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Parameter identification for structural health monitoring based on Monte Carlo method and likelihood estimate

Songtao Xue[1,2], Bo Wen[3], Rui Huang[3], Liyuan Huang[3], Tadanobu Sato[3], Liyu Xie[1], Hesheng Tang[1] and Chunfeng Wan[3]

Abstract
Structural parameters are the most important factors reflecting structural performance and conditions. As a result, their identification becomes the most essential aspect of the structural assessment and damage identification for the structural health monitoring. In this article, a structural parameter identification method based on Monte Carlo method and likelihood estimate is proposed. With which, parameters such as stiffness and damping are identified and studied. Identification effects subjected to three different conditions with no noise, with Gaussian noise, and with non-Gaussian noise are studied and compared. Considering the existence of damage, damage identification is also realized by the identification of the structural parameters. Both simulations and experiments are conducted to verify the proposed method. Results show that structural parameters, as well as the damages, can be well identified. Moreover, the proposed method is much robust to the noises. The proposed method may be prospective for the application of real structural health monitoring.

Keywords
Parameter identification, damage identification, likelihood estimate, Monte Carlo method, structural health monitoring

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Introduction

From the beginning of getting into use, civil engineering structures face the effect of load, environmental erosion, material aging, accidental bumping, and many other factors, which will ultimately lead to structural damage and destruction.[1–3] Based on this consideration, structure health monitoring (SHM) technologies have been developed in recent decades to estimate the actual values of structural parameters changed due to deterioration or damage of structures in use.[4,5] As the most important part of structural health monitoring, damage identification includes four aspects, namely detecting the existence, the location, the severity of structural damage, and service life of the structures.[6–9] Most damage identification judges the existence and degree of damage by comparing structural parameters' change rules or differences before and after the damage occurs.[10,11] Structural

[1]Research Institute of Structural Engineering and Disaster Reduction, College of Civil Engineering, Tongji University, Shanghai, P.R. China
[2]Department of Architecture, Tohoku Institute of Technology, Sendai, Japan
[3]Key Laboratory of Concrete and Prestressed Concrete Structure of Ministry of Education, Southeast University, Nanjing, P.R. China

Corresponding author:
Chunfeng Wan, Key Laboratory of Concrete and Prestressed Concrete Structure of Ministry of Education, Southeast University, Nanjing 210096, P.R. China.
Email: wan@seu.edu.cn

parameter identification plays an important role in the damage identification of structures.

For parameters identification, it is common to define the vibratory characteristics of a structure with the parameter of mass, damping, and stiffness.[12] Many researchers have made a lot of efforts in the identification of structural parameters with different methods such as Bayesian network, Kalman filter, Likelihood estimation.[13–18] Udwadia et al.[19] put forward that the stiffness and damping of a building story can be uniquely identified with the conditions of the recorded absolute accelerations. Takewaki and Nakamura[20] taken advantage of the limited earthquake records and the knowledge of the floor masses to identify the stiffness and damping of building structures. D'Amore et al.[21] investigated the capability of input–output system identification methods, using Kalman filter to identify dynamic characteristics of typical office building structures subjected to strong ground motion. There is no doubt, identifying the physical parameters of structures is the most direct and efficient way for the health monitoring of structures.

In this study, a structural parameter identification method based on Monte Carlo Method and likelihood estimate is proposed. Identification effect and its robustness to different types and levels of noises will be studied in detail. Structural damage identification will also be studied with the proposed method.

Basic principal of parameter identification

Monte Carlo method, also called statistical experimental method, is based on the probability definition that the probability of an event can be estimated from the frequency of occurrence of the event in a large number of trials. In order to identify the structural parameters such as stiffness k and damping c, we could randomly pick n groups of stiffness k_i and damping c_i in their distribution ranges, then compare the calculated responses with the observed responses. The structure model represented by certain parameters will be regarded as the identified one if the calculated responses under such parameters can match the best with the observed responses which are from the actual target structure. At the meantime, such parameters are regarded as the identified ones of the target structure. This structural parameter identification method is based on Monte Carlo method and the accurate probability will be larger when n tends to increase.

To compare the observed response and calculated response, we here apply the concept of maximum likelihood value. Through the likelihood function in statistics shown in equations (1)–(3), the likelihood value of each sample can be achieved. The sample with the maximum likelihood can be identified as well and we assume this sample to be the predicted sample. This also means that the difference between observed and calculated response reaches the minimum when the likelihood value reaches the maximum. Combining Monte Carlo method and maximum likelihood value, stiffness k_i and damping c_i nearest to the actual value of stiffness k_0 and the actual value of damping c_0, therefore, can be identified as the true values.

$$L(x_i(t)) = \max \sum_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x_i(t) - x_0(t))^2}{2\sigma^2}\right] \quad (1)$$

$$L(\dot{x}_i(t)) = \max \sum_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(\dot{x}_i(t) - \dot{x}_0(t))^2}{2\sigma^2}\right] \quad (2)$$

$$L(\ddot{x}_i(t)) = \max \sum_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(\ddot{x}_i(t) - \ddot{x}_0(t))^2}{2\sigma^2}\right] \quad (3)$$

Based on such consideration, we proposed to realize the parameter identification with the Monte Carlo method and likelihood estimates. For an unknown target structure, proper parameters such as stiffness and damping will be selected for identification. Then proper sampling range is also needed to be determined. After sampling, dynamic responses under the same excitation of the target structure will be calculated under each sample case. Likelihood will be calculated between the calculated responses and the observed ones from the target structure. Finally, maximum likelihood will be selected and corresponding parameters (samples) become the identified parameters. The flowchart of the mechanism is shown in Figure 1.

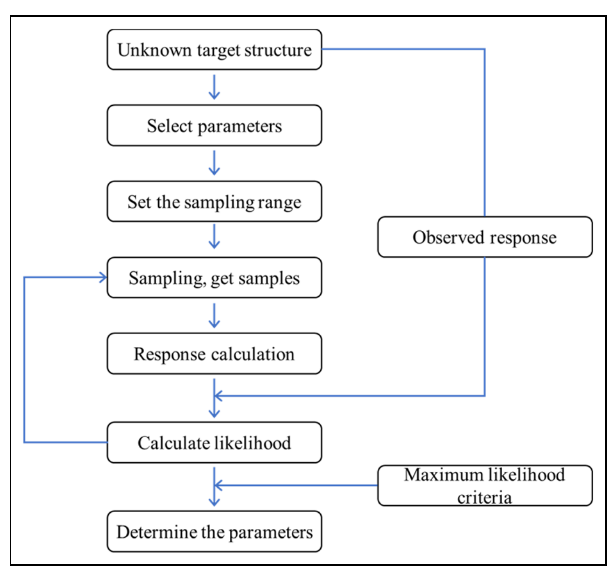

Figure 1. Flowchart of the parameter identification approach.

Numerical simulation of structural parameter identification

Single- and multiple-degree-of-freedom structure model

As the most basic and simplest system, single-degree-of-freedom (SDOF) linear system is the basis of multi-degree system, continuous system, and nonlinear system in vibration analysis. Many practical problems can be simplified to SDOF linear system to deal with. A SDOF linear structure is shown in Figure 2.

The SDOF structure in Figure 2 is adopted with some basic assumptions that the foundation is rigid and the structure is elastic, so the horizontal motion of the whole structure is consistent. $\ddot{x}_g(t)$ is the horizontal ground acceleration of Kobe earthquake.

The initial parameters of the structure in Figure 2 are assumed as mass $m_0 = 400$ kg, stiffness $k_0 = 2500$ N/m, damping ratio $\xi = 0.05$. The damping coefficient is supposed to be $c_0 = 2\sqrt{m_0 k_0}\xi = 100\,\mathrm{N\,s/m}$.

At the beginning, the displacement and velocity of the structure are both zero. Using $\ddot{x}_g(t)$ as the input acceleration at the base and its time-history curve is illustrated in Figure 3.

However, for many practical structures, it is not enough to be simplified in SDOF system for analysis. They are usually described in multi-degree-of-freedom (MDOF) structures. In this paper, multi-degree structures are considered as the following model in Figure 4.

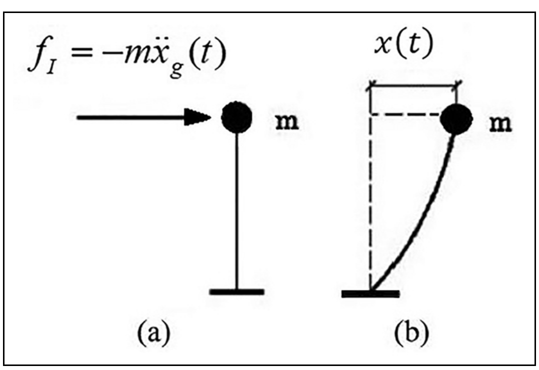

Figure 2. Lumped-mass single-degree-of-freedom structure under translational motion: (a) structure is subjected to the earthquake motion and has the inertia force at the mass point and (b) structure vibrates and has displacement at mass point.

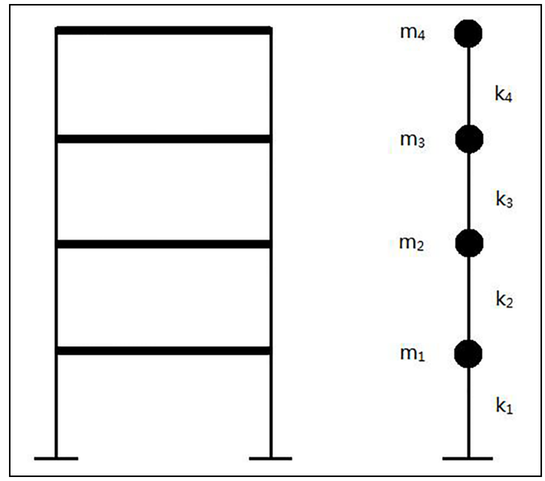

Figure 4. Multiple-degree-of-freedom structure model.

Structural parameter identification without noise

As an example, n groups of stiffness k_i and damping c_i are randomly taken according to equations (4) and (5), where $rand_i$ is a function to produce a uniform distribution of pseudo random integers. When $n = 1000$, structural parameters' (stiffness k_i and damping c_i) distribution is shown in Figure 5

$$k_i \sim rand_i(n, [1500, 3000]) \qquad i = 1, 2, \ldots, n \tag{4}$$

$$c_i \sim rand_i(n, [20, 120]) \qquad i = 1, 2, \ldots, n \tag{5}$$

During the earthquake, horizontal ground motion will cause the vibration of the upper structure, we can get n groups of time–history curve of displacement response $x_i(t)$, velocity response $\dot{x}_i(t)$, and acceleration response $\ddot{x}_i(t)$ from n groups of different k_i and c_i. The time step is taken as 0.02 s

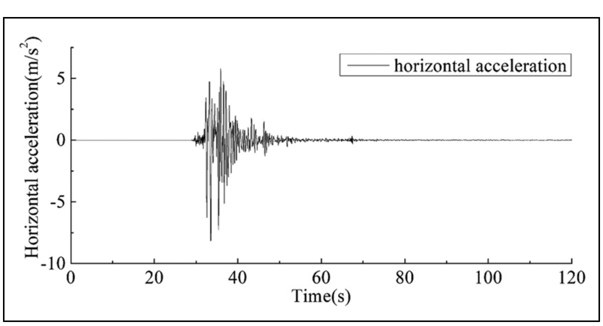

Figure 3. Time-history curve of Kobe earthquake's horizontal acceleration.

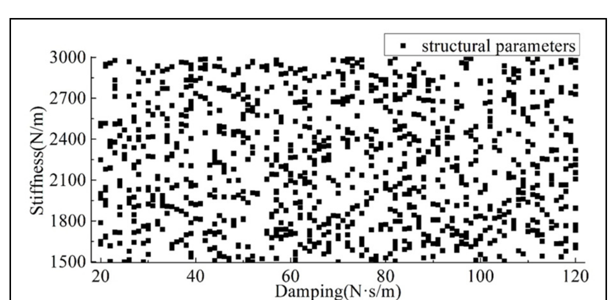

Figure 5. Structural parameters' distribution of initial predicted sample points.

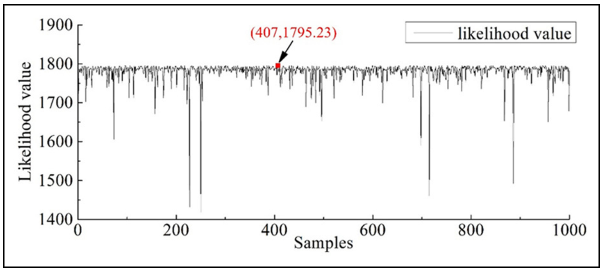

Figure 6. Likelihood value of samples.

$$m\ddot{x}(t) + c_i\dot{x}(t) + k_i x(t) = -m\ddot{x}_g(t) \quad (6)$$

Based on the actual parameters k_0 and c_0, time–history curves of displacement response $x_0(t)$, velocity response $\dot{x}_0(t)$, and acceleration response $\ddot{x}_0(t)$ can also be achieved through equation (6) and considered as the observed value. Through the maximum likelihood function in statistics, likelihood value of each sample point can be calculated in the time interval and the sample with the maximum likelihood value tends to be the predicted sample. This also means that the k_i and c_i nearest to k_0 and c_0 can be identified.

The likelihood value of 1000 samples can be calculated separately on the basis of displacement, velocity, and acceleration response. The calculated likelihood values based on the acceleration response are shown in Figure 6.

In Figure 6, the marked point is supposed to be the point with the maximum likelihood value. The horizontal and vertical ordinates respectively indicate the sample number and the maximum likelihood value.

The results of noiseless SDOF structural parameters (stiffness k and damping c) based on maximum likelihood estimation are shown in Table 1. According to the results, dynamic time-history curves of the identified parameters (stiffness k_s and damping c_s) and true values (stiffness k_0 and damping c_0) can be drawn.

More samples are chosen. For example, $n = 2000$, final results of noiseless single-degree parameters (stiffness k and damping c) based on maximum likelihood estimation are shown in Table 2.

Comparing Tables 1 and 2, it is found that the more samples we generate, the higher accuracy this identification method would have but the longer time it would take.

Table 1. Parameter identification results of the noiseless SDOF structure (n = 1000).

Parameters	True values	Identification results	Identification error (%)
k(N/m)	2500	2494	0.2
c(N s/m)	100	102	2.0

SDOF: single-degree-of-freedom.

Table 2. Parameter identification results of the noiseless SDOF structure (n = 2000).

Parameters	True values	Identification results	Identification error (%)
k(N/m)	2500	2497	0.1
c(N s/m)	100	102	2.0

SDOF: single-degree-of-freedom.

For multi-degree structures, the real parameters are assumed as follows

$$\text{Mass matrix} : [M] = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & m_3 & \\ & & & m_4 \end{bmatrix} = \begin{bmatrix} 400 & & & \\ & 400 & & \\ & & 400 & \\ & & & 400 \end{bmatrix} \text{kg}$$

$$\text{Stiffness matrix} : [k] = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} = \begin{bmatrix} 5000 & -2500 & 0 & 0 \\ -2500 & 5000 & -2500 & 0 \\ 0 & -2500 & 5000 & -2500 \\ 0 & 0 & -2500 & 2500 \end{bmatrix} \text{N/m}$$

$$\text{Damping matrix} : [C] = \begin{bmatrix} 200 & -100 & 0 & 0 \\ -100 & 200 & -100 & 0 \\ 0 & -100 & 200 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix} \text{N s/m}$$

$n = 4^8$ samples are chosen and the final results are shown in Table 3.

For the multi-degree structures, the identification error tends to become larger than that of the single-degree structures. This may due to the complexity of the multi-degree structures.

Structural parameter identification with Gaussian noise

Gaussian noise is one kind of common noises with the probability density function corresponding to Gaussian distribution. Noise with different levels was added to

Table 3. Parameter identification results of the noiseless MDOF structure (n = 4^8).

Parameters		True values	Identification results	Identification error (%)
k(N/m)	k_1	2500	2608	4.3
	k_2	2500	2419	3.2
	k_3	2500	2380	4.8
	k_4	2500	2417	3.3
c(N s/m)	c_1	100	104.2	4.2
	c_2	100	96.5	3.5
	c_3	100	95.9	5.9
	c_4	100	93.8	6.2

MDOF: multi-degree-of-freedom.

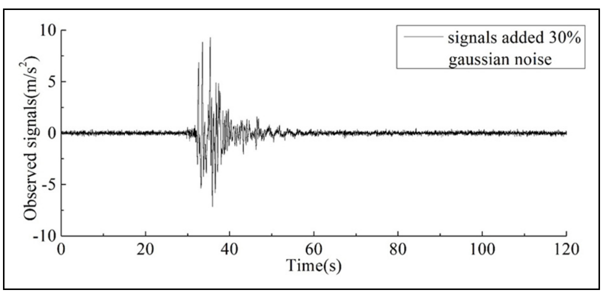

Figure 7. Observed acceleration signals with 30% Gaussian noise.

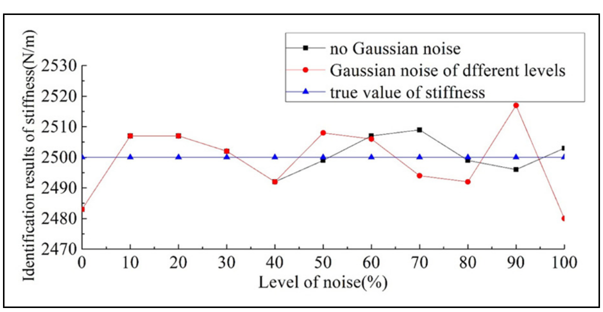

Figure 8. Identification results of stiffness under Gaussian noise of different levels.

the true values as the observed signal to study their effect. The observed acceleration signals with 30% Gaussian noise are illustrated in Figure 7.

The structural parameters of stiffness k and damping c under different levels of Gaussian noise are identified and compared. The comparison results are illustrated in Figures 8 and 9.

From Figures 8 and 9, there is no obvious rule for the identification of stiffness and damping. This is mainly because that all the samples are generated randomly while the identification results are influenced by

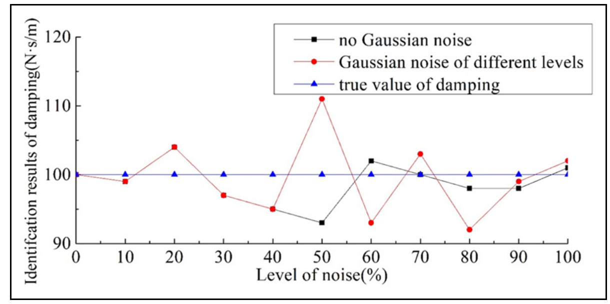

Figure 9. Identification results of damping under Gaussian noise of different levels.

Table 4. Parameter identification results of the MDOF structure under Gaussian noise (n = 4^8).

Parameters to be identified		True values	Identification results	Identification error (%)
k(N/m)	k_1	2500	2382	4.7
	k_2	2500	2520	0.8
	k_3	2500	2544	1.8
	k_4	2500	2412	3.5
c(N s/m)	c_1	100	104.3	4.3
	c_2	100	97.2	2.8
	c_3	100	101.3	1.3
	c_4	100	101.5	1.5

MDOF: multi-degree-of-freedom.

samples. However, on the whole, it is shown that this method is not so sensitive to the Gaussian noise, even when the noise level becomes fairly large, the parameters still can be identified without large error. For multi-degree structures, we choose $n = 4^8$ samples and the final results are shown in Table 4.

Structural parameter identification with non-Gaussian noise

Non-Gaussian noise can be generated from equation (7)

$$x_{n\_2}(i) = p_d \cdot \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} \left[\ddot{x}_0(i) - \frac{\sum_{i=1}^{n} \ddot{x}_0}{n} \right]^2} \cdot r(i) \qquad (7)$$

$$i = 1, 2, \ldots, n$$

In this equation, $r(i)$ obeys t distribution of which mean value is 0 and freedom is 3. With the time step of 0.02 s, the time-history curve of $r(i)$ can be illustrated in Figure 10. $x_{n\_2}(i)$ can be calculated from $r(i)$ and its time-history curve can also be drawn in Figure 11.

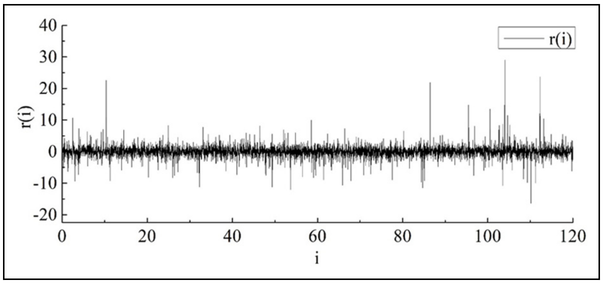

Figure 10. Time-history curve of $r(i)$.

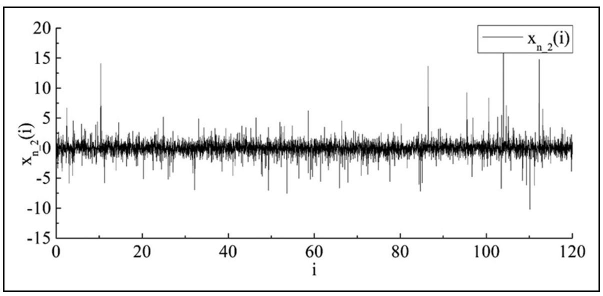

Figure 11. Time-history curve of non-Gaussian noise.

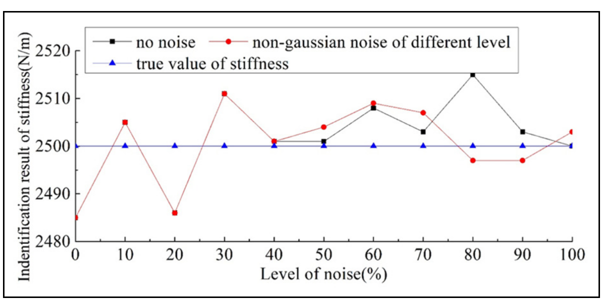

Figure 12. Identification results of stiffness under non-Gaussian noise of different levels.

The structural parameter identification with non-Gaussian noise of different levels was carried out in similar to the parameter identification with Gaussian noise, and the identification results are described in Figures 12 and 13.

It can be seen from Figures 12 and 13 that the changes of stiffness and damping have no obvious rules. For multi-degree structure, we choose $n = 4^8$ samples and the final results are shown in Table 5.

Analysis of structural parameter identification after damage happens

Considering the existence of structural damage, the true value of stiffness changes from $k_i = 2500$ N/m to $k_i = 2000$ N/m and damping from $c_i = 100$ N s/m to

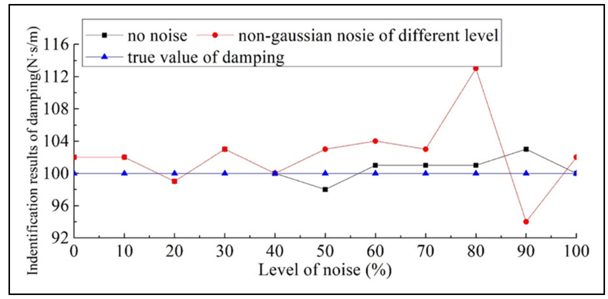

Figure 13. Identification results of damping under non-Gaussian noise of different levels.

Table 5. Parameter identification results of the MDOF structure under non-Gaussian noise ($n = 4^8$).

Parameters to be identified		True values	Identification results	Identification error (%)
k(N/m)	k_1	2500	2536	1.4
	k_2	2500	2520	0.8
	k_3	2500	2503	0.1
	k_4	2500	2429	2.8
c(N s/m)	c_1	100	104.6	4.6
	c_2	100	103.9	3.9
	c_3	100	99.2	0.8
	c_4	100	101.2	1.2

MDOF: multi-degree-of-freedom.

$c_i = 150$ N s/m. This is to say, in the time interval of [25 s, 35 s], $k_i = 2500$ N/m, $c_i = 100$ N s/m and in the time interval of [35 s, 45 s], $k_i = 2000$ N/m, $c_i = 150$ N s/m. For single-degree structures, parameter identification results with no noise are shown in Table 6.

For multi-degree structures, when $n = 4^8$, parameter identification results with no noise are shown in Tables 7 and 8.

Experimental verification

To verify the correctness of the proposed method in structural parameter identification, as well as the damage identification, experiments are conducted. An aluminum frame with four layers is adopted in the experiments, as shown in Figure 14. The size of each floor slab is 300 mm × 200 mm × 12 mm; the mass of each floor is $m_{1st} = m_{2nd} = m_{3rd} = 2.342$ kg. At each floor, there are four columns. In the experiment, damages are simulated by replacing a normal column with a thin column or totally removing that column. For a normal column, its size is 235 mm × 40 mm × 1.5 mm, while the size of a damaged column is 235 mm × 30 mm × 1.5 mm, which actually has 6.25% of stiffness decrease compared to the intact one.

Table 6. Damage identification results of the SDOF structure after damage happens.

Time interval [25 s, 35 s]			Time interval [35 s, 45 s]		
True values	Identification results	Identification error (%)	True values	Identification results	Identification error (%)
k = 2500 N/m	k = 2612 N/m	4.5	k = 2000 N/m	k = 1924 N/m	3.8
c = 100 N s/m	c = 102.3 N s/m	2.3	c = 150 N s/m	c = 167.4 N s/m	1.2

SDOF: single-degree-of-freedom.

Table 7. Stiffness identification results of the MDOF structure after damage happens (n = 4^8).

Time interval [25 s, 35 s]			Time interval [35 s, 45 s]		
True values (N/m)	Stiffness identification results (N/m)	Stiffness identification error (%)	True values (N/m)	Stiffness identification results (N/m)	Stiffness identification error (%)
k_1 = 2500	k_1 = 2562	2.5	k_1 = 2000	k_1 = 1992	0.4
k_2 = 2500	k_2 = 2593	3.7	k_2 = 2000	k_2 = 2049	2.5
k_3 = 2500	k_3 = 2541	1.6	k_3 = 2000	k_3 = 1913	4.4
k_4 = 2500	k_4 = 2466	1.4	k_4 = 2000	k_4 = 2047	2.4

MDOF: multi-degree-of-freedom.

Table 8. Damping identification results of the MDOF structure after damage happens (n = 4^8).

Time interval [25 s, 35 s]			Time interval [35 s, 45 s]		
True values (N s/m)	Damping identification results (N s/m)	Damping identification error (%)	True values (N s/m)	Damping identification results (N s/m)	Damping identification error (%)
c_1 = 100	c_1 = 106.2	6.3	c_1 = 150	c_1 = 154.9	3.3
c_2 = 100	c_2 = 103.9	3.9	c_2 = 150	c_2 = 147.3	1.8
c_3 = 100	c_3 = 101.1	1.1	c_3 = 150	c_3 = 145.3	3.1
c_4 = 100	c_4 = 100.9	0.9	c_4 = 150	c_4 = 153.6	2.4

MDOF: multi-degree-of-freedom.

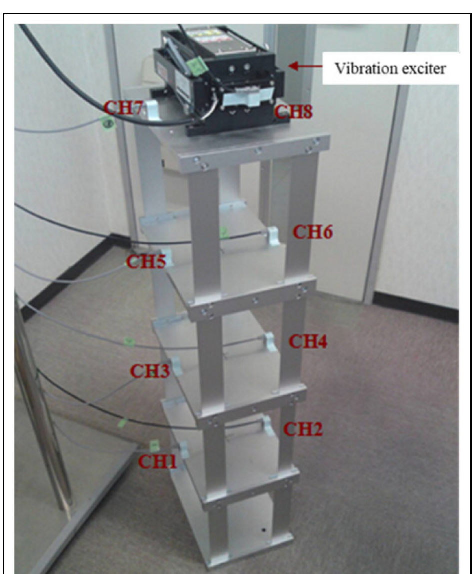

Figure 14. Test models and the distribution of acceleration sensors.

In the experiment, for the convenience, the excitation is using a vibration exciter to produce vertical force at the top, instead of putting the earthquake motions at the ground. The vibration exciter is installed firmly at the top layer of the frame. The total mass of the vibration exciter is 6.0208 kg, including the moving part of 0.6 kg. At the lower three floors, there are two accelerometers deployed at each floor slab. While at the top floor, one accelerometer is installed at the floor slab to get the floor acceleration and the other accelerometer is installed on the moving part of the vibration exciter, from which the exciting force at the top can be obtained. The acceleration at the moving part in the experiment can be shown in Figure 15.

In the experiment, totally five structural damage cases are studied, including one intact case and four damaged cases with damage occurring at the first or second stories, as shown in Table 9.

Experimental results considering different damage cases are shown in Table 9. Through the comparison

Table 9. Experimental results in different damage cases.

Test number	Damage case	Story stiffness	Real value of stiffness (N/m)	Identification results (N/m)	Identification error (%)
1	No damage	k_1	2871	2796	2.6
		k_2	2871	2813	2.0
		k_3	2871	2714	5.4
		k_4	2692	2647	1.7
2	One column in the first floor is removed (damage of 25% in the first floor)	k_1	2153	2054	4.6
		k_2	2871	2806	2.3
		k_3	2871	2761	3.8
		k_4	2692	2659	1.2
3	One column in the first floor replaced (damage of 6.25% in the first floor)	k_1	2692	2741	1.8
		k_2	2871	2798	2.5
		k_3	2871	2809	2.2
		k_4	2692	2612	3.0
4	One column in the second floor is removed (damage of 25% in the second floor)	k_1	2871	2963	6.7
		k_2	2153	2074	3.7
		k_3	2871	2716	5.4
		k_4	2692	2598	3.5
5	One column in the second floor is replaced (damage of 6.25% in the second floor)	k_1	2871	2811	2.1
		k_2	2692	2635	2.1
		k_3	2871	2934	2.2
		k_4	2692	2750	2.2

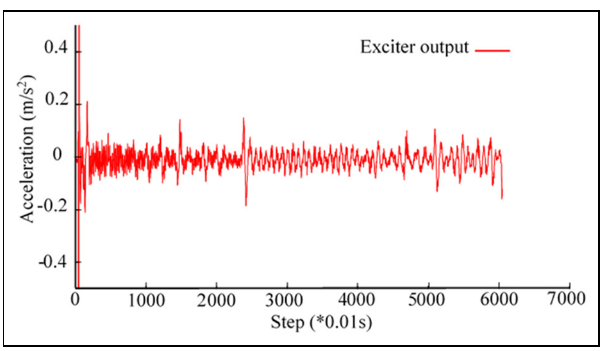

Figure 15. Acceleration at the moving part of the vibration exciter.

between the identified stiffness values and the real ones under different damage cases, the proposed method is proved to be very accurate and effective. Damages can be well detected and well quantified, which is very useful for the structural health monitoring.

Conclusion

Parameter identification of the SDOF and MDOF structures under no noise, Gaussian noise, and non-Gaussian noise of different levels are studied and discussed. Through the comparison, it can be found that the proposed method can help to identify the structural parameters. With the increase of sample numbers, the error of structural parameter identification decreases. The structural parameter identification based on Monte Carlo method and likelihood estimation shows favorable anti-noise performance. Unless the level of noise reaches a certain degree, the identification results seem to be robust to the noise.

Considering the existence of structural damage, the identification results show the good consistency with the present actual status of the structure. Through the comparison with the true values of parameters in the experiments, the stiffness identification results of each floor in the aluminum frame under different damage cases show the good performance and effectiveness of the proposed method in structural damage detection and evaluation. The proposed method is found to be effective in identifying the parameters such as stiffness and damping of structures, with high accuracy and stable performance, which could be well extended and applied in the identification of actual damaged structures.

Declaration of conflicting interests

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ORCID iD

Bo Wen https://orcid.org/0000-0002-5550-9774

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