$ randomly. And compute the weight objective of truss structure at these points.

Step 2: Evaluate the failure plausibility at each design point. As mentioned before, this step is also divided into three substeps: (1) forming the BBA structure of uncertainty variables $\boldsymbol{u}$ according to the limited 50 data (Section "Uncertainty representation"); (2) minimizing the value of g_o and g_d using the DE strategy combined with parallel technique to overcome the intensive computational difficulty (section "Uncertainty propagation"); (c) calculating the $\mathrm{Pl}(g < 0)$.

Step 3: Proceed the DEMO operation (section "Implementation of the EBDO using DEMO") to obtain the trade-off solution between the weight objective and failure plausibility.

Step 4: Check convergence, if the number of iteration is less than 500, go back to step 1 and generate the new design variables using DE operator, not produce randomly; otherwise, go back to step 5.

Step 5: Output the Pareto-optimal set.

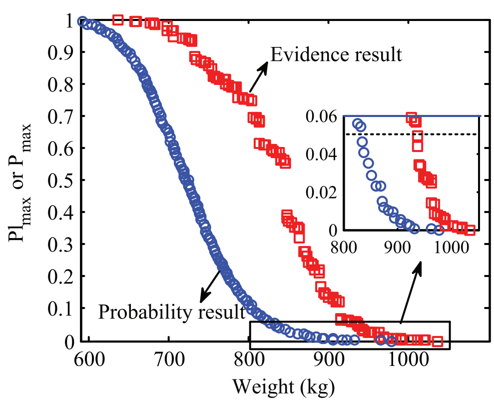

Figure 12. Comparison of the feasible Pareto front results given by probability theory and evidence theory to solve 39-bar truss design.

In addition, to compare with probability theory, we also adopt Monte Carlo simulation to achieve the RBDO with approximate PDFs as given in Figure 11. Feasible Pareto fronts are presented in Figure 12, four geometry results of the truss obtained in the evidence theory-based feasible Pareto-optimal set are plotted in Figure 13. As shown in Figure 12, similar conclusions with the first example are drawn. As expected, the discontinuous and jumped nature in the evidence-based Pareto front is revealed again due to the jumps in the BBA structure of the uncertain inputs. Yet, decision-makers still choose a preferred solution depending on their requirements for safety or economy from a set of trade-off points. Meanwhile, it can be inferred that the evidence theory-based optimization design is more robust than RBDO design.

Table 3 shows the design detail information for a particular bi-objective optimal result and single-objective optimum with the same constraint $\mathrm{Pl}_{\max} < \mathrm{P}_{\lim} = 0.05$ as mentioned in the previous example. As discussed before, DEMO exhibits good performance in terms of finding optimal trade-off solution for two conflict objectives, it also takes less time than other optimization algorithms. And the conservative nature of the evidence theory-based optimization is again observed.

Conclusion

In this article, we addressed a robust multi-objective design optimization problem based on evidence theory to quantify uncertainties inherent in the practical engineering. A DE approach also offers the possibility of parallelization and an algorithmic flexibility which can

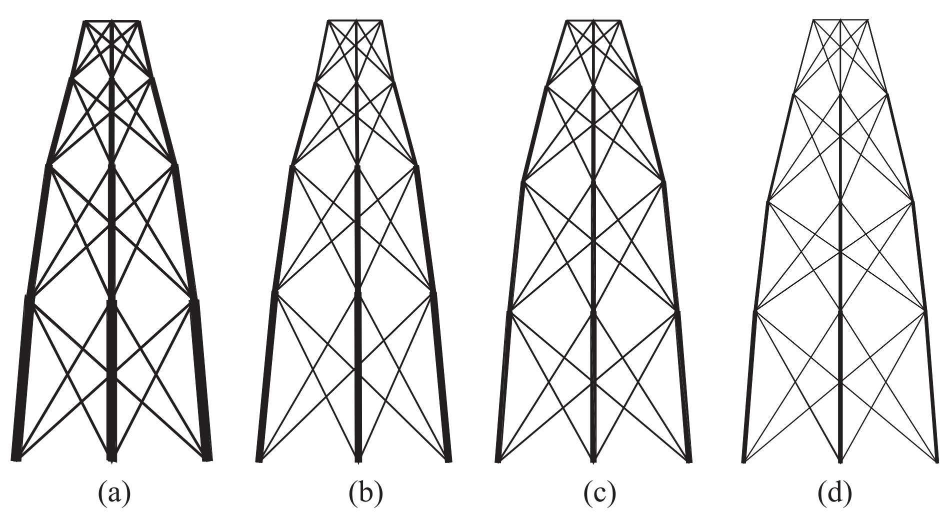

Figure 13. Geometry of 39-bar tower obtained in the feasible Pareto-optimal set: (a) $\mathrm{Pl}_{\max}$ = 0.0489, (b) $\mathrm{Pl}_{\max}$ = 0.3920, (c) $\mathrm{Pl}_{\max}$ = 0.561, and (d) $\mathrm{Pl}_{\max}$ = 0.9971.

Table 3. Comparison results of 39-bar truss.

Variable	Evidence-based optimum		Reliable optimum		Deterministic optimum
	Bi-objective	Single-objective	Bi-objective	Single-objective	
x_1 (mm^2)	2068.5	1988.1	2303.6	1992.6	2067
x_2 (mm)	3151.6	3431.8	3221.8	3459.3	3315
x_3 (mm)	4921.6	4438.1	4417.3	4412.1	4549
x_4 (mm)	3432.2	3338.9	3374.7	3360.1	3329
x_5 (mm)	2621.8	2674.9	2687.9	2709.7	2635
x_6 (mm)	1664.6	1658.5	1797.2	1676.4	1748
x_7 (mm^2)	2204.2	2234.3	1981.2	1976.6	1736
x_8 (mm^2)	1716.9	1640.8	1582.1	1449.1	1342
x_9 (mm^2)	1116.5	1098.2	951.5	961.8	879.5
x_{10} (mm^2)	388.8	361	321.4	315.6	237
x_{11} (mm^2)	275.2	276.3	245.5	249.3	196
Weight (kg)	936.9	932.6	834.5	830.1	708.6
Computing time (s)	58560 (parallel) 222,360 (non-parallel)	195,240	70,741	59,385	–
Pl(g_σ<0)	0.0	0.0	0.0	0.0	–
Pl(g_d<0)	0.0489	0.0493	0.0473	0.0497	–

drastically reduce the computation time required for analysis. As a measurement of evidence theory, plausibility of failure was considered as an additional objective to reflect the reliability. Parallelization technique combined with DE-based global optimization is adopted for reducing the computational cost while keeping the high accuracy level.

Due to the non-differentiability of plausibility of failure, DEMO was applied to solve this evidence-based bi-objective optimization problem. We used this method to balance economy (total weight) and reliability problem commonly in civil engineering and a set of optimal trade-off solutions were supplied for designers to choose. Two problems involving shape and size optimization of 10-bar truss and 39-bar tower provided a realistic example of the epistemic treatment of the design uncertainties (elastic modulus and applied loads), it can be concluded that the evidence theory-based design has the robust nature to avoid unreasonable optimal results compared with probability-based RBDO designs when the available information is incomplete and imprecise. Furthermore, due to the implement of parallel technique, the computational time of evidence-based bi-objective design is enhanced about 3–4 times compared with serial operation and shows that the proposed method is a promising and less-consuming method in real-world design.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This study was supported by the National Natural Science Foundation of China (Grant Nos 51178337 and 50708076) and Basic Research of the State Key Laboratory for Disaster Reduction in Civil Engineering of Tongji University (Grant No. SLDRCE14-B-03).

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